Generalized Two Dimensional Half Canonical Cosine Transform

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Abstract: This paper is devoted for the analytic study of two dimensional generalized half canonical cosine transform and some properties of two dimensional half canonical cosine transform.

Keywords: 2-D cosine-cosine transform, 2-D Canonical transforms.

Introduction: Integral transforms had provided a well establish and valuable method for solving problems in several areas of both Physics and Applied Mathematics. This method proved to be of great importance, in the initial and final value problems for partial differential equations. Due to wide spread applicability of this method for partial differential equations involving distributional boundary conditions, many of the integral transforms are extended to generalized functions.

In the past decade, FRFT has attracted much attention of the signal processing community. As the generalization of FT, the relevant theory has been developed including uncertainty principle, sampling theory, convolution theorem. However, FRFT can be further generalized to obtain the linear canonical transform LCT [3]. In fact LCT is not only the generalization of the FRFT, but also the generalization of the many other integral transforms, like Fresnel transform, Chirp transform etc. Later on numbers of integral transforms are extended in its fractional domain. For examples Almeida [2] had studied fractional Fourier transform, Akay [1] developed fractional Mellin transform, A. S. Gudadhe, A.V. Joshi [4], On Generalized Half Canonical Cosine Transform.

This paper emphasizes defining two dimensional half canonical cosine transform, and deriving its inversion theorem, then some properties of the two dimensional half canonical cosine transform are discussed and finally conclusions are given.

1. Testing Function Space E:

An infinitely differentiable complex valued function ϕ on R^n belongs to E(R^n), if for each compact set, $I \subset S_\alpha$ where $S_\alpha = \{t : t \in R^n, |t| \le \alpha, \alpha > 0\}$ and for $k \in R^n$,

$$\gamma_{E,k}\,\phi(t) = \sup_{t \in I} \left|D^k \phi(t)\right| < \infty.$$

Note that space E is complete and a Frechet space, let E' denotes the dual space of E.

2. Two Dimensional Half Canonical Cosine Transform:

2.1 Definition:

The two dimensional generalized Half Canonical Cosine Transform $f \in E'(R^n)$ can be defined by, {HCCT $f(t, z)$} (s, u) = < $f(t)$, $K_{HC1}(t, s)$ $K_{HC2}(z, u)$ > where,

$$K_{HC_1}(t,s) = \sqrt{\frac{2}{\pi .ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \cdot e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \cos\left(\frac{s}{b}t\right) \quad , \quad when\ b \neq 0 \ \ and$$

$$= \sqrt{d}.e^{\frac{i}{2}\left(cds^2\right)} \delta(t-ds) \quad , \quad when\ b = 0$$

$$K_{HC_2}(z,u) = \sqrt{\frac{2}{\pi .ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)u^2} \cdot e^{\frac{i}{2}\left(\frac{a}{b}\right)z^2} \cos\left(\frac{s}{b}z\right) \quad , \quad when\ b \neq 0 \ \ and$$

$$= \sqrt{d}.e^{\frac{i}{2}\left(cdu^2\right)} \delta(z-du) \quad , \quad when\ b = 0$$

Hence the two dimensional generalized half canonical cosine transform of $f \in E'(R^n)$ can be defined by,

$$\{\ HCCTf(t,z)\ \}(s,u) = \sqrt{\frac{2}{\pi ib}} \cdot \sqrt{\frac{2}{\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} e^{\frac{i}{2}\left(\frac{d}{b}\right)u^2} \int_0^\infty e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{\frac{i}{2}\left(\frac{a}{b}\right)z^2} \cos\left(\frac{s}{b}t\right) \cdot \cos\left(\frac{u}{b}z\right) \cdot f(t,z)\, dtdz$$

The two dimensional generalized half canonical cosine transform.

2.1.1 Inversion theorem for two dimensional Half Canonical Cosine Transform:

If $\{HCCT\ \ f(t,z)\}(s,u)$ two dimensional half canonical cosine transform of *f(t ,z)* is given by,

$$\{\ HCCTf(t,z)\ \}(s,u) = \sqrt{\frac{2}{\pi ib}} \cdot \sqrt{\frac{2}{\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} e^{\frac{i}{2}\left(\frac{d}{b}\right)u^2} \int_0^\infty e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{\frac{i}{2}\left(\frac{a}{b}\right)z^2} \cos\left(\frac{s}{b}t\right) \cdot \cos\left(\frac{u}{b}z\right) f(t,z)\, dtdz$$

then

$$f(t,z) = \sqrt{\frac{\pi i}{2b}} \sqrt{\frac{\pi i}{2b}} \cdot e^{-\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{-\frac{i}{2}\left(\frac{a}{b}\right)z^2} \int_0^\infty \int_0^\infty e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2} e^{-\frac{i}{2}\left(\frac{d}{b}\right)u^2} \cos\left(\frac{s}{b}t\right) \cos\left(\frac{u}{b}z\right) \{HCCT\ \ f(t,z)\}(s,u)\, dsdu$$

Proof: The two dimensional half canonical cosine transform of *f(t, z)* is given by

$$\{\ HCCTf(t,z)\ \}(s,u) = \sqrt{\frac{2}{\pi ib}} \cdot \sqrt{\frac{2}{\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} e^{\frac{i}{2}\left(\frac{d}{b}\right)u^2} \int_0^\infty e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{\frac{i}{2}\left(\frac{a}{b}\right)z^2} \cos\left(\frac{s}{b}t\right) \cdot \cos\left(\frac{u}{b}z\right) \cdot f(t,z)\, dtdz$$

$$F_{HCC}(s,u) = \sqrt{\frac{2}{\pi ib}} \sqrt{\frac{2}{\pi ib}}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} e^{\frac{i}{2}\left(\frac{d}{b}\right)u^2} \int_0^\infty \int_0^\infty e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{\frac{i}{2}\left(\frac{a}{b}\right)z^2} \cos\left(\frac{s}{b}t\right) \cos\left(\frac{u}{b}z\right)\ f(t,z)\, dt.dz$$

$$\therefore F_{HCC}(s,u) \sqrt{\frac{\pi ib}{2}} \sqrt{\frac{\pi ib}{2}}\ e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2} e^{-\frac{i}{2}\left(\frac{d}{b}\right)u^2} = \int_0^\infty \int_0^\infty e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{\frac{i}{2}\left(\frac{a}{b}\right)z^2} \cos\left(\frac{s}{b}t\right) \cos\left(\frac{u}{b}z\right)\ f(t,z) dtdz$$

$$\therefore B_1(s,u) = \int_0^\infty \int_0^\infty g(t,z) \cdot \cos\left(\frac{s}{b}t\right) \cos\left(\frac{u}{b}z\right) dtdz$$

where, $B_1(s,u) = F_{HCC}(s,u)\sqrt{\frac{\pi i b}{2}}\sqrt{\frac{\pi i b}{2}} \cdot e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2} e^{-\frac{i}{2}\left(\frac{d}{b}\right)u^2}$ and $g(t,z) = e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{\frac{i}{2}\left(\frac{a}{b}\right)zt^2} \cdot f(t,z).$

$$B_1(s,u) = \int_0^\infty\int_0^\infty g(t,z)\cdot\cos(\frac{s}{b}t)\cos(\frac{u}{b}z)dtdz = \int_0^\infty\int_0^\infty g(t,z)\cdot\cos(\eta t)\cos(\psi z)\ d\eta d\psi.$$

where, $\left(\frac{s}{b}\right) = \eta \quad and \quad \left(\frac{u}{b}\right) = \psi \Rightarrow d\eta = \frac{1}{b}ds \quad and \quad d\psi = \frac{1}{b}du$

By using inverse formula, $g(t,z) = \int_0^\infty\int_0^\infty B_1(s,u)\cdot\cos(\eta.t)\cos(\psi z)\ d\eta d\psi$

$$e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{\frac{i}{2}\left(\frac{a}{b}\right)z^2} \cdot f(t,z) = \int_0^\infty\int_0^\infty B_1(s,u)\sqrt{\frac{\pi i b}{2}}\sqrt{\frac{\pi i b}{2}}\, e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2} e^{-\frac{i}{2}\left(\frac{d}{b}\right)u^2} \cdot\cos(\eta.t)\ \cos(\psi z)\ d\eta\, d\psi$$

$$f(t,z) = e^{-\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{-\frac{i}{2}\left(\frac{a}{b}\right)z^2} \int_0^\infty\int_0^\infty F_{HCCC}(s,u)\ \sqrt{\frac{\pi i b}{2}}\sqrt{\frac{\pi i b}{2}}\ e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}\ e^{-\frac{i}{2}\left(\frac{d}{b}\right)u^2}\ \cos(\eta.t)\cos(\psi z)\ d\eta\, d\psi.$$

$$f(t,z) = e^{-\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{-\frac{i}{2}\left(\frac{a}{b}\right)z^2} \int_0^\infty\int_0^\infty \sqrt{\frac{\pi i b}{2}}\sqrt{\frac{\pi i b}{2}}\, e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2} e^{-\frac{i}{2}\left(\frac{d}{b}\right)u^2} F_{HCC}(s,u)\cos\left(\frac{s}{b}t\right)\cos\left(\frac{u}{b}z\right)\frac{1}{b}ds\frac{1}{b}du.$$

$$f(t,z) = \sqrt{\frac{\pi i}{2b}}\sqrt{\frac{\pi i}{2b}}\cdot e^{-\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ e^{-\frac{i}{2}\left(\frac{a}{b}\right)z^2} \int_0^\infty\int_0^\infty e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}\ e^{-\frac{i}{2}\left(\frac{d}{b}\right)u^2}\ \cos\left(\frac{s}{b}t\right)\cos\left(\frac{u}{b}z\right)\ \{HCCT\ f(t,z)\}(s,u)\ dsdu$$

3. Property of two dimensional half canonical cosine transform.

3.1.1 Linearity property of two dimensional half canonical cosine transform:

If P_1, P_2 are constants and {HCCT $f_1(t,z)$}(s, u), {HCCT $f_2(t,z)$}(s, u) denotes generalized two dimensional half canonical cosines transform of $f_1(t,z)$, $f_2(t,z)$ respectively then

$$\{HCCT\ (P_1 f_1(t,z) + P_2 f_2(t,z))\}(s,u) = P_1\ \{HCCT\ \ f(t,z)\}(s,u) + P_2\ \{HCCT\ \ f(t,z)\}(s,u)$$

3.1.2 Shifting property of two dimensional half canonical cosine transform:

If *{HCCT f(t,z)}(s,u)* denotes generalized two dimensional half canonical cosine transform, then,

$$\{HCCT\,[f(t-p,z-q)]\}(s,u) = e^{\frac{i}{2}\left(\frac{a}{b}\right)(p^2+q^2)}[\cos\left(\frac{s}{b}p\right)\cos\left(\frac{u}{b}q\right)\left\{HCCTf(x,y)e^{i\left(\frac{a}{b_1}\right)(xp+yq)}\right\}(s,u) - i\sin\left(\frac{s}{b}p\right)\cos\left(\frac{u}{b}q\right)\left\{HCSCTf(x,y)e^{i\left(\frac{a}{b_1}\right)(xp+yq)}\right\}(s,u)$$

$$-i\cos\left(\frac{s}{b}p\right)\sin\left(\frac{u}{b}q\right)\left\{HCCSTf(x,y)e^{i\left(\frac{a}{b_1}\right)(xp+yq)}\right\}(s,u) - \sin\left(\frac{s}{b}p\right)\sin\left(\frac{u}{b}q\right)\left\{HCSTf(x,y)e^{i\left(\frac{a}{b_1}\right)(xp+yq)}\right\}(s,u)\}]$$

3.1.3 Scaling property of two dimensional half canonical cosine transform:

If *{HCCT f(t)}(s)* denotes generalized two dimensional half canonical cosine transform, then,

$$\{HCCT\ [f(k_1 t, k_2 z)]\}(s,u)=\frac{1}{k_1 k_2}\,e^{\left(1-\frac{1}{k_1}\right)\frac{i}{2}\left(\frac{d}{b}\right)s^2}\,e^{\left(1-\frac{1}{k_2}\right)\frac{i}{2}\left(\frac{d}{b}\right)u^2}\left[HCCT\left\{f(t,z)e^{\left(\frac{1}{k_1}-1\right)\frac{i}{2}\left(\frac{a}{bk_1}\right)t^2}\,e^{\left(\frac{1}{k_2}-1\right)\frac{i}{2}\left(\frac{a}{bk_2}\right)t^2}\right\}\right]\left(\frac{s}{bk_1},\frac{u}{bk_2}\right)$$

Proof: We have,

$$\{HCCT\ \ f(k_1 t, k_2 z)\}(s,u)=\sqrt{\frac{2}{\pi ib}}\sqrt{\frac{2}{\pi ib}}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)u^2}\int_0^\infty\!\!\int_0^\infty e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ e^{\frac{i}{2}\left(\frac{a}{b}\right)z^2}\ \cos\left(\frac{s}{b}t\right)\cos\left(\frac{u}{b}z\right)\ f(k_1 t, k_2 z)\,dtdz$$

Putting, $k_1 t = T \Rightarrow dt=\frac{1}{k_1}dT,\ \ k_2 z = Z \Rightarrow dz=\frac{1}{k_2}dZ$

$$\{HCCT\ \ f(k_1 t, k_2 z)\}(s,u)=\sqrt{\frac{2}{\pi ib}}\sqrt{\frac{2}{\pi ib}}\,e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)u^2}\int_0^\infty\!\!\int_0^\infty e^{\frac{i}{2}\left(\frac{a}{b}\right)\frac{T^2}{k_1^2}}\ e^{\frac{i}{2}\left(\frac{a}{b}\right)\frac{Z^2}{k_2^2}}\ \cos\left(\frac{s}{bk_1}T\right)\cos\left(\frac{u}{bk_2}Z\right)f(T,Z)\,\frac{dT}{k_1}\,\frac{dZ}{k_2}$$

$$=\sqrt{\frac{2}{\pi ib}}\sqrt{\frac{2}{\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\ e^{-\frac{i}{2}\left(\frac{d}{bk_1}\right)s^2}\ e^{\frac{i}{2}\left(\frac{d}{bk_1}\right)s^2}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)u^2}\ e^{-\frac{i}{2}\left(\frac{d}{bk_{21}}\right)u^2}\ e^{\frac{i}{2}\left(\frac{d}{bk_{21}}\right)u^2}$$

$$\int_0^\infty\!\!\int_0^\infty e^{\frac{i}{2}\left(\frac{a}{bk_1^2}\right)T^2}\ e^{-\frac{i}{2}\left(\frac{a}{bk_1}\right)T^2}\ e^{\frac{i}{2}\left(\frac{a}{bk_1}\right)T^2}\ e^{\frac{i}{2}\left(\frac{a}{bk_2^2}\right)Z^2}\ e^{-\frac{i}{2}\left(\frac{a}{bk_2}\right)Z^2}\ e^{\frac{i}{2}\left(\frac{a}{bk_2}\right)Z^2}\ \cos\left(\frac{s}{bk_1}T\right)\cdot\cos\left(\frac{u}{bk_2}Z\right)\ \ f(T,Z)\frac{dT}{k_1}\,\frac{dZ}{k_2}$$

$$\{HCCT\ [f(k_1 t, k_2 z)]\}(s,u)=\frac{1}{k_1 k_2}\,e^{\left(1-\frac{1}{k_1}\right)\frac{i}{2}\left(\frac{d}{b}\right)s^2}\,e^{\left(1-\frac{1}{k_2}\right)\frac{i}{2}\left(\frac{d}{b}\right)u^2}\left[HCCT\left\{f(t,z)e^{\left(\frac{1}{k_1}-1\right)\frac{i}{2}\left(\frac{a}{bk_1}\right)t^2}\,e^{\left(\frac{1}{k_2}-1\right)\frac{i}{2}\left(\frac{a}{bk_2}\right)t^2}\right\}\right]\left(\frac{s}{bk_1},\frac{u}{bk_2}\right)$$

4. Parseval's Identity for two dimensional half canonical cosine transforms:

If *f(t, z)* and *g(t, z)* are the inversion canonical two dimensional half cosine transform of F_{2DHCC}(s, u) and G_{2DHCC}(s, u) respectively, then

(1) $\int_0^\infty\!\!\int_0^\infty f(t,z).\overline{g(t,z)}\ dtdz=\frac{\pi^2}{4}\int_0^\infty\!\!\int_0^\infty F_{HCC}(s,u)\overline{G_{HCC}(s,u)}\ .dsdu$ and

(2) $\int_0^\infty\!\!\int_0^\infty |\,f(t,z)\ |^2\ dtdz=\frac{\pi^2}{4}\int_0^\infty\!\!\int_0^\infty |F_{HCC}(s,u)\ |^2 dsdu$

Proof: By definition of two dimensional HCCT,

$$\{HCCT\ \ g(t,z)\}(s,u)=\sqrt{\frac{2}{\pi ib}}\sqrt{\frac{2}{\pi ib}}\,e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)u^2}\int_0^\infty\int_0^\infty e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ e^{\frac{i}{2}\left(\frac{a}{b}\right)z^2}\ \cos\left(\frac{s}{b}t\right)\cos\left(\frac{u}{b}z\right)\cdot g(t,z)\,dt.dz\ \text{ --}$$

(4.1.1)

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Using the inversion formula

$$g(t,z)=\sqrt{\frac{\pi i}{2b}}\sqrt{\frac{\pi i}{2b}}\ e^{-\frac{i}{2}\left(\frac{d}{b}\right)t^2}\ e^{-\frac{i}{2}\left(\frac{d}{b}\right)z^2}\int_0^\infty\int_0^\infty e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}\ e^{-\frac{i}{2}\left(\frac{d}{b}\right)u^2}\ \cos\left(\frac{s}{b}t\right)\cos\left(\frac{u}{b}z\right)G_{HCC}(s,u)\,dsdu$$

Taking complex conjugate we get,

$$\overline{g(t,z)}=\sqrt{\frac{-\pi i}{2b}}\sqrt{\frac{-\pi i}{2b}}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)t^2}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)z^2}\int_0^\infty\int_0^\infty e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)u^2}\ \cos\left(\frac{s}{b}t\right)\cos\left(\frac{u}{b}z\right)\overline{G_{HCC}(s,u)}\,dsdu$$

$$\int_0^\infty\int_0^\infty f(t,z).\overline{g(t,z)}\,dtdz=\int_0^\infty\int_0^\infty f(t,z)dtdz\left(\sqrt{\frac{-\pi i}{2b}}\sqrt{\frac{-\pi i}{2b}}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)t^2}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)z^2}\int_0^\infty\int_0^\infty e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)u^2}\ \cos\left(\frac{s}{b}t\right)\cos\left(\frac{u}{b}z\right)\overline{G_{HCC}(s,u)}\,dsdu\right)$$

Changing the order of integration, we get,

$$\int_0^\infty\int_0^\infty f(t,z).\overline{g(t,z)}\,dtdz=\sqrt{-\frac{\pi i}{2b}}\sqrt{-\frac{\pi i}{2b}}\int_0^\infty\int_0^\infty\overline{G_{HCC}(s,u)}\,dsdu\,\frac{1}{\sqrt{\frac{2}{\pi ib}}\sqrt{\frac{2}{\pi ib}}}\left(\sqrt{\frac{2}{\pi ib}}\sqrt{\frac{2}{\pi ib}}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)t^2}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)z^2}\int_0^\infty\int_0^\infty e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)u^2}\ \cos\left(\frac{s}{b}t\right)\cos\left(\frac{u}{b}z\right)f(t,z)dtdz\right)$$

$$\int_0^\infty\int_0^\infty f(t,z).\overline{g(t,z)}\,dtdz=\frac{\pi^2}{4}\int_0^\infty\int_0^\infty\overline{G_{HCC}(s,u)}\,.F_{HCC}(s,u)dsdu$$

$$\int_0^\infty\int_0^\infty f(t,z).\overline{g(t,z)}\,dtdz=\frac{\pi^2}{4}\int_0^\infty\int_0^\infty F_{HCC}(s,u)\overline{G_{HCC}(s,u)}\,.dsdu \qquad \text{---------(4.1.2)}$$

Hence proved

(ii) Putting $f(t,z)=g(t,z)$ in equation (4.1.2), we get

$$\int_0^\infty\int_0^\infty|f(t,z)|^2\,dtdz=\frac{\pi^2}{4}\int_0^\infty\int_0^\infty|F_{HCC}(s,u)|^2dsdu$$

Table for two dimensional half canonical cosine transform

$f(t, z)$	F_{HCCT}(s, u)
$\{P_1 f_1(t,z)+P_2 f_2(t,z)\}(s,u)$	$P_1\,\{HCCT\ \ f(t,z)\}(s,u)+P_2\,\{HCCT\ \ f(t,z)\}(s,u)$
$\{HCCT\,[f(t-p,z-q)]\}(s,u)$	$e^{\frac{i}{2}\left(\frac{a}{b}\right)(p^2+q^2)}\begin{bmatrix}\cos\left(\frac{s}{b}p\right)\cos\left(\frac{u}{b}q\right)\left\{HCCTf(x,y)e^{i\left(\frac{a}{b_1}\right)(xp+yq)}\right\}(s,u)-i\sin\left(\frac{s}{b}p\right)\cos\left(\frac{u}{b}q\right)\left\{HCSCTf(x,y)e^{i\left(\frac{a}{b_1}\right)(xp+yq)}\right\}(s,u)\\-i\cos\left(\frac{s}{b}p\right)\sin\left(\frac{u}{b}q\right)\left\{HCCSTf(x,y)e^{i\left(\frac{a}{b_1}\right)(xp+yq)}\right\}(s,u)-\sin\left(\frac{s}{b}p\right)\sin\left(\frac{u}{b}q\right)\left\{HCSTf(x,y)e^{i\left(\frac{a}{b_1}\right)(xp+yq)}\right\}(s,u)\end{bmatrix}$
$[f(k_1t,k_2z)]\}(s,u)$	$\frac{1}{k_1k_2}e^{\left(1-\frac{1}{k_1}\right)\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\left(1-\frac{1}{k_2}\right)\frac{i}{2}\left(\frac{d}{b}\right)u^2}\left[HCCT\left\{f(t,z)e^{\left(\frac{1}{k_1}-1\right)\frac{i}{2}\left(\frac{a}{bk_1}\right)t^2}e^{\left(\frac{1}{k_2}-1\right)\frac{i}{2}\left(\frac{a}{bk_2}\right)t^2}\right\}\right](\frac{s}{bk_1},\frac{u}{bk_2})$

Conclusion: In this paper, brief introduction of the generalized two dimensional half canonical cosine transform is given and Inversion theorem for two dimensional half canonical cosine transform proved. Properties of two dimensional half canonical cosine transform are also obtained which will be useful in solving differential equations occurring in signal processing and many other branches of engineering.

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