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GPS Attitude Determination with a Multiple Antenna using Carrier Phase

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***Abstract*--Three GPS antennas are mounted properly on a platform and differences of GPS signals measurements are collected simultaneously, the baselines vectors between antennas can be determined and the platform orientation defined by these vectors can be calculated. Thus, the prerequisite for attitude determination technique based on GPS is to calculate baselines between antennas to centimeter level of accuracy. For accurate attitude solutions to be attained, carrier phase double differences are used as main type of measurements. The use of carrier phase measurements leads to the problem of precise determination of the ambiguous integer number of cycles in the initial carrier phase (integer ambiguity). This integer ambiguity must be solved for the precise result. For this aims at defining the search space for the ambiguity candidates and identifying the correct candidate, mainly three sequential steps were involved (Hofmann-Wellenhof et al.2001). Platform orientation was obtained using Least Square Method, to resolve the three dimensional unknown Euler angles.**

Keywords—Gps; Attitude determination; Carrier Phase; Multiple Antenna; Least Square Method

I. INTRODUCTION

The Global Positioning System (GPS) is a space-based satellite navigation system that provides location and time information in all weather conditions, anywhere on or near the Earth where there is an unobstructed line of sight to four or more GPS satellites. The satellite-based Global Positioning System (GPS) creates a new era for navigation, surveying and geodesy. The development of GPS multi-antenna systems, which integrate three or more GPS antennas into one system with a proper antenna configuration in a plane or in space, has been resulted another leap in GPS applications. The necessary parameters like transmitter and receiver, accurate position, attitude and velocity of the any platforms can be obtained by using GPS system. Besides that, GPS multi-antenna system has become a high-accurate approach for attitude determination (Cohen et al. 1994; Van Grass and Braasch 1991). In comparison with the traditional inertial sensors, the GPS multi-antenna system provides attitude results without drift effects, and it has the advantages due to the cost-effectiveness and the flexible installation.

If three or more GNSS antennas properly mounted on a platform and differences of GNSS signals measurements are collected simultaneously, baselines vectors formed between antennas can be determined, and orientation of the platform defined by these vectors can be calculated. Thus, the prerequisite for the attitude determination technique based on GNSS systems is to calculate the baselines between the antennas. Accurate attitude solutions can be obtained using carrier phase double difference observables as the main type of measurements, including all independent combinations of antenna positions. Baselines between antennas must be determined in centimeter level of accuracy. The use of carrier phase measurements leads to the problem of determining precisely the ambiguous initial carrier phase integer number of cycles (integer ambiguity). The double-differenced carrier phase observation equation is an underdetermined equation and the ambiguities cannot be solved directly. Typically, the distance between the antennas is a few meters or less, and all spatially correlated errors between the antennas are almost eliminated in differencing (single and double) process, including orbital, ionospheric, and tropospheric errors. Therefore, main error sources affecting attitude determination to derive the true integer values of ambiguities are the multipath, receiver internal noise, and antenna phase center variation instantaneously.

In GPS attitude determination, fast and reliable on-the-fly (OTF) ambiguity searching methodology is always expected. Most ambiguity resolution strategies borrow ambiguity searching algorithms from static or rapid kinematic positioning, such as the fast ambiguity search filter (FASF) (Chen and Lachapelle 1994), the least squares ambiguity search technique (LSAST)(Hatch 1989) and the fast ambiguity resolution approach (FARA) (Freiand Beutler 1990). Any of these ambiguity search techniques includes three main common steps (Hofmann-Wellenhof et.al 2001). The first step is to resolve the float ambiguities, namely to calculate the float values of ambiguities through a proper mathematical model. The second step is to generate integer candidates around the float values and choose the best one. The first two steps determine the center and the size of the search space. The final step is the ambiguity validation, namely to verify whether or not the best ambiguity candidate is correct one. With a closely spaced antenna configuration, the inter-antenna distance in GPS attitude determination systems can be easily determined by conventional surveying methods and used either as an observable or an extra constraint in ambiguity resolution. Once ambiguities have been correctly determined and the antenna vector has been transformed from WGS-84 frame into the local level frame, attitude parameters can be derived from the rotation matrix using least squares estimation methods based on the known antenna coordinates both in the body and local level frames.

II. OBSERVATION EQUATIONS

GPS observation equations are the basis for the GPS data processing. The carrier phase measurements reflect the difference between the phase of the incoming signal from a GPS satellite and the generated signal in the receiver. A carrier phase measurement is occupied with four parts.

$$\phi_{output}(t) = \phi_f(t) + \phi_i(t) + N(t) + e(t) \qquad (1)$$

Where,
Φ_{output} represents the carrier phase measurements from the receiver,
Φ_f is the fractional part of the measured carrier phase,
Φ_i is the number of integer cycles accumulated from the first observation epoch to the current epoch,
e is the error term,
N is the integer phase ambiguity.

According to (Leick 2004), N refers to the first epoch of observation and remains constant during the period of observation. During this period, the receiver accumulates the phase differences between arriving phases and internally generated receiver phases. The receiver, therefore, effectively generates an accumulated carrier phase observables that reflects the changes in distance to the satellite. The integer ambiguity should be resolved a priori and subtracted from the carrier phase measurements obtained from the receiver. As the integer ambiguity remains constant epoch by epoch, the time dependence can therefore be dropped. Therefore, the carrier phase observation equation can be obtained as,

$$\lambda_{Li}\phi_{Li}(t) = \rho(t) + \lambda_{Li}N_{Li} - \frac{\lambda_{Li}^2}{\lambda_{L1}^2} I_{Li}(t) + T(t) + S(t)$$
$$+ c.t_r(t) + c.t^s(t) + e_{Li}(t) + M_{Li}(t)$$
(2)

Where,
Li (subscript) indicates the corresponding signal,
λ is the wavelength of the corresponding GPS signal,
Φ is the phase measurement,
ρ is the geometric distance from the GPS receiver's antenna phase center at the epoch of signal reception to the GPS satellite's antenna phase center at the epoch of signal transmission,
I is the ionospheric delay,
T is the tropospheric delay,
S is the satellite orbit bias,
c is the speed of light,
t^s is the satellite clock bias in units of time,
t_r is the receiver clock bias in units of time,
e is the thermal noise contained in the phase data,
M is the multipath error,
Φ and N are expressed in units of cycles and all terms except for clock biases are given in units of length.

Some common errors can be cancelled or reduced by differencing the measurements between the satellites and the receivers by using the differencing carrier phase techniques. Two or more receivers are included in this mechanism as in figure (1) below

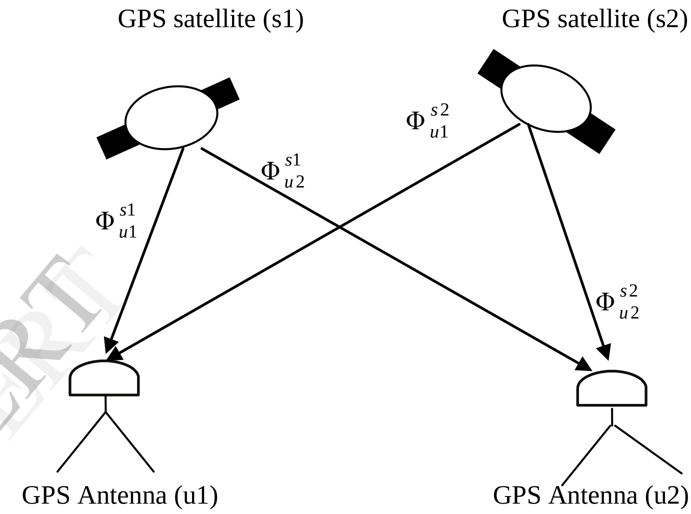

Figure (1): An demonstration for differential positioning

The carrier phase measurement of both antennas to a common satellite s1 using equation 2 we can get,

$$\lambda_{Li}\phi_{u1}^{s1} = \rho_{u1}^{s1} + \lambda_{Li}N_{u1,Li}^{s1} - \frac{\lambda_{Li}^2}{\lambda_{L1}^2} I_{u1,Li}^{s1} + T_{u1}^{s1} + S^{s1} + c.t_{u1} + c.t^{s1} + e_{u1}^{s1} + M_{u1}^{s1}$$
(3)

$$\lambda_{Li}\phi_{u2}^{s1} = \rho_{u2}^{s1} + \lambda_{Li}N_{u2,Li}^{s1} - \frac{\lambda_{Li}^2}{\lambda_{L1}^2} I_{u2,Li}^{s1} + T_{u2}^{s1} + S^{s1} + c.t_{u2} + c.t^{s1} + e_{u2}^{s1} + M_{u2}^{s1}$$

Where,
$u1$ and $u2$ indicate the user (antenna),
$s1$ indicate the satellite.
The satellite clock error t^{s1} and orbits error S^{s1} are common errors in equation (3) so the resultant of equation (3) gives:

$$\lambda_{Li}\Delta\phi_{u1-u2}^{s1} = \Delta\rho_{u1-u2}^{s1} + \lambda_{Li}\Delta N_{u1-u2,Li}^{s1} - \frac{\lambda_{Li}^2}{\lambda_{L1}^2}\Delta I_{u1-u2,Li}^{s1} + \qquad (4)$$
$$\Delta T_{u1-u2}^{s1} + c.\Delta t_{u1-u2} + \Delta e_{u1-u2}^{s1} + \Delta M_{u1-u2}^{s1}$$

Where the operator Δ indicates the differencing, for example $\Delta\rho^{s1}_{u1-u2}$ stands for $\Delta\rho^{s1}_{u1}$ - $\Delta\rho^{s1}_{u2}$

If both antennas are located closely to each other on the ground plane, the atmospheric effects on the observation equations in (3) are approximately same, so that the differenced tropospheric error ΔT and ionospheric error ΔI might be neglected.
Then equation (4) can be formulated as;

$$\lambda_{Li}\Delta\phi^{s1}_{u1-u2} = \Delta\rho^{s1}_{u1-u2} + \lambda_{Li}\Delta N^{s1}_{u1-u2,Li} + c.\Delta t_{u1-u2} + \Delta e^{s1}_{u1-u2} + \Delta M^{s1}_{u1-u2} \quad (5)$$

Equation (5) is referred as the "***single-differential positioning.*** However, to remove the receiver clock error Δt_{u1-u2} we first apply the single-differential positioning to satellite s2;

$$\lambda_{Li}\Delta\phi^{s2}_{u1-u2} = \Delta\rho^{s2}_{u1-u2} + \lambda_{Li}\Delta N^{s2}_{u1-u2,Li} + c.\Delta t_{u1-u2} + \Delta e^{s2}_{u1-u2} + \Delta M^{s2}_{u1-u2} \quad (6)$$

In equation (5) and (6), the receiver clock error (Δt_{u1-u2}) is a common error term and hence can be eliminated by further differencing both equations, so that we have:

$$\lambda_{Li}\Delta\nabla\phi^{s1-s2}_{u1-u2} = \Delta\nabla\rho^{s1-s2}_{u1-u2} + \lambda_{Li}\Delta\nabla N^{s1-s2}_{u1-u2,Li} + \Delta\nabla e^{s1-s2}_{u1-u2} + \Delta\nabla M^{s1-s2}_{u1-u2} \quad (7)$$

where, $\Delta\nabla$ denotes a further difference between the "single-differential" measurements associated to a common receiver. For eg. $\Delta\nabla\rho^{s1-s2}_{u1-u2}$ express as ($\Delta\rho^{s1}_{u1}$ - $\Delta\rho^{s1}_{u2}$) - ($\Delta\rho^{s2}_{u1}$ - $\Delta\rho^{s2}_{u2}$). This procedure is called "***double-differential positioning***". So far, the satellite clock error, satellite orbit error and receiver clock error have been eliminated. In case of short antenna baseline, the remaining ionospheric and tropospheric errors can also be neglected. For these reasons, the differential positioning leads to the accuracy improvement. However, a drawback of the differential positioning is that the thermal noise and multipath errors will be accumulated.

III. Attitude Determination

After the ambiguities are resolved, the carrier phase measurement can be used for attitude determination. The antenna body application (ABF) is formed by the GPS antennas. The antennas are mounted on a rigid platform, i.e. the relative distances between the antennas remain constant. One antenna is chosen as the master antenna, and the other antennas are called slave antennas. Actually, three antennas are sufficient to determine the antenna body frame. The origin is chosen as the position of the phase center of antenna 1, namely the master antenna.

A three-dimensional rotation can be decomposed into three individual rotations with each around a single axis. Euler angles represent the rotation angles with respect to three axes and usually comprise of yaw, pitch and roll angles. Note that in the right-handed frame, the Euler angles describe counter-clockwise rotations when viewed from the end of the positive axes and clockwise rotation when viewed from the origin of the positive axes. Each rotation can be described by a Direction Cosine Matrix (DCM). A three-dimensional rotation can be obtained by multiplying the three DCMs into a specific order, yielding the combined rotation matrix. An example using the yaw-pitch-roll sequence is given below

$$X_{ABF} = \begin{bmatrix} c(r)c(y)-s(r)s(p)s(y) & c(r)s(y)+s(r)s(p)c(y) & -s(r)c(p) \\ -c(p)s(y) & c(p)c(y) & s(p) \\ s(r)c(y)+c(r)s(p)s(y) & s(r)s(y)-c(r)s(p)c(y) & c(r)c(p) \end{bmatrix} X_{LLF} \quad (8)$$

This is the general mathematic model for attitude determination. Where LLF=Local Level Frame; y, p and r are short-form notation for yaw, pitch and roll angles, respectively; c and s denote the cosine and sine operators, respectively. In order to resolve the three dimensional unknown Euler angles using the nonlinear model given in equation (8), more than three baseline vectors are needed. Each master-slave antenna baseline provides three baseline vectors, and hence we need a minimum of two non-collinear slave antennas. Based on the linearization of the DCM around the proper attitude parameters y0, r0 and p0, we have the following model to construct the least-squares attitude estimation (Lu 1995)

$$\begin{bmatrix} A_2 \\ A_3 \\ \cdots \\ A_n \end{bmatrix}\begin{bmatrix} \nabla y \\ \nabla p \\ \nabla r \end{bmatrix} + \begin{bmatrix} [R_0 \;\; -I] & 0 & \cdots & 0 \\ 0 & [R_0 \;\; -I] & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & [R_0 \;\; -I] \end{bmatrix}\left(\begin{bmatrix} l_2 \\ b_2 \\ l_3 \\ b_3 \\ \cdots \\ l_n \\ b_n \end{bmatrix} - \begin{bmatrix} \Delta l_2 \\ \Delta b_2 \\ \Delta l_3 \\ \Delta b_3 \\ \cdots \\ \Delta l_n \\ \Delta b_n \end{bmatrix}\right) = 0 \quad (9)$$

In order to describe the matrix **A**i for i=2, 3, …, n, we express the combined rotation matrix R in terms of row vectors, i.e. **R**=[**r**1 **r**2 **r**3]^T, then the matrix **A**i has the form:

$$A_i = \begin{bmatrix} \frac{\partial(r_1 l_i)}{\partial y} & \frac{\partial(r_1 l_i)}{\partial p} & \frac{\partial(r_1 l_i)}{\partial r} \\ \frac{\partial(r_2 l_i)}{\partial y} & \frac{\partial(r_2 l_i)}{\partial p} & \frac{\partial(r_2 l_i)}{\partial r} \\ \frac{\partial(r_3 l_i)}{\partial y} & \frac{\partial(r_3 l_i)}{\partial p} & \frac{\partial(r_3 l_i)}{\partial r} \end{bmatrix} \quad (10)$$

From equation (9), R_0 is the DCM at y_0, r_0 and p_0; $\Delta\mathbf{b}_i$ and $\Delta\mathbf{l}_i$ are the errors contained in the antenna body frame and the local level frame of the antenna i, respectively; **I** denotes the identity matrix and **O** the zero matrix. Based on this equation, the least-squares adjustment can be carried out. The correction values for the three Euler angles corresponding to a rotation matrix $\mathbf{R}_0$ are computed by;

$$\begin{bmatrix}\Delta y & \Delta p & \Delta r\end{bmatrix}^T = -\left[\sum_{i=2}^{n} A_i^T (R_0^T Cov(l_i) R_0 + Cov(b_i))^{-1} A_i\right]^{-1} \times \left[\sum_{i=2}^{n} A_i^T (R_0^T Cov(l_i) R_0 + Cov(b_i))^{-1} (\Delta l_i - \Delta b_i)\right] \tag{11}$$

Where the short-form notation $Cov(\cdot)$ denotes the error covariance matrix. The least-squares adjustment proceeds until the correction values converge to a certain threshold or the maximal iteration number is reached.

IV. RESULTS

A set of data obtained from a static experiment is provided to demonstrate the performance. The GPS measurements are acquired by using a NovAtel GPS simulator. The GPS simulator will generate the GPS RF signals according to the antenna position in ECEF frame specified by the user. The signals are then transferred to the GPS receiver and the measurements will be output. By setting 3 antenna reference points we can obtain an antenna frame composed of three distributed antennas, and we also know the true baselines. The observation session takes about 10 minutes with 1 Hz data rate. The result below is based on the carrier phase data with resolved phase ambiguities.

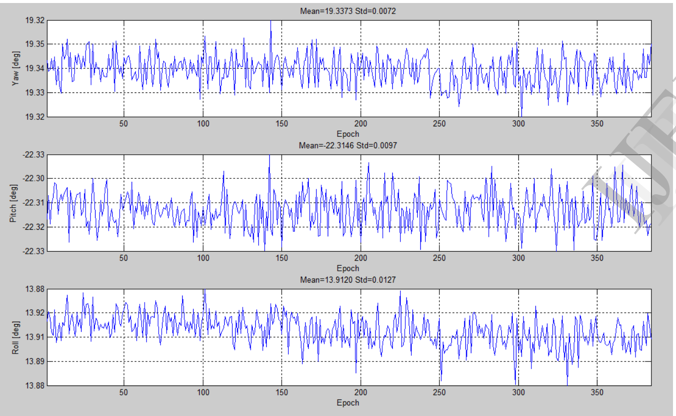

Figure (2) Least Square Attitude Determination

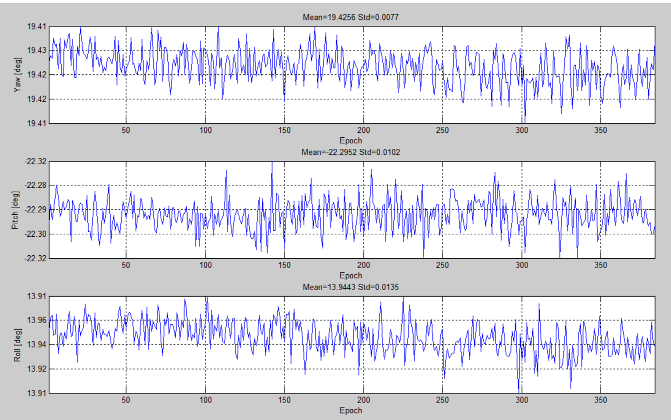

Figure (3) Direct Method

In the results, the X-axis shows the epochs and the Y-axis shows the estimated Euler angles in units of degrees. The title for each subplot also shows the mean value and the standard deviation of the results. Here Figure (2) and Figure (3) is obtained by Least Square Attitude Determination and Direct Attitude Determination. From the standard deviation (std) value, we can get the result that Least Square Attitude Determination is more efficient than Direct Computation Method.

V. CONCLUSION

In comparison with the traditional inertial sensors, the GPS multi-antenna system provides attitude results without drift effects, and it has the advantages due to the cost-effectiveness and the flexible installation. A successful ambiguity resolution of carrier phase measurements enables precise attitude estimation. Using the different observations for carrier phase measurement and using the least square method obtained attitude determination in GPS multi antenna system developing the accuracy of the estimated value of the baseline error from carrier phase data at centimeter level. The use of Least Square Attitude Determination is more efficient than Direct Computation Method, where Standard deviation (std) value obtained is lower in Least Square Attitude Determination than Direct Method as shown in Figure in result.

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