Heat And Mass Transfer With Chemical Reaction And Exponential Mass Diffusion

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ABSTRACT

An analysis is performed to study heat and mass transfer with chemical reaction and exponential mass diffusion, in the presence of a homogeneous chemical reaction of first order. The dimensionless governing equations are solved using the Laplace transform techniques'. The results were obtained for velocity, temperature and concentration profiles, and computed for physical parameters such as, chemical reaction parameter K, thermal Grashof number Gr, mass Grashof number Gc, Schmidt number Sc, Prandtl number Pr, time t, and acceleration a. It is observed that the velocity increases with increasing values of K, Gr, Gc, a and t, It was also observed that velocity decreases with increasing Pr and Sc respectively.

Key word: mass transfer, chemical reaction, exponential, mass diffusion.

1. INTRODUCTION

Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed system, the reaction is heterogeneous if it place at an inter face, and homogeneous if it takes place in solution. In most chemical reactions the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order if the rate of reaction is directly proportional to concentration. In many chemical engineering processes there is a chemical reaction between a foreign mass and fluid. The processes takes place in numerous industrial applications such as manufacturing of ceramics, food processing and polymer production. Chamber and Young (1958) have analyzed a first order chemical reaction in the neighborhood of a horizontal plate, Gupta et al. (1979) have studied free convective effects flow past accelerated vertical plate in incompressible dissipative fluid, Mass transfer and free convection effects on the flow past an accelerated vertical plate with variable suction or injection, Singh and Kumar (1984) was studied free convection effects on flow past an exponentially accelerated vertical plate, further researchers in this area were done by Jha et al. (1991) analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Das et al. (1994) have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer, Raptis and Massalas (1998) have analyzed magneto hydrodynamic flow past by the presence of radiation. Chamkha and Soundalgekar (2001) have analyzed radiation effects on free convection flow Past a semi-infinite vertical plate with mass transfer, Chaudhary and Jain (2006) analyzed Influence of fluctuating surface temperature and velocity on medium with heat absorption, Muthucumaraswamy et al. (2009) examined the exact Solution of flow past an accelerated infinite vertical plate with heat and mass flux. Muthucumaraswamy and Valliammal (2010) have studied chemical reaction effects on flow past an exponentially accelerated vertical plate with variable temperature.

2. PROBLEM FORMULATION:

Governing equation for heat and mass transfer with chemical reaction parameter and exponential mass diffusion. Then under usual Boussinesq's approximation the unsteady flow equations are presented as momentum equation, energy equation, and mass equation respectively.

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \qquad (1)$$

$$\rho C_\rho \frac{\partial T}{\partial t'} = K\frac{\partial^2 T}{\partial y^2} \qquad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - KC' \qquad (3)$$

The initial and boundary conditions are:

$$\left.\begin{array}{ll} U = 0, T = T_\infty, \quad C' = C'_\infty, & for\ all\ \ y, t' \leq 0 \\ t' > 0 : U = u_0 t', T = T_w, \ C' = C'_\infty + (C'_w - C'_\infty)\, e^{a't'}, & at\ \ y = 0 \\ U \to 0, T \to T_\infty, \ C' \to C'_\infty, & as \quad y \to \infty \end{array}\right\} \qquad (4)$$

$$where \qquad A = \frac{{u_0}^2}{\nu}$$

Where u is the velocity of the fluid, T is the fluid temperature, C' is the concentration, g is gravitational constant, $\beta\, and\, \beta^*$ are the thermal

expansion of fluid, t' is the time, ρ is the fluid density, C_ρ is the specific heat capacity, V is the velocity of the fluid.

The non-dimensional quantities are:

$$\left.\begin{array}{l} U=\dfrac{u}{u_0},\ t=\dfrac{t' u_0^{\ 2}}{\nu},Y=\dfrac{y u_0}{\nu},\ \theta=\dfrac{T-T_\infty}{T_w-T_\infty} \\ \Pr=\dfrac{\mu C_p}{k},\ a=\dfrac{a'\nu}{u_0^{\ 2}},\ Sc=\dfrac{\nu}{D}\ ,K=\dfrac{k\nu}{u_0^{\ 2}}, \\ Gr=\dfrac{g\beta\nu(T_w-T_\infty)}{u_0^{\ 3}},\ C=\dfrac{C'-C'_\infty}{C'_w-C'_\infty}, \\ Gc=\dfrac{\nu g\beta^*(C'_w-C'_\infty)}{u_0^{\ 3}} \end{array}\right\} \qquad (5)$$

Substituting the non-dimensional quantities of (5) in to (1) to (4) leads to dimensionless equations as:

$$\frac{\partial u}{\partial t}=Gr\theta+GcC+\frac{\partial^2 u}{\partial y^2} \qquad (6)$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \qquad (7)$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-KC \qquad (8)$$

Where Sc is the Schmidt number, Pr is Prandtl number, and Gr is thermal Grashof number, Gc is the mass Grashof number, K is the chemical reaction parameter.

The initial and boundary conditions are reduces to:

$$\left.\begin{array}{lllll} & U=0, & \theta=0, & C=0, & \text{for all } y,t\le 0 \\ t>0: & U=t, & \theta=1, & C=e^{at}, & \text{at}\quad y=0 \\ & U\to 0, & \theta\to 0, & C\to 0, & \text{as}\quad y\to\infty \end{array}\right\} \qquad (9)$$

3. METHOD OF SOLUTION

The dimensionless governing equations (6) to (8) with initial boundary conditions are solved using Laplace transform techniques and the results for temperature, concentration and velocity in terms of exponential and complementary error function:

$$L(\theta)=\frac{e^{-y\sqrt{S\Pr}}}{s} \qquad (10)$$

$$L(C)=\frac{e^{-y\sqrt{Sc(s+K)}}}{s-a} \qquad (11)$$

$$L(U)=\frac{e^{-y\sqrt{s}}}{s^2}+\frac{Gr}{s^2(\Pr-1)}\left(e^{-y\sqrt{s}}-e^{-y\sqrt{s\Pr}}\right)$$

$$- \frac{Gc(e^{-y\sqrt{s}} - e^{-y\sqrt{Sc(s+K)}})}{(1-Sc)(a-b)(s-a)} - \frac{Gc(e^{-y\sqrt{s}} - e^{-y\sqrt{Sc(s+K)}})}{(1-Sc)(b-a)(s-b)} \quad (12)$$

The Laplace inversion gives,

$$\theta = erfc\left(\eta\sqrt{\Pr}\right) \quad (13)$$

$$C = \frac{\exp(at)}{2}\left\{\begin{array}{l}\exp\left(2\eta\sqrt{Sc(a+K)t}\right) \\ erfc\left(\eta\sqrt{Sc} + \sqrt{(a+K)t}\right) \\ +\exp\left(-2\eta\sqrt{Sc(a+K)t}\right) \\ erfc\left(\eta\sqrt{Sc} - \sqrt{(a+K)t}\right)\end{array}\right\} \quad (14)$$

$$U = t\left[(1+2\eta^2)\, ercf\,\eta - \frac{2\eta\exp\left(-\eta^2\right)}{\pi}\right]$$

$$+ \frac{Gr\,t}{(\Pr-1)}\left\{\begin{array}{l}\left(1+2\eta^2\right) erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp\left(-\eta^2\right) \\ +\left(1+2\eta^2 \Pr\right) erfc\left(\eta\sqrt{\Pr}\right) \\ \qquad - \frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp\left(-\eta^2 \Pr\right)\end{array}\right\}$$

$$- \frac{Gc\exp(at)}{2(1-Sc)(a-b)}\left\{\begin{array}{l}\exp\left(2\eta\sqrt{at}\right) \\ erfc\left(\eta + \sqrt{at}\right) \\ +\exp\left(-2\eta\sqrt{at}\right) \\ erfc\left(\eta - \sqrt{at}\right)\end{array}\right\}$$

$$+\ \frac{Gc\exp(at)}{2(1-Sc)(a-b)}\left\{\begin{array}{l}\exp\left(2\eta\sqrt{Sc(a+K)t}\right)\\ erfc\left(\eta\sqrt{Sc}+\sqrt{(a+K)t}\right)+\\ \exp\left(-2\eta\sqrt{Sc(a+K)t}\right)\\ erfc\left(\eta\sqrt{Sc}-\sqrt{(a+K)t}\right)\end{array}\right\}$$

$$-\ \frac{Gc\exp(bt)}{2(1-Sc)(b-a)}\left\{\begin{array}{l}\exp\left(2\eta\sqrt{bt}\right)\\ erfc\left(\eta+\sqrt{bt}\right)+\\ \exp\left(-2\eta\sqrt{bt}\right)\\ erfc\left(\eta-\sqrt{bt}\right)\end{array}\right\} \quad (15)$$

$$where \quad b=\frac{ScK}{(1-Sc)}, \quad \eta=\frac{y}{2\sqrt{t}}$$

4. RESULTS AND DISCUSSION

The problem of heat and mass transfer with chemical reaction has been formulated, analyzed and solved analytically, for physical understanding to the problems numerical computations were carried out for different physical parameters such as chemical reaction parameter K, thermal Grashof number Gr, mass Grashof number Gc, Schmidt number Sc, Prandtl number Pr, time t, and acceleration a, upon the nature of flow and transport, the value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor, also the value of Prandtl number Pr are chosen such that they represent air (Pr=0.71). It is observed that the velocity increases with increasing values of K, Gr, Gc, and a.
To access the effects of the various parameters in the flow fields, graphs are presented as follows:

4.1 Velocity profiles

Figures 1 to 5 represent velocity profile for the flow

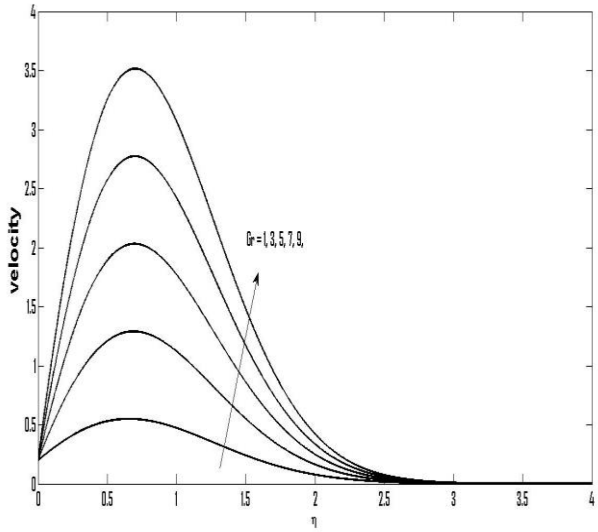

Figure 1 : Velocity profiles for different Gr

The velocity profiles for different values of thermal Grashof number, (Gr=1, 3, 7, 9) is presented in figure 1. It observed that velocity increases with increasing Gr.

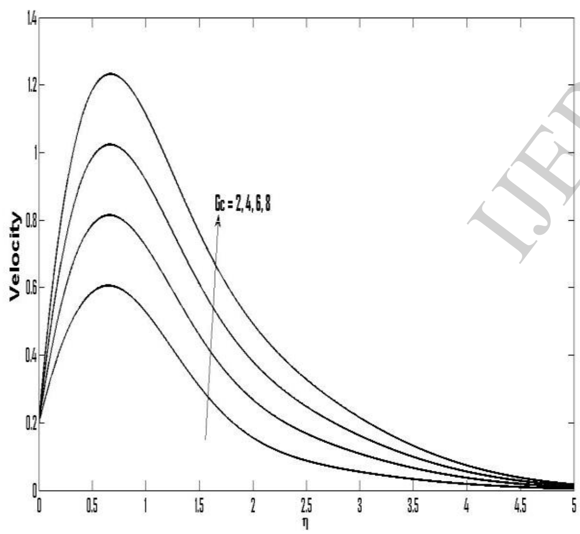

Figure 2 : Velocity profiles for different Gc

The velocity profiles for different values of mass Grashof number (Gc=2, 4, 6, 8) is presented in figure 2. It observed that velocity increases with increasing Gc.

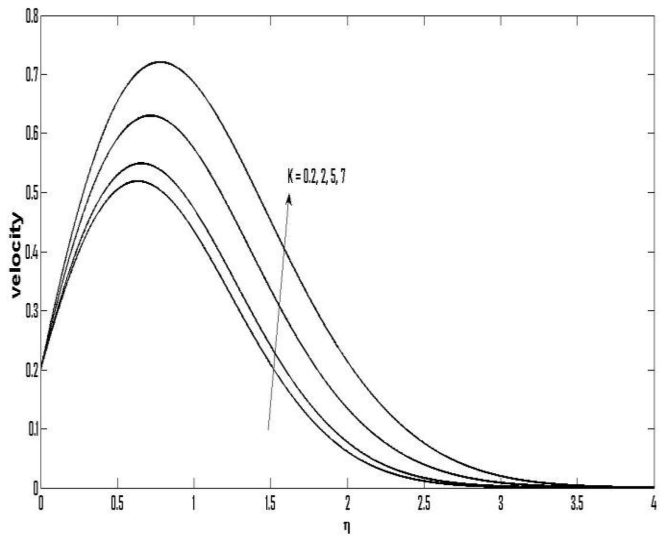

Figure 3 : Velocity profiles for different K

The velocity profiles for different values of chemical reaction parameter (K=0.2, 2, 5, 7) is presented in figure 3. It observed that velocity increases with increasing K.

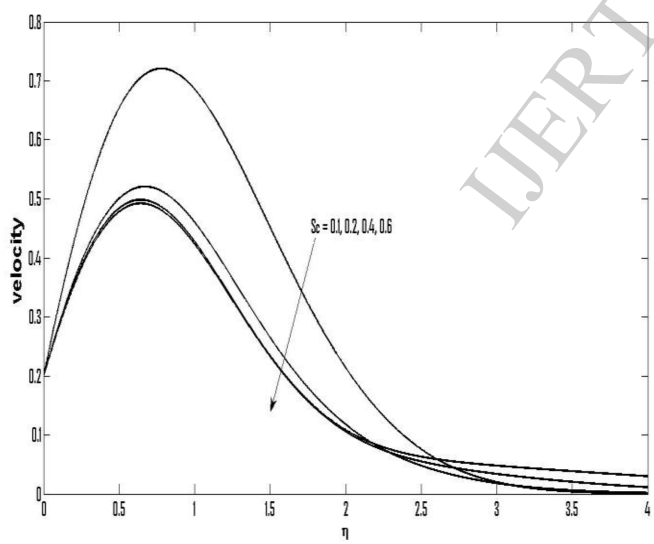

Figure 4 : Velocity profiles for different Sc

The velocity profiles for different values of Schmidt number (Sc= 0.1, 0.2, 0.4, 0.6) is presented in figure 4.

It observed that velocity decreases with increasing Sc.

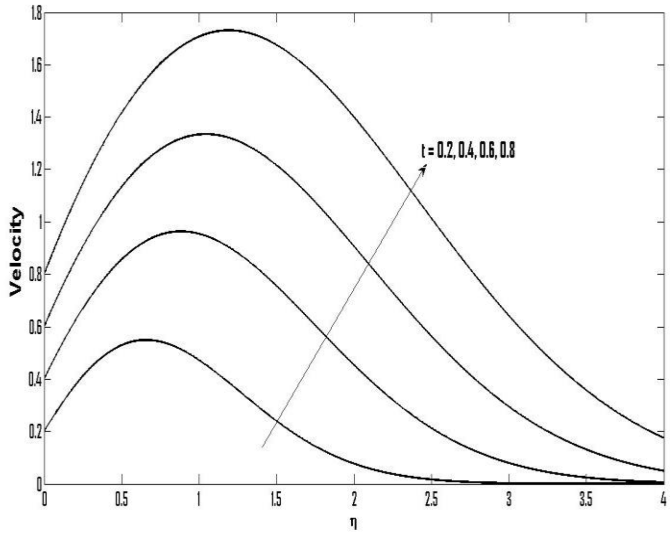

Figure 5: Velocity profiles for different t

The velocity profiles for different values of time (t= 0.2, 0.4, 0.6, 0.8) is presented in figure 5. It observed that velocity increases with increasing t.

4.2 Temperature profiles

Figures 6 and 7 represent temperature profiles for the flow

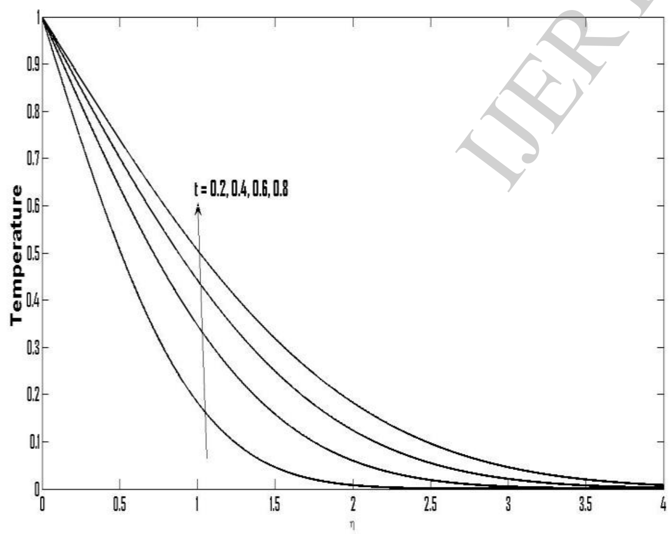

Figure 6: Temperature profiles for different t

The temperature profiles for different values of time (t=0.2, 0.4, 0.6, 0.8) is presented in figure 6. It is observed that temperature increases with increasing t.

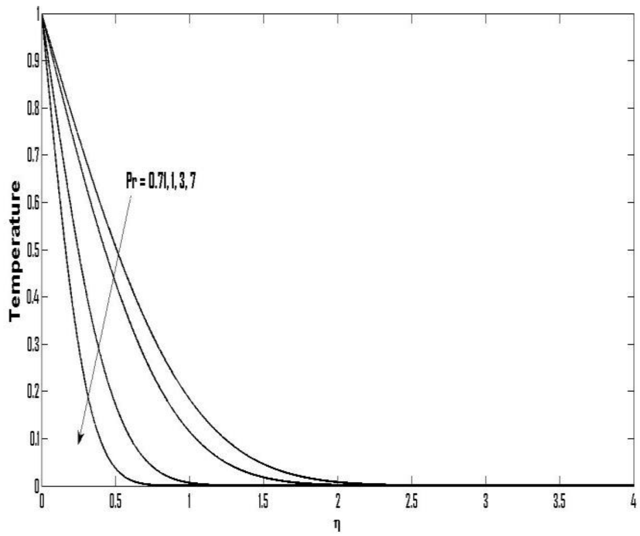

Figure 7 Temperature profiles for different Pr

The temperature profiles for different values of prandtl number (Pr= 0.71, 1, 3, 7) is presented in figure 7. It is observed that temperature decreases with increasing Pr.

4.3 Concentration profiles

Figures 8 and 9 represent concentration profiles for the flow

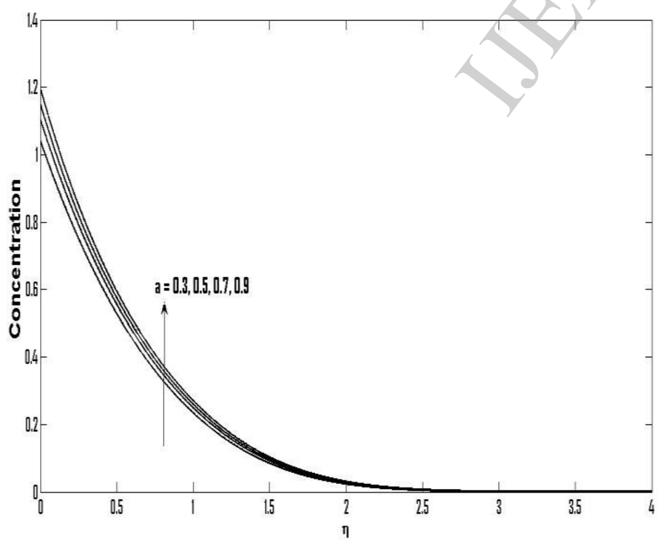

Figure 8: Concentration profiles for different a

The concentration profiles for different values of a (a=0.3, 0.5, 0.7, 0.9) is presented in figure 8. It is observed that concentration increases with increasing a.

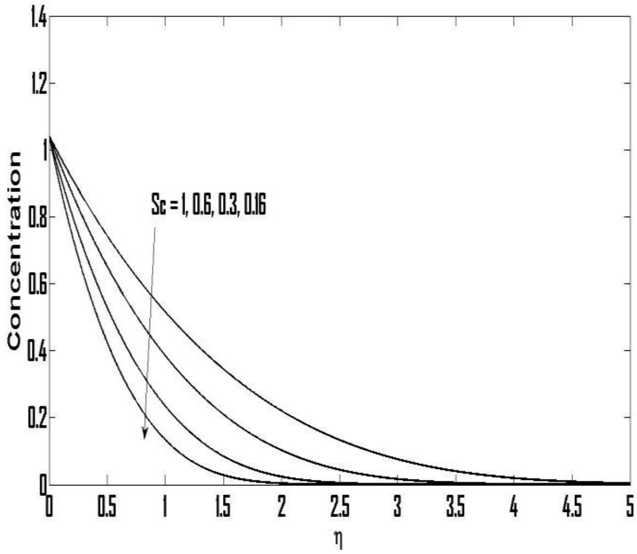

Figure 9: Concentration profiles for different Sc

The concentration profiles for different values of Schmidt number (Sc=1, 0.6, 0.3, 0.16) is presented in figure 9. It is observed that concentration decreases with increasing Sc.

CONCLUSION:

Analytical solutions of heat and mass transfer with chemical reaction and exponential mass diffusion have been studied. The dimensional governing equations are solved by Laplace transform technique. The effect of different parameters such as Chemical reaction parameter, Schmidt number, Prandtl number, mass Grashof number, thermal Grashof number, and time are presented graphically. It is observed that velocity profile increases with increasing parameter k, t, Gc, and Gr and also decreases with increasing Sc and Pr respectively, it is also observed that temperature and concentration profile increases with increasing k, and inversely, decreases as Sc and Pr increases respectively.

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6 ABBREVIATIONS

C' Species concentration in the fluid $kg{:}\,m^{-3}$
C dimensionless concentration
C_p Specific heat at constant pressure $J{:}kg^{-1}{:}K$
D mass diffusion coefficient m^2, s^{-1}
Gc mass Grashof number

Gr thermal Grashof number

g acceleration due to gravity $m\,s^{-2}$

k thermal conductivity *W:* $m^{-1}s^{-1}$

Pr Prandtl number

Sc Schmidt number

T temperature of the fluid near the plate *K*

t' times

t dimensionless time

u velocity of the fluid in the x'-direction $m\,s^{-1}$

u_0 velocity of the plate $m\,s^{-1}$

u dimensionless velocity

y coordinate axis normal to the plate *m*

Y dimensionless coordinate axis normal to the plate

α thermal diffusivity $m^{-2}s^{-1}$

β volumetric coefficient of thermal expansion k^{-1}

β^* volumetric coefficient of expansion with concentration k^{-1}

μ coefficient of viscosity *Ra.s*

kinematic viscosity $m^{-2}s^{-1}$

ρ density of the fluid $kg\,m^{-3}$

T dimensionless skin-friction *kg,* $m^{-1}s^{-2}$

θ dimensionless temperature

η similarity parameter

ercf complementary error function