Gradient Descent Optimization Approach for Optimal Synthesis of Path Generator Linkage

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Abstract

A very important task in a design process is how to make a mechanism which will satisfy desired characteristics of motion of a member, i.e. a mechanism in which one part will surely perform desired motion. There are three common requirements in kinematic synthesis of mechanisms: path generation, function generation and motion generation..The gradient descent (also known as steepest descent) optimisation algorithm has been used in the optimisation process. A computer programme based on this algorithm is implemented in MATLAB to obtainthe optimum dimension of four-bar linkage.All this is illustrated on an example of Path synthesis problem having 10 points and 19 variables.

Keyword-optimal synthesis, four bar linkage, objective function,steepest descent optimization algorithm.

1. Introduction

There are various tools are available for the kinematic synthesis of a mechanism like Graphical approach, Matrix method approach, Complex number modelling but these approaches are valid for three to four precision points. So, it is essential to select as many as points on the given trajectory (path) to get the generated coupler curve very close to the desired coupler curve. Steepest descent approaches are applicable for many precision points so that we will get the exact desired coupler curve. Therefore this Approach is selected for path generation problem. Many techniques for the synthesis of linkages are invented in recent years. Most of these approaches are involved techniques and are mathematically complicated. Only few of them allow a closed from solution. Of these, optimization procedures attempting to minimize an objective function play an important role. A set of inequality constraints that limit the range of variation of parameters may be included in the calculation. The new values of linkage parameters are generated with each iteration step according to particular optimization scheme used. The closest achievable fit between the calculated points and

desired points is sought. Even the desired points willnot exactly match but this is considered as acceptable result for most engineering tasks. Each optimization approach has as own advantages and disadvantages in term of convergence accuracy, reliability, complexity and speed. Some methods converge even to a minimum value of objective they may not be the best solution. Based on this points there is a lot of scope for application of new methods of optimization for four-bar synthesis problem

Thispaper explains the basis path synthesis problem in terms of objective function to be minimized and constraints to be satisfied, describes the history and algorithm of steepest descent optimization method adopted in present work which gives the methodology implemented in MATLAB for handling constraints and minimizing the objective function

2. Coupler Point Coordinates

In the problem of four-bar linkage synthesis there is some number of precision points to be traced by the coupler point P. To trace the coupler point, the dimension of the links (a, b, c, d, L_x, L_y) is to be determined along with the input crank angle θ_2, so that the average error between these specified precision points (Px_{di}, Py_{di}), (where i=1, 2,…N with N as number of precision points given) and the actual points to be traced by the coupler point P gets minimized. The objective or error function can be calculated when the actual traced points (Px_i, Py_i) is evaluated which is traced by the coupler point P with respect to the main coordinate from X,Y as shown in Fig.2.1.
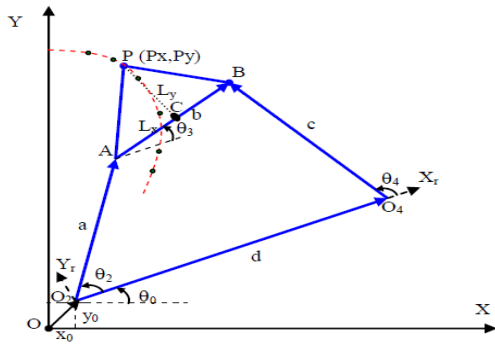
Fig.4.1 Four-bar linkage with ABP as coupler link

The position vector of the coupler point P reference frame X_r-Y_r can be expressed as a vector equation:

$$\vec{r}^{\,P} = \vec{a} + \vec{L}_x + \vec{L}_y \qquad (2.1)$$

whichcan be represented in its components according to:

$$Px_r = a\cos\theta_2 + L_x\cos\theta_3 + L_y(-\sin\theta_3) \qquad (2.2)$$
$$Py_r = a\sin\theta_2 + L_x\sin\theta_3 + L_y\cos\theta_3 \qquad (2.3)$$

Here, for calculation the coupler point coordinates (Px, Py), we have to first compute the coupler link angle θ_3 using the following vector loop equation for the four-bar linkage:

$$\vec{a} + \vec{b} - \vec{c} - \vec{d} = 0 \qquad (2.4)$$

This equation also can be expressed in its components with respect to relative coordinates:

$$a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d \qquad (2.5)$$
$$a\sin\theta_2 + b\sin\theta_3 - c\sin\theta_4 = 0$$

We can compute the angle θ_3 for known values of θ_2 and eliminating θ_4 so, the result will be

$$K_1\cos\theta_3 + K_4\cos\theta_2 + K_5 = \cos(\theta_2-\theta_3) \qquad (2.7)$$

where

$$K_1 = d/a, \quad K_4 = d/b \text{ and } K_5 = \frac{c^2 - d^2 - a^2 - b^2}{2ab} \qquad (2.8)$$

For this equation following two solutions are obtained:$\theta3^1 =$

$$2\tan^{-1}\left(\frac{-E + \sqrt{E^2 - 4DF}}{2D}\right) \qquad (2.9)$$

$\theta3^2$

$$= 2\tan^{-1}\left(\frac{-E - \sqrt{E^2 - 4DF}}{2D}\right) \qquad (2.10)$$

where,

$$D = \cos\theta_2 - K_1 + K_4\cos\theta_2 + K_5, \quad E = -2\sin\theta_2$$

$$\text{and} F = K_1 + (K_4-1)\cos\theta_2 + K_5 \qquad (2.11)$$

These solutions may be (i) real and equal (ii) real and unequal and (iii) complex conjugates. If the discriminates E^2-4DF is negative, then solution is complex conjugate, which simply means that the

link lengths chosen are not capable of connection for the chosen value of the input angle θ_2. This can occur either when the link lengths are completely incapable of connection in any position. Except this there are always two values of θ_3 corresponding to any one value of θ_2. These are called, (i) crossed configuration (plus solution) and (ii) Open configuration of the linkage (minus solution) and also known as the two circuits of the linkage. The other methods such as Newton-Raphson solution technique can also be used to get approximate solution for θ_3. The positionof coupler P, with respect to world coordinate system XOY is finally defined by:

$$Px = x_0 + Px_r\cos\theta_0 - Py_r\sin\theta_0 \qquad (2.12)$$
$$Py = y_0 + Px_r\sin\theta_0 + Py_r\cos\theta_0 \qquad (2.13)$$

3. Position Errors as objective Function

The objective function is usually used to determine the optimal link lengths and the coupler link geometry. In path synthesis problems, this part is the sum squares which computes the position error of the distance between each calculated precision point (Px_i, Py_i) and the desired points (Pxd_i,Pyd_i) which are the target points indicated by the designer. This is written as:

$$f(X) = \sum_{i=1}^{N}[(Pxd_i - Px_i)]^2 + (Pyd_i - Py_i)^2 \quad (2.14)$$

where X is set of variables to be obtained by minimizing this function. Some authors have also considered additional objective functions such as the deviation of minimum and maximum transmission angles μ_{min} and μ_{max} from 90°, for all the set of initial solutions considered.

4. The constraints of the linkages

The synthesis of the four-bar mechanism greatly depends upon the choice of the objective function and the equality or the inequality constraints which is imposed on the solution to get the optimal dimensions. Generally the objective function is minimized under certain conditions so that the solution is satisfied by a set of the given constraints. The bounds for variables considered in the analysis are treated as 20 one set of constraints, while the other constraints include: Grashof condition, input link order constraint and the transmission angle constraint.

I) Grashof criterion

For Grashof criterion, it is required that one of the links of mechanism, should revolve fully by 360° angle. There are three possible Grashof linkages for a four-bar crank chain: (a) Two crank-rocker mechanisms (adjacent link to shortest is fixed) (b) One double crank mechanism (shortest link is fixed) and (c) One double rocker mechanism (opposite to shortest link is fixed). Of all these, in

the present task, only crank-rocker mechanism configuration is considered. Here, the input link of the four-bar mechanism to be crank. Grashof criterion states that the sum (Ls+Ll) of the shortest and the longest links must be lesser than the sum (La+Lb) of the rest two links. That is:

$$(Ls+Ll \leq La+Lb)$$

(or) $2(Ls+Ll) \leq a+b+c+d$

   (or) $g1 = 2(Ls + Ll)/(a + b + c + d) - 1 \leq 0$ (2.15)

In the present work violation is defined as follows:

Grashof's = 1 if $g1 > 0$

Or =0 if $g1 \leq 0$

II) Input link angle order Constraint

Usually a large combination of the mechanisms exists that generates the coupler curves passing through the desired points, but those solutions may not satisfy the desired order. To ensure that the final solution honors the desired order, testing for any order violation is imposed. This is achieved by requiring that the direction of rotation of the crank as defined by the sign of its angular increments $\Delta\theta_2^i = (\theta_2^i - \theta_2^{i-1})$, between the two positions i and i-1, where i=3,4,5….,N, have same direction as that between the 1st and the 2nd positions ($\theta_2^2 - \theta_2^1$). That checks the following:

 Is sign $(\Delta\theta_2^i)$ = sign $(\theta_2^1 - \theta_2^1)$ for all i=3 to N?   (2.16)
where,   sign(Z)=1 if $Z \geq 0$
= -1 if Z<0   (2.16a)If this condition is not satisfied the solution is rejected.

III) Transmission Angle Constraint

For a crank-rocker mechanism generally the best results the designers recognize when the transmission angle is close to 90 degree as much as possible during entire rotation of the crank. Alternatively, the transmission angle during entire rotation of crank should lie between the minimum and maximum values. This can be written as one of the constraints as follows. First of all, the expressions for maximum and minimum transmission angles for crank-rocker linkage are defined.

$$\mu_{max} = 2\cos^{-1}\left(\frac{b^2 - (d+a)^2 + c^2}{2bc}\right)$$
$$\mu_{min} = 2\cos^{-1}\left(\frac{b^2 - (d-a)^2 + c^2}{2bc}\right)$$ (2.17)

The actual value of transmission angle at any crankθ_2^i angle is given by:

$$\mu_{max} = 2\cos^{-1}\left(\frac{b^2 - a^2 - d^2 + c^2 + 2ad \cos\theta_2^i}{2bc}\right)(2.18)$$

The condition to be satisfied is:$\mu_{min} \leq \mu \leq \mu_{max}$(2.19)

The constraint given by equations (2.15), (2.16) and (2.19) are handled by penalty method. That is the non-dimensional constraint deviation is directly added to the objective function for minimization. For example, constraint eq.(2.19) if not satisfied, the penalty term is given as follows:

$$Trans = \sum_{i=0}^{n}(1 - Trans\text{min})(\mu - \mu_{min})^2$$
$$+ (1 - Trans\text{max})(\mu - \mu_{max})^2$$

Where,

Transmin   =   sign $(b^2 + c^2 - (d-a)^2 - 2bc$ $cos\mu_{min})$
Transmax =sign $(2bc \, cos\mu_{max} - b^2 + c^2(a+d)^2)$

Thus the solution seeks to obtain a feasible set of optimum values.

3)   Variable Bounds

All variables considered in the design vector should be defined within pre specified minimum and maximum values. Often, this depends on the type of problem. For example, if we have 19 variables in a 10 point optimization problem, all the variables may have different values of minimum and maximum values. Generally, in non-conventional optimization techniques starting with set of initial vectors, this constraint is handled at the beginning itself, while defining the random variable values. That is we use the following simple generation rule:

X=Xmin +rand (Xmax-Xmin)

Where, rand is a random number generator between 0 and 1.

5.   OVERALL OPTIMIZATION PROBLEM

The objective function is the sum of the error function and the penalties assessed to violation the constraints as follows:

F (k) = f(X) + W1 × Grashof + W2 × Tran , whereas

W1 is the weighting factor of the Grashof's criteria anW2

is the weighting factor of the Transmission angle constraints .these additional terms acts as scaling factors to fix the order of magnitude of the different variables present in the problem or the objective function.

6.   STEEPEST DESCENT METHOD

The method of steepest descent is the simplest of the gradient methods. Imagine that there's a function F(x), which can be defined and differentiable within a given boundary, so the direction it decreases the fastest would be the negative gradient of F(x). To find the local minimum of F(x), The Method of The Steepest Descent is employed, where it uses a zigzag like path from an arbitrary point $X_0$and gradually slide down the gradient, until it converges to the actual point ofminimum.
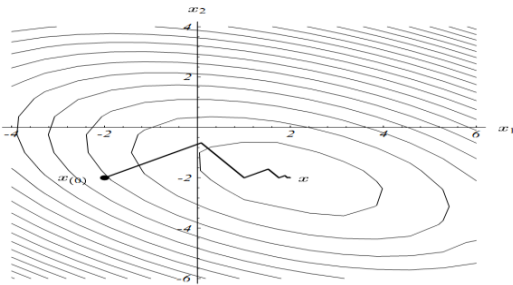

FIG. 4.1 The Method of Steepest Descent finds the local minimum through iterations, as the figure shows, it starts with an arbitrary point x_0 and taking small steps toward the direction of Gradient since it is the direction of fastest changesand stops at the minimum.

It is not hard to see why the method of steepest descent is so popular among many mathematicians: it is very simple, easy to use, and each repetition is fast. But the biggest advantage of this method lies in the fact that it is guaranteed to find the minimum through numerous times of iterations as long as it exists. However, this method also has some big flaws: If it is used on a badly scaled system, it will end up going through an infinite number of iterations before locating the minimum, and since each of steps taken during iterations are extremely small, thus the convergence speed is pretty slow, and this process can literally take forever. Although a larger step size will increase the convergence speed, but it could also result in an estimate with large error.

I) Algorithm

The algorithm is initialized with a guess x_1, a maximum iteration count N_{max}, a gradient norm tolerance ϵ_g that is used to determine whether the algorithm has arrived at a critical point, and a step tolerance ϵ_x to determine whether significant progress is being made. It proceeds as follows.

1   For, $k = 1, 2. . . N_{max}$:

$2x^{k+1} \leftarrow x_k - \alpha_k \nabla f(x^k)$

3.   If $\|\nabla f(x^{k+1})\| < \epsilon_g$ then return "Converged on critical point"123

4.   If $\|x_k - x^{k+1}\| < \epsilon_x$ then return "Converged on an x value"

5.   If $f(x^{k+1}) > f(x^k)$ then return "Diverging"

6.   Return "Maximum number of iterations reached"

The variable α_k is known as the step size, and should be chosen to maintain a balance between convergence speed and avoiding divergence. Note that $\alpha_k$may depend on the step k. Note that the steepest decent direction at each iteration is orthogonal to previous one. Therefore the method zigzags in the design space and is rather inefficient. The algorithm is guaranteed to converge, but it may take an infinite number of iterations. The rate of converge is linear. Usually, a substantial decrease is observed in the few iteration, but the method is very slow. Where,

x^k, x^{k+1} = Values of variable in k and k+1 iteration

f(x)   = Objective function to be minimized

$\nabla f(x) = $ **Gradient of objective function**

α_k = The size of the step in direction of travel
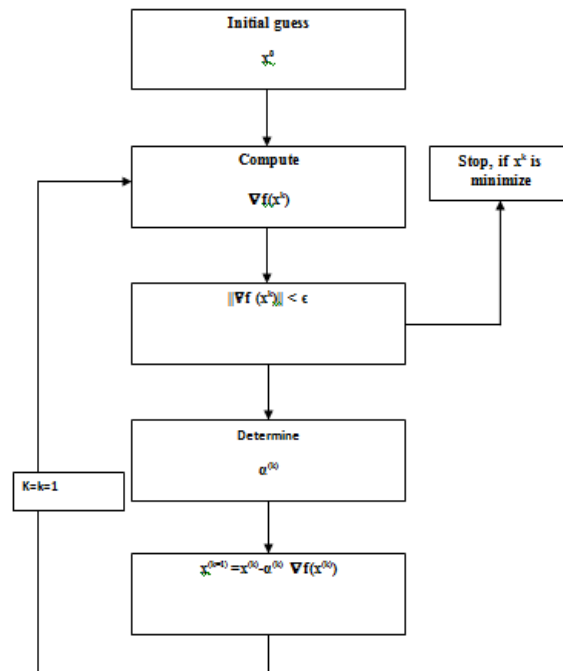
II) FLOW CHART OF STEEPEST DESCENT METHOD:

FIG 5.3 Flow chart of steepest descent method.

7. RESULT AND DISCUSSION

7.1) Path synthesis problem having 10 points and 19 variables

Ten Points Path Generation and 19 design variables:

Design variables are:

$$X=[a,b,c,d,l_y,l_x,,\theta_2^3,\theta_2^4,\theta_2^5,\theta_2^6,\theta_2^7,\theta_2^8,\theta_2^9,\theta_2^{10},\theta_0,$$

$$ly_o, lx_o]$$

Target Points: $C_d^i=$

[(20,10),(17.66,15.142),(11.736,17.878),(5,16.928), (0.60307,12.736),(0.60307,7.2638), (5, 3.0718),(11.736,2.1215),(17.66,4.8577),(20,10)]

Limits of the variable:

$$a, b, c, d, \in [5,60];$$

$$ly_o,lx_o, l_y, l_x, \in [-60,60];$$

$$\theta_2^1,\theta_2^2,\theta_2^3,\theta_2^4,\theta_2^5,\theta_2^6 ... ,\theta_2^{10},\theta_0 \in [0, 2\pi];$$

The synthesized geometric parameters and the corresponding values of the precision points (Pxd, Pyd) and the traced points by the coupler point (Px,Py) and the difference between them are shown in table (7.1) and table (7.2) respectively. Although the constraint of the sequence of the input angles during the evolution is ignored in this case. The accuracy of the solution in case has been remarkably improved using the present method.fig (7.1) shows the convergence of steepest descent algorithm. Fig 7.2 shows ten target points and the coupler curve obtained using the steepest descent search method.

7.2) Optimized values for the ten target point's problem

a	b	c	d	l_x	l_y	θ_2^1	θ_2^2	θ_2^3	θ_2^4
65.4921	78.6161	65.69999	10.1748	36.9961	46.8083	4.3049	4.7145	1.0563	1.704

θ_2^5	θ_2^6	θ_2^7	θ_2^8	θ_2^9	θ_2^{10}	θ_0	lx_0	ly_0
2.2251	2.6147	3.0282	3.4990	3.9342	4.3049	1.2727	9.8114	7.9970

Table 7.1 Optimized values for the ten target point's problem

7.3) Actual points which is traced by the coupler link and the precision points

SL NO	Px	Pxd	(Px – Pxd)	(Px – Pxd)²	% Error	Py	Pyd	(Py – Pxd)	(Py – Pyd)²	% Error
1	20.0775	20.0000	0.0775	0.0060	1.13	10.1632	10.0000	0.1682	0.02666	4.56
2	19.3961	17.6600	1.7361	3.0139	25.33	16.1774	15.1400	0.0374	1.0761	1.01
3	9.8768	11.7360	-1.8592	3.4567	27.13	16.4873	17.8700	-1.3827	1.9118	37.49
4	4.8166	5.0000	-0.1834	0.0336	2.67	16.7262	16.9200	-0.1935	0.0374	5.24
5	1.5592	0.6030	0.9562	0.9144	13.95	12.3071	12.7300	-0.4229	0.1788	11.46
6	1.9250	0.6030	1.3220	1.7478	19.29	7.6251	7.2600	0.3651	0.1333	9.90
7	5.3854	5.0000	0.3854	0.1486	5.62	3.5220	3.0700	0.4520	0.2043	12.25
8	11.7911	11.7360	0.0551	0.0030	0.80	2.2532	2.1200	0.1332	0.0177	3.61
9	17.4602	17.6600	0.1998	0.0399	2.91	5.2196	4.8500	0.3696	0.1366	10.02
10	20.0775	20.0000	0.0775	0.0060	1.13	10.1632	10.0000	0.1632	0.0266	4.42

Table 7.2 Actual points which is traced by the coupler link and the precision points

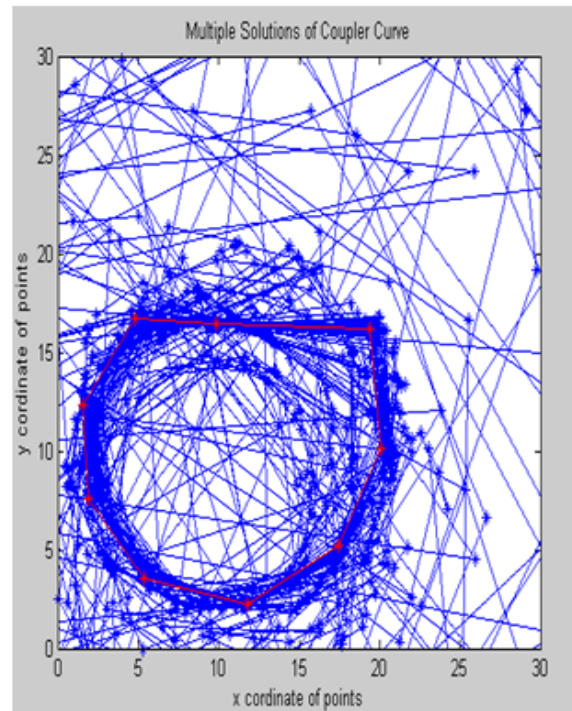### 7.4) Multiple solution of coupler curve

Fig (7.1)multiple solution of coupler curve

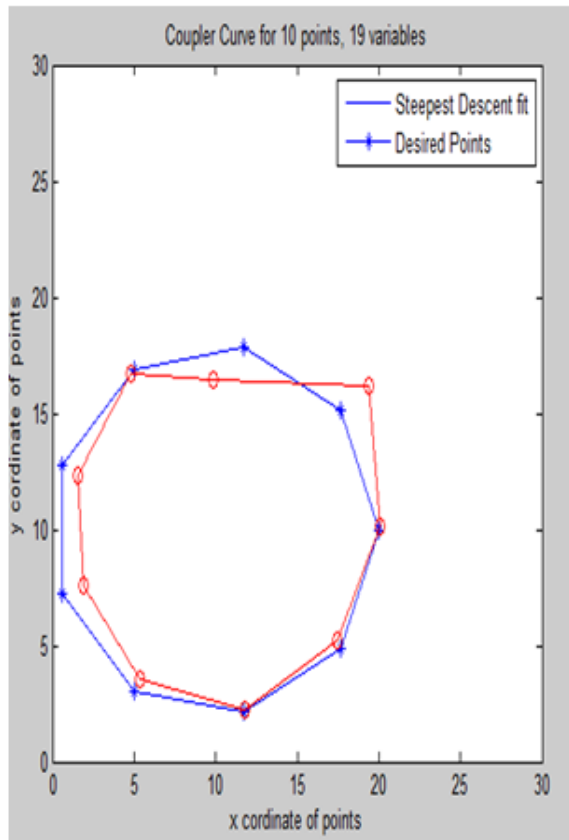### 7.5) Ten target points and the coupler curve obtained

Fig 7.2 (Ten target points and the coupler curve obtained).

7.6) The Matlab program shows the final output as final minimize error for 10 points and 21 variables

X =

Columns 1 through 16

69.7541	83.4937	57.7889	12.8368	29.0996
49.9909	11.0996	10.4460	-0.5152	3.7542
6.7239	7.8453	8.1098	8.7228	10.1396
11.0996				

Columns 17 through 19

34.4372   3.8311   7.9678

f =4.0008e+003

msg

X =

Columns 1 through 16

65.9145	79.0994	64.9069	10.4386	36.2136
47.1237	4.9782	5.2825	0.9006	1.9093
2.6710	3.1331	3.5318	4.0167	4.5492	4.9782

Columns 17 through 19

4.5593   9.2188   7.9941

f =3.3802e+003

msg

X =

Columns 1 through 16

65.6375	78.7892	65.3948	10.2563	36.7092
46.9506	4.1708	4.5727	0.9279	1.5803
2.1124	2.5205	2.9677	3.4441	3.8369	4.1708

Columns 17 through 19

0.4439   9.6930   7.9628

f =

794.5327

msg =

X =

Columns 1 through 16

65.6375	78.7892	65.3948	10.2563	36.7092
46.9506	4.1708	4.5727	0.9279	1.5803
2.1124	2.5205	2.9677	3.4441	3.8369	4.1708

Columns 17 through 19

0.4439   9.6930   7.9628

f =794.5327

msg

Solver stopped prematurely.

fminunc stopped because it exceeded the function evaluation limit,
options.MaxFunEvals = 1900 (the default value).

x =

Columns 1 through 16

65.4921	78.6161	65.6899	10.1748	36.9961
46.8083	4.3049	4.7145	1.0563	1.7064
2.2251	2.6147	3.0282	3.4990	3.9342	4.3049

Columns 17 through 19

1.2727   9.8114   7.9970

fval =13.1193

mymsg =

My final minimised error

final error = 13.1193

8. Conclusion

In this paper we have consider a crank rocker mechanism of four-bar linkage. The objective function namely path error varies with respect to the number of precision points specified. The two different cases were considered. It is found that in each case the computational time for convergence of 10,000 cycles changes. In some examples even the constraint violation is maintained, the minimum value of the objective function is found to be close to the published results available in the literature by other methods. In each case the convergence of multiple iteration graph, coupler curves & tables of optimum dimensions and final coupler point coordinate were reported.

9. Scope of Future work

Even this work has concentrated on path synthesis part with some important constraints, some more constraints like mechanical advantage of the linkage, and flexibility effects can be also considered to get the accuracy. Also as in hybrid synthesis approach, the same linkage may be adopted both for path synthesis applications as well as motion synthesis applications. The objective function should be modified so as to get a different optimum link dimensions. Finally fabrication of the proto-type of this linkage may be done to know the difference between theoretically obtained coupler coordinates and actual values achieved.

These approaches are very useful for the kinematician those who are working in the field of mechanism synthesis, robotics, assembly line, automation material handling and conveyors.

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