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Geometric non-linear analysis of thin flat membrane

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Abstract

This paper present the model analysis for the predicating the behaviour of inflatable membrane structure of general L-shape with a thickness in millimetre using the various smart material which optimally within structural member subjected to pre-stressed rather than bending or moments. A numerical solution for membranes may also be found using the finite element method. In this paper flat thin membrane choose to analysis the behavioural effect of the membranes using the properties of different smart material and compare their results in terms of frequency and generalized mass with mode shape. This analysis makes more effective to selects the smart material in the space technology. Geometrically non-linear Vibration analysis of arbitrary L-shape membrane is also done using a finite element package, ABAQUS. The analysis shows good agreement between finite element and analytical solutions.

Keywords*: *Boundary condition, pre-stressed, natural frequency, material property, membrane shape, static displacement, mode shape, finite element.

1. Introduction

In the field of Engineering and Architecture, membrane structures play a vital role in many ways Examples include textile covers and roofs, aircraft and space structures, parachutes, automobile airbags, sails, windmills, human tissues and long span structures. They are typically built with very light materials which are optimally used. These structures are characterized because they are only subjected to in-plane axial forces. Even in the field of architectures and civil engineering, both pre-stressed membranes and cable networks constitute a very remarkable group. A membrane is essentially a thin shell with no flexural stiffness. Consequently a membrane cannot resist any compression at all. However, membrane theory accounts for tension and compression stresses, and the need for a computational procedure that takes into account tension stresses only is needed. In membrane theory only the in-plane stress resultants are taken into account. A numerical solution for membranes may be found using the finite element method [1–3].

The deployable space structures consist of thin polymer films that offer a wider range of packaging configurations than structures with traditional deployment mechanisms. Due to the flexibility of such deployable structure like shell or membrane shows greater importance for space application and hold great promise. The material constitutive behaviour and the analytical tools to analyze them are required to make advances in building cheaper, lighter and more reliable structures. Many structures are in the developing stage and the materials that are meant to serve to make these applications possible are not yet within reach. Future missions depend much on new discoveries, mainly in material manufacturing. There are several different space applications in which the use of thin membrane structures are used or being considered. Due to their light weight, high strength-to-weight ratio and ease of stowing and deploying, membranes are especially attractive for space applications. Inflatable reflectors, space-based radar, space based communication systems such as antennae and solar power collection panels on spacecraft, etc are the examples included. [4–5].

The membrane material used in the numerical analysis was assumed inextensible and its weight was neglected in the determination of the equilibrium shape. They found that the membrane's mass density is of little influence on the computed natural frequencies. Other researchers used finite elements and boundary elements to model and compute natural frequencies and mode shapes of a single-anchor inflatable dam [6]. This study makes impact on finding the vibration aspect on the flat membrane using the various smart materials. The pressure in an inflatable structure can also play a critical role in the suppression of vibration [7]. Literature that exists on 'pure' structural membrane components has concentrated mostly on inflated components such as beams [8], torus (Main), and inflated lenticular concentrators [9]. The dynamics of the membrane themselves are of great interest though, as it is the membrane itself that is performing the 'useful' work, and in some applications they could be attached to more traditional aerospace structures. Therefore improving understanding the behaviour of the membranes appears to be important. A membrane is essentially a thin shell with no flexural stiffness. Consequently a membrane cannot resist any compression at all. However, membrane theory accounts for tension and compression stresses. In membrane theory only the in-plane stress resultants are taken into account [10].

This paper present the model analysis for the predicating the behaviour of various inflatable membrane structure of general L-shape with a thickness in millimetre using the various smart material which optimally within structural member subjected to pre-stressed rather than bending or moments. A numerical solution for membranes may be found using the finite element method. Finite element analysis of membrane structures for small deformations can be found in [11] but with only single material. In this paper, the L-shaped general sketch of flat thin membrane is chosen & analysis the behavioural effect of the membranes using different properties for different smart material. Comparing the various parameters like frequency, Eigen values, displacement, etc. This analysis is more effective in future to selects the suitable smart material in the design of the space technology. The geometrically non-linear vibration due to pre-stressed is modelled and analysed using finite element package, ABAQUS [12]. A numerical solution is also presented.

2. Membrane material Properties

Membrane structures consist of thin membrane or fabric as a major structural element. A membrane has no compression or bending stiffness, therefore it has to be pre-stressed to act as a structural element [13]. The analysis, design and construction of such structures are a field that has developed very considerably during the last 30 years. The type of structures that is of interest in the present study is high-precision deployable for spacecraft, where is a growing requirement for furlable reflecting surfaces for antennae, reflectors and solar arrays. The conjurations that are being considered include at pre-stress membrane panels and parabolidal pre-stressed membranes formed by contiguous cylindrical pieces.

The performance efficiency of these reflective surfaces depends not only on the geometric accuracy of the surface but also on its vibration characteristics. The vibrations of lightweight structures are afflicted considerably by the surrounding medium. Thus, spacecraft structures should be tested in a vacuum chamber, but this would be too costly for a large structure. The efficiency and stability of the membrane structures depends on their dynamic controls in the deployed configuration, thus it is necessary to have a detailed understanding of vibration characteristics of these membrane structures.

Material	Kevlar [14]	Kapton [15]	Mylar [14]
Density [ρ] (Kg/m^3)	1450	1420	1070
Young's Modulus[E] (N/m^2)	131e9	2.5e9	3.5e9
Poisson's ratio [μ]	0.30	0.34	0.35
Thickness [t] (mm)	0.1	0.1	0.1

Table 1: Membrane material properties

3. Governing Equation

The plate in which the ratio $a/h \geq 80,....100.$, where 'a' is a typical dimension of a plate in a plane and 'h' is a plate thickness is maintained such plates are referred to as membranes and they are devoid of flexural rigidity. Membranes carry the lateral loads by axial tensile forces N (and shear forces) acting in the plate middle surface as shown in Fig. 1. These forces are called membrane forces; they produce projection on a vertical axis and thus balance a lateral load applied to the plate membrane. The fundamental assumptions of the linear, elastic, small-deflection for thin membrane structure may be stated as the material of the plate is elastic, homogeneous, isotropic and initially remain flat. The deflection of the mid-plane is very small compared to that of membrane thickness. Middle surface remains unstrained even after bending, since the deflection is too small.

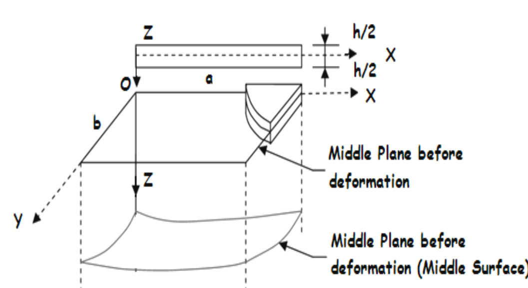

Figure 1(a): A load free membrane

To derive the equation of motion of a membrane, consider the membrane to be bounded by a plane curve S in the XY plane, as shown in fig 1(b). Let f (x, y, t) denote the pressure loading acting in the Z direction and P the intensity of tension at a point that is equal to product of the tensile stress and the thickness of the membrane.

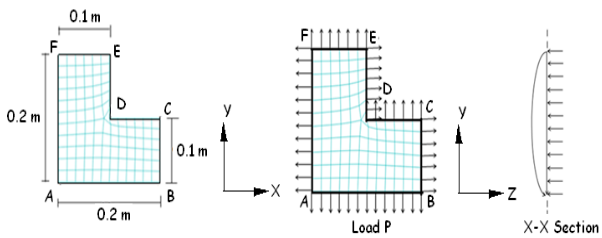

Figure 1(b): Geometry and pre-stress of L-shaped

The magnitude of P is usually constant throughout the membrane. If we consider an elemental area dx dy. Forces of magnitude Pdx and Pdy act on the sides parallel to the Y and X axes respectively as shown in figure 1(b).
The net forces acting along the Z direction due to these forces are

$$(P\,\frac{\partial^2 w}{\partial y^2}\,dx\,dy) \quad \text{and} \quad (P\,\frac{\partial^2 w}{\partial x^2}\,dx\,dy) \qquad (1)$$

The pressure force along the Z direction is f(x, y, t) dxdy and the inertia force is

$$\rho(x,y)\,\frac{\partial^2 w}{\partial t^2}\,dx\,dy \qquad (2)$$

where, $\rho(x, y)$ is the mass per unit area. The equation of motion for free transverse vibration of the membrane can be obtained as

$$P\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=\rho\,\frac{\partial^2 w}{\partial t^2} \qquad (3)$$

The above equation can be expressed as,

$$P\,\nabla^2 w=\rho\,\frac{\partial^2 w}{\partial t^2} \qquad (4)$$

where, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the Laplacian operator.

4. Initial and boundary condition

Since the equation of motion Eq. (3) involves second order partial derivatives with respect to each of t, x and y. we need to specify two initial conditions and four boundary condition to find a unique solution of the problem. Usually, the displacement and velocity of the membrane at t=0 are specified as $w_0(x, y)$ and $\dot{w}_o$ (x, y). Hence the initial conditions are given by,

$$w\,(x, y, 0) = w_0\,(x, y) \qquad (5)$$

$$\frac{\partial w}{\partial t}(x,y,0)=\dot{w}_0(x,y) \qquad (4)$$

The boundary conditions are as follows:

1. If the membrane is fixed at any point (x1, y1) on a segment of the boundary, we have

$$w(x_1,y_1,t)=0 \qquad t\ \geq\ 0 \qquad (7)$$

2. If the membrane is free to deflect transversely (in the z direction) at a different point (x_2,y_2) of the boundary, then the force component in the Z direction must be zero.

$$P\,\frac{\partial w}{\partial n}\,(x_2,y_2,t)=\ 0 \qquad t\ \geq\ 0 \qquad (8)$$

where, $\frac{\partial w}{\partial n}$ represents the derivative of w with respect to a direction n normal to the boundary at point (x_2, y_2).
The free vibration solution of the thin flat membrane can be obtained by using the method of separation of variables w(x, y, t) can be assumed as

$$w(x, y, t) = W(x, y)\,T(t) = X(x)\,Y(y)\,T(t) \qquad (9)$$

By using the equation of motion Eq. (3), we obtain,

$$\frac{d^2\,X(x)}{dx^2}+\ \alpha^2\,X(x)=\ 0 \qquad (10)$$

$$\frac{d^2\,Y(y)}{dy^2}+\ \beta^2\,Y(y)=\ 0 \qquad (11)$$

$$\frac{d^2\,T(t)}{dt^2}+\ \omega^2\,T(t)=\ 0 \qquad (12)$$

Where α^2 and β^2 are constants related to ω^2 as follows:

$$\beta^2=\frac{\omega^2}{C^2}-\ \alpha^2 \quad and \quad C^2=(\frac{P}{\rho}) \qquad (13)$$

The solutions of the above Eq. (10) to Eq. (12) are given by;

$$X(x) = C1\ Cos\ cx + C2\ Sin\ \alpha x$$
$$Y(y) = C3\ Cos\ \beta y + C4\ Sin\ \beta y$$
$$T(t) = A\ Cos\ \omega t + B\ Sin\ \omega t$$

Where, the constants C_1, C_2, C_3, C_4 and A, B can be determined from the boundary conditions;

Table 2: Constant values for (xi, yi), i = 1, 2, 3…

x	y	α	β
0	0.1	-	$\pi/0.1$
0	0.2	-	$\pi/0.2$
0.1	0.1	$\pi/0.1$	$\pi/0.1$
0.1	0.2	$\pi/0.1$	$\pi/0.2$

5. Finite element method

The geometric model with orientation is shown in fig. 1(a) The orientation of the element normal has to be representative for the whole element. The number of elements was set approximate to 1000 elements and it is concluded. The flat membranes having square have been analyzed using the FE package. The L-shaped flat membrane has a side length of 0.2 m and a thickness of 0.1 mm; it is supported on knife edges, which act as simple supports along all six edges. Different plan stress elements are available in ABAQUS to model the membrane structure. Since the M3D4 membrane element having 4 nodes quadrilateral elements are more dominant than the M3D3 membrane element which is a 3 node triangular element. Hence, M3D4 is chosen for the modelling analysis. These elements are surface elements that transmit in-plane forces only (no moments) and have no bending stiffness. This means that it is necessary to pre-stress these elements before any vibration analysis is carried out. The various different materials are taken whose properties are tabulated in Table 1.

The meshing is shown below fig. 1(a) consisting quadrilateral elements. The meshing element set aprox. to 1000 for general L-shaped flat thin membrane. The variation in frequency or other variables have been obverse due to higher computation times. The following boundary conditions opt: The nodes along the edge AB are restrained in the 'y' direction to simulate simple supports. The nodes along the edges BC, CD, DE & EF restrained in the 'z' direction to simulate simple supports. The nodes along the edges FA restrained in the 'x' direction to simulate simple supports. All other nodes have three degrees of freedom. The pre-stress of 10 N/m is applied along the nodes of all the edges of the L-shaped flat membrane.

6. Results and Discussion:

When the pre-stressed of 10 N/m is applied to the three material membranes the various obtained result are given below in Table 3 and 4. The graphical comparison between the three materials (Kevlar, Mylar and Kapton) is corresponding to Natural frequency; Mode shape and its generalised mass are shown in the Figure 2 and 3.

Table 3: Natural Frequency and Mode shapes.

Mode No.	Kevlar (Hz)	Kapton (Hz)	Mylar (Hz)
1	14.55	11.03	11.32
2	22.60	17.34	17.99
3	24.40	18.83	19.62
4	30.93	24.16	25.45
5	31.83	24.82	26.10
6	38.44	30.30	32.15
7	38.88	30.58	32.41
8	44.71	35.30	37.44
9	45.10	35.69	37.99
10	48.43	36.91	38.07

Table 4: Generalised mass and Mode shapes

Mode No.	Kevlar (kg)	Kapton (kg)	Mylar (kg)
1	0.00027	0.00051	0.00052
2	0.00054	0.00099	0.00100
3	0.00023	0.00039	0.00038
4	0.00036	0.00065	0.00064
5	0.00009	0.00016	0.00015
6	0.00023	0.00041	0.00040
7	0.00006	0.00010	0.00010
8	0.00005	0.00005	0.00003
9	0.00006	0.00017	0.00029
10	0.00004	0.00024	0.00031

Low frequency stabilizes the oscillation for the data transmission in the space technology and hence the Kapton membrane possesses the lower natural frequency range as compared to others as shown in the fig. 2. The generalised mass acting on the membrane node corresponding to the different mode shapes which helps to stabilise the entire model due to the pre-stressed of 10 N/m. Here also, Kapton membrane plays the major role (fig. 3).

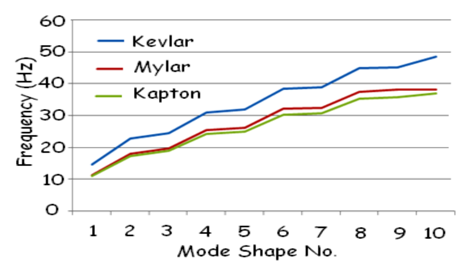

Figure – 2: Frequency Vs Mode Shape

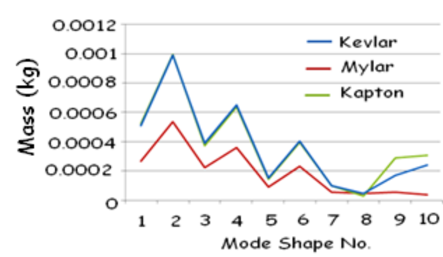

Figure – 3: Generalised mass vs Mode Shape

The few mode shape(s) variation of the flat thin membrane material under the given pre-stressed of 10 N/m within the prescribed boundary condition are shown below which have been evaluated from FE tool as:

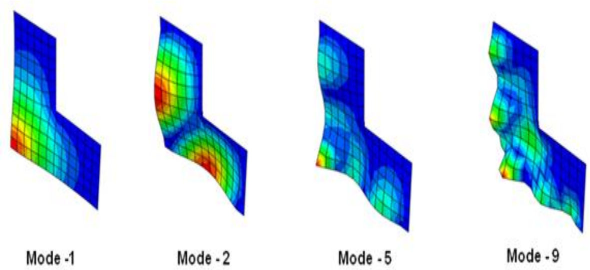

Figure 4: Mode shapes (Kevlar)

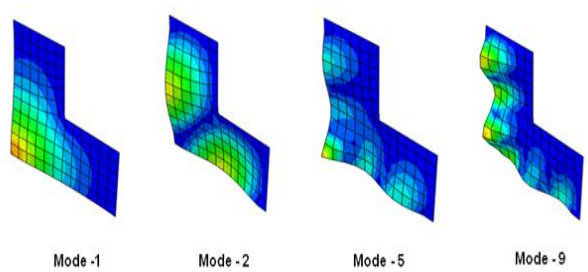

Figure 5: Mode shapes (Kapton)

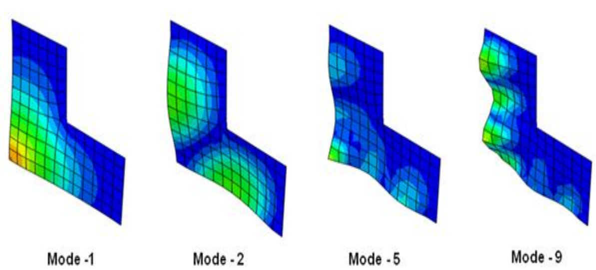

Figure 6: Mode shapes (Mylar)

The results obtain from the finite element method (FEM) for flat thin flat membrane is being validated using the analytical method. The few results validation is shown below in the tabulated form for various membrane materials.

Table 5: Results (Kevlar material)

SN	Mode Shape	Analytical solution	FEM solution	Absolute error
1	1	18.148	14.54	3.608
2	2	23.748	22.60	1.148
3	5	32.718	31.82	0.898
4	6	39.073	38.44	0.633
5	8	45.088	44.70	0.388

Table 6: Results (Kapton material)

SN	Mode Shape	Analytical solution	FEM solution	Absolute error
1	1	13.269	11.03	2.239
2	2	17.805	17.34	0.465
3	5	25.359	24.92	0.439
4	6	30.446	30.30	0.146
5	8	35.619	35.38	0.239

Table 7: Results (Mylar material)

SN	Mode Shape	Analytical solution	FEM solution	Absolute error
1	1	13.411	11.32	2.091
2	2	18.531	17.99	0.541
3	5	26.506	26.10	0.406
4	6	32.415	32.15	0.265
5	8	37.99	37.44	0.550

7. Conclusion

In the field of engineering application, thin membrane structures with very light materials are demandable due to non flexural stiffness and optimally within structural member subjected to pre-stressed rather than bending or moments. In this paper, the dynamic behaviour of the *L*-shaped flat thin membrane is being analyzed in terms of the mode shape and natural frequency using the different types of smart materials such as Kevlar, Kapton and Mylar. Using the pre-stressed of 10 N/m to the outer edges along the plane is applied and the encaster boundary condition to the inner edges of the flat *L*-shaped membrane shows the symmetric variation. This analysis makes more effective to selects the smart material in the space technology. The analysis shows good agreement between FEM and analytical solutions as shown in Table 5 to 7.

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