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# Graphoidal Covering Number Of Product Graphs

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## Abstract

*A Graphoidal cover of a graph G is a collection $\psi$ of paths in G such that every path in $\psi$ has at least two vertices, every vertex of G is an internal vertex of at most one path in $\psi$ and every edge of G is in exactly one path in $\psi$. The minimum cardinality of a graphoidal cover of G is called the graphoidal covering number of G and is denoted by $\eta(G)$. Also, if every member in graphoidal cover is an open path then it is called an acyclic graphoidal cover. The minimum cardinality of an acyclic graphoidal cover of G is called the acyclic graphoidal covering number of G and is denoted by $\eta_a(G)$ or $\eta_a$. Here we find minimum graphoidal covering number and minimum acyclic graphoidal covering number of Cartesian product, weak product and strong product of some graphs.*



## 1. Introduction

A Graph is a pair G= (V, E) where V is the set of vertices and E is the set of edges. Here we consider only nontrivial, finite, connected and simple Graphs. $|V(G)| = p$ and $|E(G)| = q$.[3]

The concept of graphoidal cover was introduced by B.D Acharaya and E. Sampathkumar [1].

**Definition 1.1 [1] —** A graphoidal cover of a graph G is called a collection $\psi$ of (not necessarily open) paths in G satisfying the following conditions:

(i) Every path in $\psi$ has at least two vertices.

(ii) Every vertex of G is an internal vertex of at most one path in $\psi$.

(iii) Every edge of G is in exactly one path in $\psi$

The minimum cardinality of a graphoidal cover of G is called the graphoidal covering number of G and is denoted by $\eta(G)$.

**Definition 1.2 [2] —** A graphoidal cover $\psi$ of a graph G is called an acyclic graphoidal cover if every member of $\psi$ is an open path. The minimum cardinality of an acyclic graphoidal cover of G is called the acyclic graphoidal covering number of G and is denoted by $\eta_a(G)$ or $\eta_a$.

**Definition 1.3 [6] —** A graphoidal cover $\psi$ of a graph G is called an induced acyclic graphoidal cover if every member of $\psi$ is an induced path. The minimum cardinality of an induced acyclic graphoidal cover of G is called the induced acyclic graphoidal covering number of G and is denoted by $\eta_{ia}(G)$ or $\eta_{ia}$.

**Definition 1.4 [1]** — Let $\psi$ be a collection of internally edge disjoint paths in G. A vertex of G is said to be an internal vertex of $\psi$ if it is an internal vertex of some path in $\psi$, otherwise it is called an external vertex of $\psi$.

**Definition 1.5[7] —** For two graphs G and H, their Cartesian product $G \times H$ has vertex set $V(G)\times V(H)$ in which $(g_1, h_1)$ is joined $(g_2, h_2)$

iff $g_1 = g_2$ and $h_1h_2 \varepsilon E(H)$ or $h_1 = h_2$ and $g_1g_2 \varepsilon E(G)$.

**Definition 1.6[7] —** For two graphs G and H, their Weak product $G \circ H$ has vertex set $V(G)\times V(H)$ in which $(g_1, h_1)$ is joined $(g_2, h_2)$ iff $g_1g_2 \varepsilon E(G)$ and $h_1h_2 \varepsilon E(H)$ .

**Definition 1.7[7]—** For two graphs G and H, their strong product $G \otimes H$ has vertex set $V(G)\times V(H)$ and edge set is $E(G\times H) \cup E(G \circ H)$.

**Theorem 1.8 [1] —** For any graphoidal cover $\psi$ of G , let $t_\psi$ denote the number of exterior vertices of $\psi$ .Let t = min $t_\psi$ where the minimum is taken over all graphoidal covers of G. Then $\eta = q - p + t$ .

**Corollary 1.9 [1] —** For any graph G**,** $\eta \geq q - p$. Moreover the following are equivalent

(i) $\eta = q - p$

(ii) There exists a graphoidal cover without exterior vertices.

(iii) There exists a set of internally disjoint and edge disjoint paths without exterior vertices.

**Theorem 1.10[1] —** For any graph G, $\delta \geq 3$ , $\eta = q - p$ .

## 2. Main Results

**Theorem 2.1[4]—** For $p_m \times p_n$ **,** the acyclic graphoidal covering number is $\eta_a = q - p$ .

**Proof**: Let p = mn and q = m(n-1)+n(m-1)

The acyclic graphoidal cover of $p_m \times p_n$ is as follows:

$$P_1 = g_1h_2, g_1h_1, g_2h_1, g_3h_1, \ldots, g_mh_1, g_mh_2$$

$$P_2 = g_1h_3, g_1h_2, g_2h_2, g_3h_2, \ldots, g_mh_2, g_mh_3$$

$$P_3 = g_1h_4, g_1h_3, g_2h_3, g_3h_3, \ldots, g_mh_3, g_mh_4$$

$$\vdots$$

$$P_{n-1} = g_2h_n, g_1h_n, g_2h_{n-1}, g_3h_{n-1}, \ldots, g_mh_{n-1}, g_mh_n$$

$$P_n = g_2h_{n-1}, g_2h_n, g_3h_n, g_3h_1, \ldots, g_{m-1}h_n, g_{m-1}h_{n-1}$$

$P_{n+1}$ = The remaining edges

From above we see that all the vertices of $p_m \times p_n$ are internal vertices.

Therefore $\eta_a = q - p$

**Theorem 2.2[5] —** For $p_m \circ p_n$ ,the acyclic graphoidal covering number is $\eta_a = q - p + 6$

**Proof**: ***Case (i)***: m is even

$$P_1 = g_1h_1, g_2h_2, g_3h_1, g_4h_2, \ldots, g_{m-1}h_1, g_mh_2, g_{m-1}h_3$$

$$P_2 = g_2h_1, g_1h_2, g_2h_3, g_3h_2, g_4h_3, \ldots, g_{m-1}h_2, g_mh_3, g_{m-1}h_4$$

$$P_3 = g_2h_2, g_1h_3, g_2h_4, g_3h_3, g_4h_4, \ldots, g_{m-1}h_3, g_mh_4, g_{m-1}h_5$$

$$\vdots$$

$$P_{n-1} = g_2h_{n-2}, g_1h_{n-1}, g_2h_n, g_3h_{n-1}, \ldots, g_{m-1}h_{n-1}, g_mh_n$$

$$LP_1 = g_3h_2, g_4h_1, g_5h_2$$

$$LP_2 = g_5h_2, g_6h_1, g_7h_2$$

$$\vdots$$

$$LP_{\frac{(m-3)-1}{2}} = g_{m-3}h_2, g_{m-2}h_1, g_{m-1}h_2$$

$$RP_1 = g_2h_{n-1}, g_3h_n, g_4h_{n-1}$$

$RP_2 = g_4 h_{n-1}, g_5 h_n, g_6 h_{n-1}$

$\vdots$

$RP_{\frac{(m-3)-1}{2}} = g_{m-2} h_{n-1}, g_{m-3} h_n, g_{m-4} h_{n-1}$

S = The remaining edges

From above we see that all the vertices of $p_m \circ p_n$ are internal vertices expect $g_1 h_1, g_2 h_1, g_n h_n, g_1 h_n, g_{m-1} h_n, g_m h_n$ .

Therefore $\eta_a = q - p + 6$

***Case (ii):*** m is odd.

$P_1 = g_1 h_1, g_2 h_2, g_3 h_1, g_4 h_2, \ldots, g_{m-1} h_2, g_m h_1$

$P_2 = g_2 h_1, g_1 h_2, g_2 h_3, g_3 h_2, g_4 h_3, \ldots, g_{m-1} h_3, g_m h_2, g_{m-1} h_1$

$P_3 = g_2 h_2, g_1 h_3, g_2 h_4, g_3 h_3, g_4 h_4, g_5 h_3, \ldots, g_{m-1} h_4, g_m h_3, g_{m-1} h_2$

$\vdots$

$P_{n-1} = g_2 h_{n-2}, g_1 h_{n-1}, g_2 h_n, g_3 h_{n-1}, g_4 h_n \ldots, g_{m-1} h_{n-1}, g_m h_n, g_{m-1} h_{n-2}$

$LP_1 = g_3 h_2, g_4 h_1, g_5 h_2$

$LP_2 = g_5 h_2, g_6 h_1, g_7 h_2$

$\vdots$

$LP_{\frac{(m-4)-1}{2}} = g_{m-2} h_2, g_{m-3} h_1, g_{m-4} h_2$

$RP_1 = g_2 h_{n-1}, g_3 h_n, g_4 h_{n-1}$

$RP_2 = g_4 h_{n-1}, g_5 h_n, g_6 h_{n-1}$

$\vdots$

$RP_{\frac{(m-4)+1}{2}} = g_{m-1} h_{n-1}, g_{m-2} h_n, g_{m-3} h_{n-1}$

S = The remaining edges

From above we see that all the vertices of $p_m \circ p_n$ are internal vertices expect $g_1 h_1, g_2 h_1, g_{m-1} h_1, g_m h_1, g_1 h_n, g_m h_n$ .

Therefore $\eta_a = q - p + 6$

**Theorem 2.3**— For $p_m \otimes p_n$ , the acyclic graphoidal covering number is $\eta_a = 2q - 2p + 6$

**Theorem 2.4**— For $C_m \times p_n$ , the acyclic graphoidal covering number is $\eta_a = q - p$

**Proof**:

The acyclic graphoidal cover of $c_m \times p_n$ is as follows:

$P_1 = g_1 h_2, g_1 h_1, g_2 h_1, g_3 h_1, \ldots, g_m h_1, g_m h_2$

$P_2 = g_1 h_3, g_1 h_2, g_2 h_2, g_3 h_2, \ldots, g_m h_2, g_m h_3$

$P_3 = g_1 h_4, g_1 h_3, g_2 h_3, g_3 h_3, \ldots, g_m h_3, g_m h_4$

$\vdots$

$P_{n-1} = g_2 h_n, g_1 h_n, g_2 h_{n-1}, g_3 h_{n-1}, \ldots, g_m h_{n-1}, g_m h_n$

$P_n = g_2 h_{n-1}, g_2 h_n, g_3 h_n, g_3 h_1, \ldots, g_{m-1} h_n, g_{m-1} h_{n-1}$

$P_{n+1}$ = The remaining edges

From above we see that all the vertices of $p_m \times p_n$ are internal vertices.

Therefore $\eta_a = q - p$

$P_{n+1}$ = The remaining edges

From above we see that all the vertices of $p_m \times p_n$ are internal vertices.

Therefore $\eta_a = q - p$

**Theorem 2.5[6]—** For $p_m \times p_n$ **,** the induced acyclic graphoidal covering number is $\eta_{ia} = q - p$ .

**Proof**: Let p = mn and q = m(n-1)+n(m-1)

The acyclic graphoidal cover of $p_m \times p_n$ is as follows:

$$P_1 = g_1h_2, g_1h_1, g_2h_1, g_3h_1, \ldots, g_mh_1$$

$$P_2 = g_1h_3, g_1h_2, g_2h_2, g_3h_2, \ldots, g_mh_2$$

$$P_3 = g_1h_4, g_1h_3, g_2h_3, g_3h_3, \ldots, g_mh_3$$

$$\vdots$$

$$P_{n-1} = g_1h_n, g_1h_{n-1}, g_2h_{n-1}, g_3h_{n-1}, \ldots, g_mh_{n-1}$$

$$P_n = g_mh_1, g_mh_2, g_mh_3, \ldots, g_mh_n, g_{m-1}h_n, \ldots, g_1h_n$$

S = The remaining edges not covered by $P_1, P_2, P_3, \ldots, P_{n-1}, P_n$

From above we see that all the vertices of $p_m \times p_n$ are internal vertices expect $g_mh_1$ and $g_1h_n$ (t=2)

Therefore $\eta_{ia} = q - p + 2$

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