Heat Transfer in A Coupled-Fluid Flow Over A Highly Porous Medium Layer in the Presence of Heat Source

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Abstract

A viscous fluid flow over a highly porous layer of thickness 'a' is considered. Porous layer is fluid saturated and has a permeable bottom where a transverse sinusoidal suction velocity is applied and the permeable bottom of the porous layer is kept at constant temperature Ta. Since the porous layer is infinite in the x - direction, all physical quantities will be independent of x, however, the flow remains three - dimensional due to the variation of the suction velocity which is applied at the permeable bottom. The governing equations are solved using a perturbation series expansion method. The effects of various flow parameters such as Prandtl number (Pr), suction parameter (λ), Permeability of the porous medium (K), Heat source parameter (S) and viscosity ratio (ϕ1), are investigated on temperature distribution and rate of heat transfer at the porous medium interface, and discussed graphically.

Key words: *Heat transfer, coupled flow, porous medium, permeability, heat source.*

1. Introduction

The Viscous fluid flow through and across porous media is a subject of common interest and has emerged as a separate intensive research area because heat and mass transfer in porous medium is very much prevalent in nature and can also be encountered in many technological processes. It has its applications in a variety of engineering processes such as heat exchanger devices, petroleum reservoirs, chemical catalytic reactors and processes, geothermal and geophysical engineering, moisture migration in a fibrous insulation and nuclear waste disposal and others. Such problems of flow and heat transfer through a wall- bounded porous medium in various types of ducts and channels, have been modeled using some variation of extended Darcy's equation, which describes a balance among pressure gradient, viscous transfer of momentum, linear or/Quadratic drag forces, by several researchers. E.g. Durlofsky and Brady (1987), Kladias and Prasad (1991), Vafai and Kim (1989), Nakayama et al. (1988), Nield at al. (1996), Al-Hadhrami et al. (2002), Kim and Russell (1985), Nield at al. (2004), Hooman et al. (2007) and Chauhan and Kumar (2009). Bejan and Khair (1985) investigated the free convection boundary layer flow in a porous medium owing to combined heat and mass transfer. Lai and Kulacki (1990) used the series expansion method to investigate coupled heat and mass transfer in natural convection from a sphere in a porous medium. The suction and blowing effects on free convection coupled heat and mass transfer over a vertical plate in a saturated porous medium were studied by Raptis et al. (1981) and Lai and Kulacki (1991), respectively.

Free convective flow in presence of heat source has been a subject of interest of many researchers because of its possible application to geophysical sciences, astrophysical sciences, and in cosmical studies. Such flows arise either due to unsteady motion of the boundary or the boundary temperature. Therefore, many researchers have paid their attention towards the fluctuating flow of viscous incompressible fluid past an infinite plate. Singh et al. (2003) have analyzed the heat and mass transfer in MHD flow of viscous fluids past a vertical plate under oscillatory suction velocity. Sharma and Singh (2008) have reported the unsteady MHD-free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. The effect of temperature-dependent heat sources has been studied by Moalem (1976) taking into account the steady state heat transfer within porous medium. Aziz (2009) theoretically examined a similarity solution for a laminar thermal boundary layer over a flat plate with a convective surface boundary condition. Recently, the combined effects of an exponentially decaying internal heat generation and a convective boundary condition on the thermal boundary layer over a flat plate are investigated by Olanrewaju et al. (2012). Similar analysis had been carried out by with heat source Bakr, A. A(2011) neglecting chemical reaction effect. There has been considerable interest in studying the effect of chemical reaction and heat source effect on the boundary layer flow problem with heat and mass transfer of an electrically conducting fluid in different geometry Gangadhar et al. (2011-2012).

$$at\ y=0, u=U, w=W, \left(\tau_{yz}\right)_I=\left(\tau_{yz}\right)_{II}, v=V, p=P, t=T, k\left(\frac{\partial t}{\partial y}\right)=\bar{k}\left(\frac{\partial T}{\partial y}\right),$$

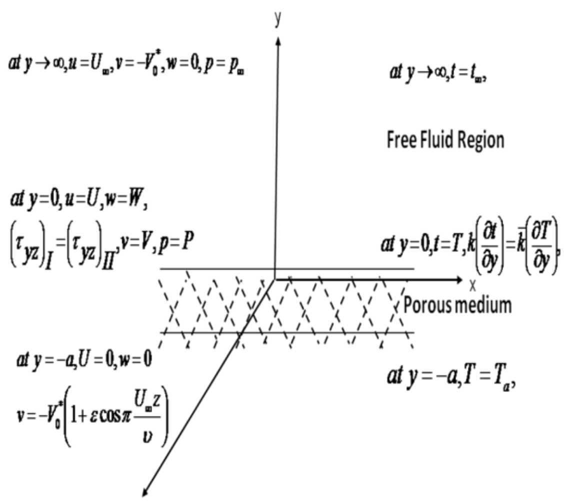

Fig. 1. Schematic diagram.

2. Formulation of the problem

A viscous fluid flow over a highly porous layer of thickness ‘a’ is considered. Porous layer is fluid saturated and has a permeable bottom where a transverse sinusoidal suction velocity is applied and the permeable bottom of the porous layer is kept at constant temperature Ta. The surface of the porous layer is taken horizontal in x-z plane. The x-axis is taken in the direction of the flow, and the y-axis is taken normal to the porous surface directed into the fluid flowing with free stream velocity U∞. Let (u,v,w,t) and (U,V,W,T) are the velocity and temperature components in free fluid and porous regions in the directions x, y, z respectively. Since the porous layer is infinite in the x - direction, all physical quantities will be independent of x, however, the flow remains three - dimensional due to the variation of the suction velocity distribution which is applied at the permeable bottom. This applied suction velocity is consisting of a basic steady distribution with a superimposed weak transversally varying distribution.

The governing equations for the free fluid region-I ($0 \le y < \infty$), are:

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (1)$$

$$\rho\left[v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=\mu\left[\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right] \qquad (2)$$

$$\rho\left[v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right]=-\frac{\partial p}{\partial y}+\mu\left[\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right] \qquad (3)$$

$$\rho\left[v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right]=-\frac{\partial p}{\partial z}+\mu\left[\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right] \qquad (4)$$

$$\rho Cp\left[v\frac{\partial t}{\partial y}+w\frac{\partial t}{\partial z}\right]=k\left[\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}\right]+\mu\left\{\begin{array}{l}2\left[\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]\\+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\end{array}\right\}+Q\left(t-t_\infty\right) \qquad (5)$$

And for porous region-II($-a \le y < o$) are :

$$\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=0 \qquad (6)$$

$$0=\frac{\bar{\mu}}{\rho}\left[\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right]-\frac{\mu}{K\rho}U \qquad (7)$$

$$0=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\bar{\mu}}{\rho}\left[\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}\right]-\frac{\mu}{K\rho}V \qquad (8)$$

$$0=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\frac{\bar{\mu}}{\rho}\left[\frac{\partial^2 W}{\partial y^2}+\frac{\partial^2 W}{\partial z^2}\right]-\frac{\mu}{K\rho}W \qquad (9)$$

$$\rho Cp\left[V\frac{\partial T}{\partial y}+W\frac{\partial T}{\partial z}\right]=\bar{k}\left[\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right]+\mu\left\{\begin{array}{l}2\left[\left(\frac{\partial V}{\partial y}\right)^2+\left(\frac{\partial W}{\partial y}\right)^2\right]\\+\left(\frac{\partial U}{\partial y}\right)^2+\left(\frac{\partial W}{\partial y}+\frac{\partial V}{\partial z}\right)^2+\left(\frac{\partial U}{\partial z}\right)^2\end{array}\right\}+Q\left(T-t_\infty\right) \qquad (10)$$

The boundary conditions are:

$$at\ y=0,\ u=U, w=W, \left(\tau_{yx}\right)_I=\left(\tau_{yx}\right)_{II},$$
$$v=V, p=P, t=T, k\left(\frac{\partial t}{\partial y}\right)=\bar{k}\left(\frac{\partial T}{\partial y}\right)$$

$$at\, y=-a, U=0, v=-V_0^*\left(1+\varepsilon\cos\pi\frac{U_\infty z}{\upsilon}\right),$$
$$w=0, T=T_a,$$

$$y\to\infty, u=U_\infty, v=-V_0^*, w=0, p=p_\infty, t=t_\infty \tag{11}$$

Here, p and P are the pressures in the free and porous regions respectively, ρ, the density; μ, the viscosity; $\bar{\mu}$, the effective viscosity in the porous region; υ, the Kinematic viscosity; K, the permeability of the porous medium; V0*>0, is the mean suction velocity and ε< <1, is the modulation parameter. Cp, k, $\bar{k}$, are the specific heat at constant pressure, thermal conductivity, effective thermal conductivity in porous region and Q is the heat source/sink respectively.

Solution for the flow problem is taken from Chauhan and Sahai [2004]. Thus we have

$$u_0=1+A_1e^{-\lambda y},\ v_0=-\lambda,\ w_0=0, p_0=p_\infty$$

$$U_0=B_1e^{\frac{1}{\sqrt{\varphi_1 K}}y}+B_2e^{-\frac{1}{\sqrt{\varphi_1 K}}y}, V_0=-\lambda,\ W_0=0,$$

$$P_0=\frac{\lambda}{K}y+P_\infty$$

And

$$u_1(y,z)=\left[A_4e^{-r_1y}-\frac{A_1A_2e^{-(\lambda+\pi)y}}{\pi}-\frac{A_1A_3e^{-(\lambda+r_1)y}}{2r_1}\right]\cos\pi z$$

$$v_1(y,z)=\left[A_2e^{-\lambda y}+A_3e^{-r_1y}\right]\cos\pi z$$

$$w_1(y,z)=\left[A_2e^{-\pi y}+\frac{r_1A_3e^{-r_1y}}{\pi}\right]\sin\pi z$$

$$p_1(y,z)=\left[\lambda A_2e^{-\pi y}+\left(r_1\pi^2+\lambda r_1^2-r_1^3\right)\frac{A_3e^{-r_1y}}{\pi^2}\right]\cos\pi z$$

$$U_1(y,z)=\left[B_7e^{-\beta_1y}+a_{40}e^{-\beta_1y}\right]\cos\pi z$$

$$V_1(y,z)=\left[B_3e^{\pi y}+B_4e^{-\pi y}+B_5e^{\beta_1y}-B_6e^{-\beta_1y}\right]\cos\pi z$$

$$W_1(y,z)=-\frac{1}{\pi}\left[\begin{array}{l}B_3\pi e^{\pi y}-B_4\pi e^{-\pi y}\\+B_5\beta_1e^{\beta_1y}-B_6\beta_1e^{-\beta_1y}\end{array}\right]\sin\pi z$$

$$P_1(y,z)=\frac{1}{\pi K}\left[B_4e^{-\pi y}-B_3e^{\pi y}\right]\cos\pi z$$

Making use of the following non-dimensional quantities:

$$\bar{t}=\frac{t-t_\infty}{T_a-t_\infty}, \Pr=\frac{\mu C_p}{k}, Ec=\frac{U_\infty^2}{C_P\left(T_a-T_\infty\right)}$$

$$\lambda=\frac{V_0^*}{U_\infty}, \bar{T}=\frac{T-t_\infty}{T_a-t_\infty}, S=\frac{Qt\upsilon}{\rho C_P U_\infty^2}$$

where λ, suction parameter, Pr, Prandtl number, Ec, Eckert number and S, the heat source parameter.

The non-dimensional energy equations for the free fluid region–I, is

$$v\frac{\partial t}{\partial y}+w\frac{\partial t}{\partial z}=\frac{1}{\Pr}\left[\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}\right]+Ec\left\{\begin{array}{l}2\left[\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]\\+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\end{array}\right\}+St \tag{12}$$

and for the porous region – II is :

$$V\frac{\partial T}{\partial y}+W\frac{\partial T}{\partial z}=\frac{\varphi_2}{\Pr}\left[\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right]+Ec\left\{\begin{array}{l}2\left[\left(\frac{\partial V}{\partial y}\right)^2+\left(\frac{\partial W}{\partial z}\right)^2\right]\\+\left(\frac{\partial U}{\partial y}\right)^2+\left(\frac{\partial W}{\partial y}+\frac{\partial V}{\partial z}\right)^2+\left(\frac{\partial U}{\partial z}\right)^2\end{array}\right\}+ST \qquad (13)$$

The boundary conditions are

$$at\ y=\infty, t=0$$

$$at\ y=0, \mathrm{t}=\mathrm{T}, \frac{\partial t}{\partial y}=\varphi_2\left(\frac{\partial T}{\partial y}\right),$$

$$at\ \ y=-a, T=1, \qquad (14)$$

$$where\,\phi_2=\frac{\overline{\mathrm{k},}}{k}$$

3. Solution of the Problem

The energy equations (12) to equations (13) can be solved by perturbation series method, for very small values of the parameter ε (<<1). We write
$t(y,z)=t_o(y)+\varepsilon\, t_1(y)\cos\pi z,$

$$T(y,z)=T_o(y)+\varepsilon\, T_1(y)\cos\pi z, \qquad (15)$$

Using equations (15) in the equations (12) and (13) and the corresponding boundary conditions, comparing the coefficients of equal power of ε on both sides, and then solving the resulting set of ordinary differential equations under the corresponding boundary conditions, we obtain

$$t(y,z)=B_8e^{a_{46}y}-\frac{Ec\Pr\lambda^2A_1^2e^{-2\lambda y}}{\left(4\lambda^2-2\lambda^2\Pr+S\Pr\right)}+\varepsilon\left[B_{11}e^{a_{49}y}+a_{50}e^{-\left(\lambda+r_1\right)y}+a_{51}e^{-\left(2\lambda+\pi\right)y}+a_{52}e^{-\left(2\lambda+r_1\right)y}\right]\cos\pi z$$

$$T(y,z)=B_9e^{a_{53}y}+B_{10}e^{a_{54}y}+a_{55}e^{\frac{2y}{\sqrt{\varphi_1K}}}+a_{56}e^{-\frac{2y}{\sqrt{\varphi_1K}}}+\frac{2B_1B_2Ec}{\varphi_1KS}$$
$$+\varepsilon[B_{12}e^{a_{57}y}+B_{13}e^{a_{58}y}+a_{59}e^{\left(\beta_1+\frac{1}{\sqrt{\varphi_1K}}\right)y}+a_{60}e^{\left(-\beta_1+\frac{1}{\sqrt{\varphi_1K}}\right)y}$$
$$+a_{61}e^{\left(\beta_1-\frac{1}{\sqrt{\varphi_1K}}\right)y}+a_{62}e^{-\left(\beta_1+\frac{1}{\sqrt{\varphi_1K}}\right)y}+a_{63}e^{\left(\pi+a_{53}\right)y}$$
$$+a_{64}e^{\left(\pi+a_{54}\right)y}+a_{65}e^{\left(\pi+\frac{2}{\sqrt{\varphi_1K}}\right)y}+a_{66}e^{\left(\pi-\frac{2}{\sqrt{\varphi_1K}}\right)y}$$
$$+a_{67}e^{\left(-\pi+a_{53}\right)y}+a_{68}e^{-\left(\pi+a_{54}\right)y}+a_{69}e^{\left(-\pi+\frac{2}{\sqrt{\varphi_1K}}\right)y}$$
$$+a_{70}e^{-\left(\pi+\frac{2}{\sqrt{\varphi_1K}}\right)y}+a_{71}e^{\left(\beta_1+a_{53}\right)y}+a_{72}e^{\left(\beta1+a_{54}\right)y}$$
$$+a_{73}e^{\left(\beta_1+\frac{2}{\sqrt{\varphi_1K}}\right)y}+a_{74}e^{\left(\beta_1-\frac{2}{\sqrt{\varphi_1K}}\right)y}+a_{75}e^{\left(-\beta_1+a_{53}\right)y}$$
$$+a_{76}e^{\left(-\beta_1+a_{54}\right)y}+a_{77}e^{\left(-\beta_1+\frac{2}{\sqrt{\varphi_1K}}\right)y}+a_{78}e^{-\left(\beta_1+\frac{2}{\sqrt{\varphi_1K}}\right)y}]\cos\pi z$$

Where A1, A2, A3, A4, B1, B2, B3, B4, B5, B6, B7, B8, B9, B10, B11, B12, B13 are constants of integrations. These constants have been obtained by the boundary conditions and matching conditions and are reported in the appendix.
The dimensionless rate of heat transfer at the permeable surface is given by

$$\left(\frac{\partial t}{\partial y}\right)_{y=0}=B_8a_{46}+\frac{2Ec\Pr\lambda^3A_1^2}{\left(4\lambda^2-2\lambda^2\Pr+S\Pr\right)}+\varepsilon\left[\begin{array}{l}B_{11}a_{49}-\left(\lambda+r_1\right)a_{50}\\-\left(2\lambda+\pi\right)a_{51}-\left(2\lambda+r_1\right)a_{52}\end{array}\right]\cos\pi z$$

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4. Results and discussions

The variations in the temperature profiles for different values of the permeability K is shown in Figure 2. On comparing the various curves in the figure it is observed that the effect of permeability is to decrease the temperature at all points in the flow field. In fact, the thermal conduction, in flow field of both the regions, is lowered as we increase the value of permeability parameter K and consequently the temperature falls.

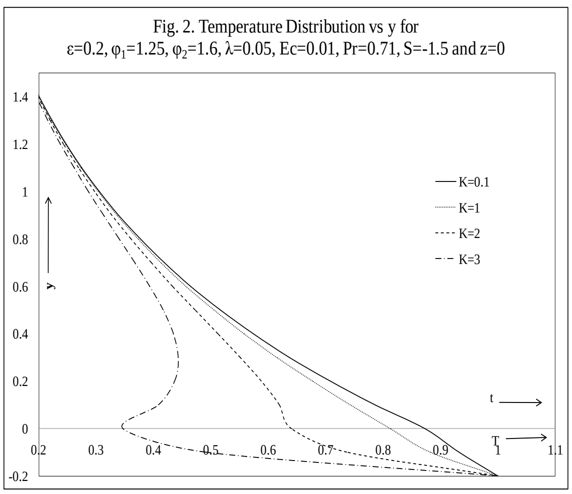


Fig. 2. Temperature Distribution vs y for ε=0.2, φ_1=1.25, φ_2=1.6, λ=0.05, Ec=0.01, Pr=0.71, S=-1.5 and z=0

Figure 3, depicts the temperature distribution for different values of viscosity ratio ϕ_1. It is observed that temperature increases by increasing the viscosity ratio ϕ_1.

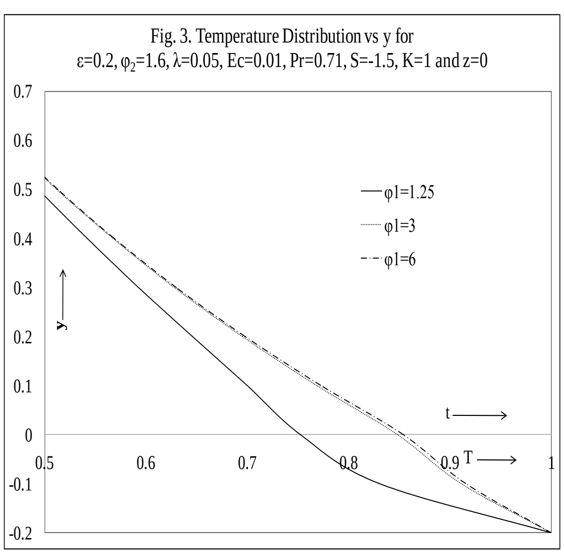


Fig. 3. Temperature Distribution vs y for ε=0.2, φ_2=1.6, λ=0.05, Ec=0.01, Pr=0.71, S=-1.5, K=1 and z=0

Figure 4, shows the effect of suction parameter λ on the temperature distribution. When λ is very small, the temperature profile is nearly linear. Increase in the value of λ causes a decrease in temperature in both regions.

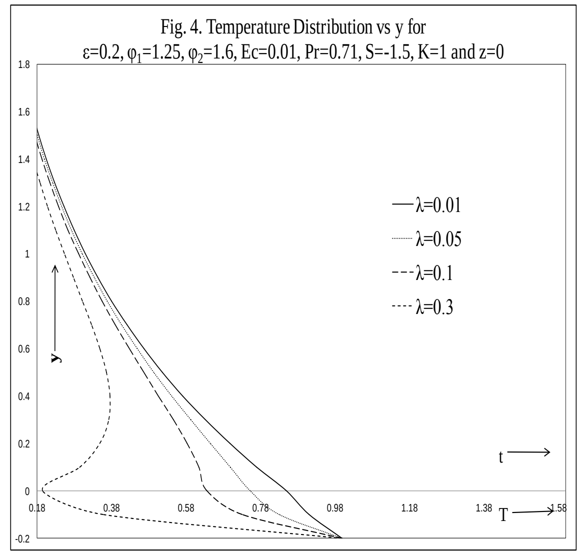


Fig. 4. Temperature Distribution vs y for ε=0.2, φ_1=1.25, φ_2=1.6, Ec=0.01, Pr=0.71, S=-1.5, K=1 and z=0

Figure 5, depicts the temperature distribution for different values of Prandtl number Pr. On comparing various curves in the figure, it is observed that the effect of Prandtl number is to decrease temperature at all points in the region.

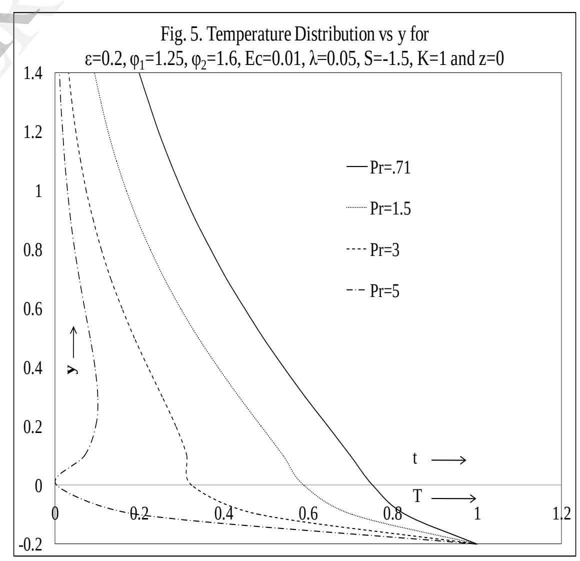
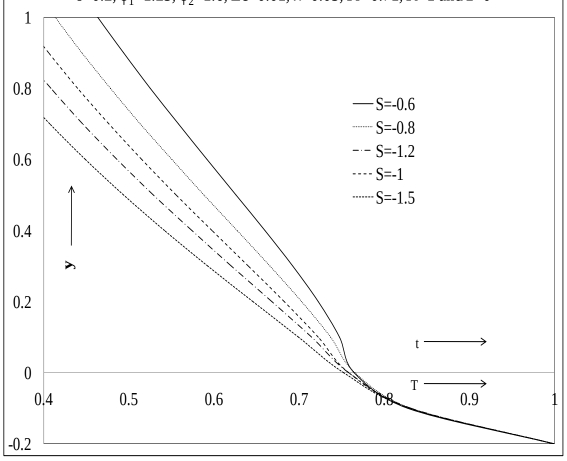


Fig. 5. Temperature Distribution vs y for ε=0.2, φ_1=1.25, φ_2=1.6, Ec=0.01, λ=0.05, S=-1.5, K=1 and z=0

Figure 6, however shows that the source parameter S increases the temperature in the porous and free fluid region at all points.

Fig. 6. Temperature Distribution vs y for
ε=0.2, φ_1=1.25, φ_2=1.6, Ec=0.01, λ=0.05, Pr=0.71, K=1 and z=0



In Figure 7, Rate of heat transfer at the porous interface is plotted against the suction parameter λ for various values of K. It is observed that rate of heat transfer increases with the increase in the suction parameter λ. It also increases with the increase in K.

Fig. 7. Rate of heat transfer vs λ for
ε=0.2, φ_1=1.25, φ_2=1.6, Ec=0.01, S=-1.5, Pr=0.71 and y=0

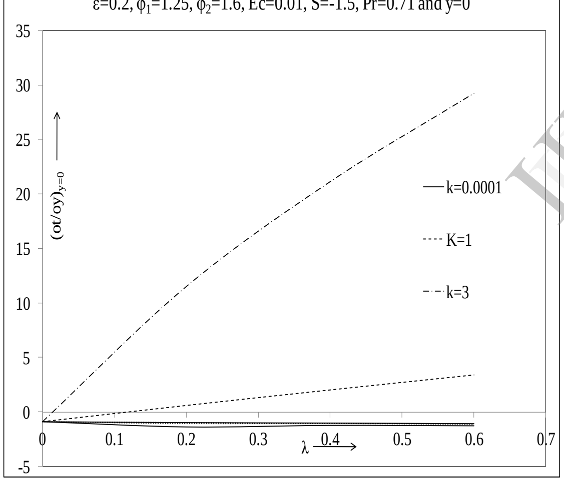
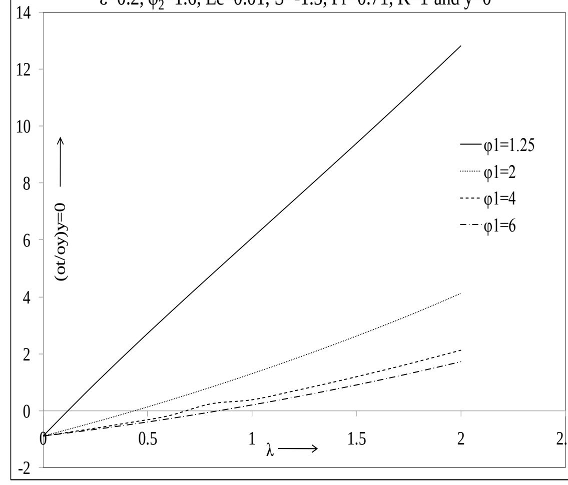


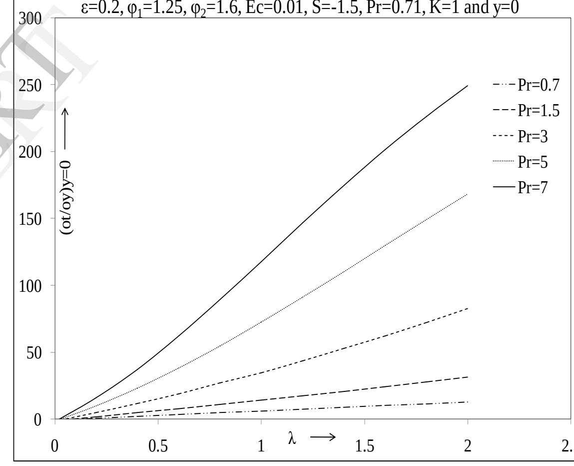
Figure 8, shows variation in rate of heat transfer at porous interface for different values of ϕ_1. The effect of viscosity parameter ϕ_1 is to decrease rate of heat transfer.

Fig. 8. Rate of heat transfer vs λ for
ε=0.2, φ_2=1.6, Ec=0.01, S=-1.5, Pr=0.71, K=1 and y=0



Rate of heat transfer for different values of Pr is shown in Figure 9. It is observed that increase in Prandtl number results in the increase in rate of heat transfer.

Fig. 9. Rate of heat transfer vs λ for
ε=0.2, φ_1=1.25, φ_2=1.6, Ec=0.01, S=-1.5, Pr=0.71, K=1 and y=0



5. Conclusion

Heat transfer characteristics in the three dimensional steady flow of a viscous incompressible fluid over a highly porous layer is investigated in the presence of heat source, when a transverse sinusoidal suction velocity is applied at the permeable bottom of porous medium. Figures 2 to 6 shows variation of temperature distribution for various values of permeability K, suction parameterλ, viscosity ratio ϕ_1, Prandtl number Pr and Heat source parameter S. It is found that temperature decreases in both porous region and free fluid region with the increase in K or λ or Pr, whereas it increases by increasing ϕ_1 or S. In figures 7 to 9, rate of heat transfer at the porous interface is plotted against the suction parameter λ for various values of K, Pr and ϕ_1. It is observed that the magnitude of the rate of heat transfer increases with the increase in K or Pr whereas it decreases with increase in ϕ_1.

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Appendix

$$A_1 = B_1 + B_2 - 1$$

$$A_2 = \left[\frac{B_3(r_1+\pi)+B_4(r_1-\pi)+B_5(r_1+\beta_1)+B_6(r_1-\beta_1)}{(r_1-\pi)}\right]$$

$$A_3 = \left[\frac{2\pi B_3 + B_5(\beta_1+\pi)+B_6(\pi-\beta_1)}{(\pi-r_1)}\right]$$

$$A_4 = B_7 + a_{40} - a_{38}$$

$$B_1 = -B_2 e^{\frac{0.4}{\sqrt{\phi_1 K}}}$$

$$B_2 = \frac{\lambda}{\left[\left(\lambda-\sqrt{\frac{\phi_1}{K}}\right)-\left(\lambda+\sqrt{\frac{\phi_1}{K}}\right)\right]e^{\frac{0.4}{\sqrt{\phi_1 K}}}}$$

$$B_3 = \left[-\lambda - B_4 e^{a_{11}} - B_5 e^{-a_{12}} - B_6 e^{a_{12}}\right]e^{a_{11}}$$

$$B_4 = \frac{[\lambda\pi - a_{22}B_5 - a_{33}B_6]}{a_{21}}$$

$$B_5 = \frac{[a_{34} - a_{31}B_6]}{a_{30}}$$

$$B_6 = \frac{a_{37}}{a_{36}}$$

$$B_7 = -a_{40}e^{0.4\beta_1}$$

$$B_8 = a_{81} + B_9 + B_{10}$$

$$B_9 = \frac{[a_{83} - a_{85}a_{44}]}{[a_{82} - a_{84}a_{44}]}$$

$$B_{10} = \left[a_{79} - B_9 e^{-0.2a_{53}}\right]e^{0.2a_{54}}$$

$$B_{11} = \frac{[a_{90} + a_{89}B_{12}]}{a_{49}}$$

$$B_{12} = \frac{a_{90} - \left[a_{86} + a_{88}e^{-0.2a_{58}}\right]a_{49}}{\left[a_{49}\left(1-e^{-0.2a_{57}}e^{0.2a_{58}}\right)-a_{89}\right]}$$

$$B_{13} = \left[a_{88} - B_{12}e^{-0.2a_{57}}\right]e^{0.2a_{58}}$$

$$\phi_1 = \frac{\bar{\mu}}{\mu}$$

$$r_1 = \frac{\lambda + \sqrt{\lambda^2 + 4\pi^2}}{2}$$

$$\beta_1 = \sqrt{\pi^2 + \frac{1}{K\phi_1}}$$

$$a_{11} = 0.2\pi$$

$$a_{12} = 0.2\beta_1$$

$$a_{13} = 2\pi^2 r_1 - 2r_1^2\pi - 2\phi_1(r_1-\pi)\pi^2$$

$$a_{14} = 2\pi^2(r_1-\pi)(1-\phi_1)$$

$$a_{15} = \pi^2(2r_1+\beta_1-\pi) - r_1^2(\pi+\beta_1) - \phi_1(r_1-\pi)\left(2\pi^2 + \frac{1}{K\phi_1}\right)$$

$$a_{16} = \pi^2(2r_1-\beta_1-\pi) - r_1^2(\pi-\beta_1) - \phi_1(r_1-\pi)\left(2\pi^2 + \frac{1}{K\phi_1}\right)$$

$$a_{17} = \lambda\pi^2(r_1+\pi) + \frac{\pi}{K}(r_1-\pi)$$

$$a_{18} = \left(\lambda\pi^2 - \frac{\pi}{K}\right)(r_1-\pi)$$

$$a_{19} = \lambda\pi^2(r_1+\beta_1)$$

$$a_{20} = \lambda\pi^2 \left(r_1 - \beta_1\right)$$

$$a_{21} = -2\pi e^{a_{11}}$$

$$a_{22} = \left(\beta_1 - \pi\right) e^{-a_{12}}$$

$$a_{23} = -\left(\beta_1 + \pi\right) e^{a_{12}}$$

$$a_{24} = a_{14} e^{-a_{11}} - a_{13} e^{a_{11}}$$

$$a_{25} = a_{15} e^{-a_{11}} - a_{13} e^{-a_{12}}$$

$$a_{26} = a_{16} e^{-a_{11}} - a_{13} e^{a_{12}}$$

$$a_{27} = a_{18} e^{-a_{11}} - a_{17} e^{a_{11}}$$

$$a_{28} = a_{19} e^{-a_{11}} - a_{17} e^{-a_{12}}$$

$$a_{29} = a_{20} e^{-a_{11}} - a_{17} e^{a_{12}}$$

$$a_{30} = a_{21}a_{25} - a_{22}a_{24}$$

$$a_{31} = a_{21}a_{26} - a_{23}a_{24}$$

$$a_{32} = a_{21}a_{28} - a_{22}a_{27}$$

$$a_{33} = a_{21}a_{29} - a_{23}a_{27}$$

$$a_{34} = \lambda\left(a_{13}a_{21} - \pi a_{24}\right)$$

$$a_{35} = \lambda\left(a_{17}a_{21} - \pi a_{27}\right)$$

$$a_{36} = a_{30}a_{33} - a_{31}a_{32}$$

$$a_{37} = a_{30}a_{35} - a_{32}a_{34}$$

$$a_{38} = -\frac{A_1A_2}{\pi} - \frac{A_1A_3}{2r_1}$$

$$a_{39} = \frac{\left(\lambda+\pi\right)A_1A_2}{\pi} + \frac{\left(\lambda+r_1\right)A_1A_3}{2r_1}$$

$$a_{40} = \frac{\left(a_{39} + r_1 a_{38}\right)}{\left[\left(r_1 - \beta_1\right) - \left(r_1 + \beta_1\right)e^{0.4\beta_1}\right]}$$

$$a_{41} = -\frac{Ec\Pr B_1^2}{\phi_1\phi_2 K\left(\frac{4}{\phi_1 K} - \frac{2\lambda\Pr}{\phi_2\sqrt{\phi_1 K}}\right)}$$

$$a_{42} = -\frac{Ec\Pr B_2^2}{\phi_1\phi_2 K\left(\frac{4}{\phi_1 K} - \frac{2\lambda\Pr}{\phi_2\sqrt{\phi_1 K}}\right)}$$

$$a_{43} = \frac{2EcB_1B_2}{\phi_1 K\lambda}$$

$$a_{44} = -\frac{Ec\Pr A_1^2}{2\left(2-\Pr\right)}$$

$$a_{45} = \left(\frac{-\lambda\Pr + \sqrt{\lambda^2\Pr^2 - 4S\Pr}}{2}\right)$$

$$a_{46} = \left(\frac{-\lambda\Pr - \sqrt{\lambda^2\Pr^2 - 4S\Pr}}{2}\right)$$

$$a_{47} = \frac{2\lambda^3 Ec\Pr A_1^2}{\left(4\lambda^2 - 2\lambda^2\Pr + S\Pr\right)}$$

$$a_{48} = \left(\frac{-\lambda\Pr + \sqrt{\lambda^2\Pr^2 - 4\left(S\Pr - \pi^2\right)}}{2}\right)$$

$$a_{49} = \left(\frac{-\lambda\Pr - \sqrt{\lambda^2\Pr^2 - 4\left(S\Pr - \pi^2\right)}}{2}\right)$$

$$a_{50}=-\frac{2\lambda A_1 A_4 Ec\Pr r_1}{\left[\left(\lambda+r_1\right)^2-\lambda\Pr\left(\lambda+r_1\right)+\left(S\Pr-\pi^2\right)\right]}$$

$$a_{51}=\frac{2\lambda A_1^2 A_2 Ec\Pr\left(\lambda+\pi\right)}{\left[\pi\left(2\lambda+\pi\right)^2-\lambda\Pr\left(2\lambda+\pi\right)+\left(S\Pr-\pi^2\right)\right]}$$

$$a_{52}=\frac{\lambda A_1^2 A_3 Ec\Pr\left(\lambda+r_1\right)}{r_1\left[\left(2\lambda+r_1\right)^2-\lambda\Pr\left(2\lambda+r_1\right)+\left(S\Pr-\pi^2\right)\right]}$$

$$a_{53}=\left(\frac{-\lambda\Pr+\sqrt{\lambda^2\Pr^2-4S\Pr\phi_2}}{2\phi_2}\right)$$

$$a_{54}=\left(\frac{-\lambda\Pr-\sqrt{\lambda^2\Pr^2-4S\Pr\phi_2}}{2\phi_2}\right)$$

$$a_{55}=-\frac{Ec\Pr B_1^2}{\phi_1\phi_2 K\left(\frac{4}{\phi_1 K}+\frac{2\lambda\Pr}{\phi_2\sqrt{\phi_1 K}}+\frac{S\Pr}{\phi_2}\right)}$$

$$a_{56}=-\frac{Ec\Pr B_2^2}{\phi_1\phi_2 K\left(\frac{4}{\phi_1 K}-\frac{2\lambda\Pr}{\phi_2\sqrt{\phi_1 K}}+\frac{S\Pr}{\phi_2}\right)}$$

$$a_{57}=\left(\frac{-\lambda\Pr+\sqrt{\lambda^2\Pr^2+4\phi_2\left(\phi_2\pi^2-S\Pr\right)}}{2}\right)$$

$$a_{58}=\left(\frac{-\lambda\Pr-\sqrt{\lambda^2\Pr^2+4\phi_2\left(\phi_2\pi^2-S\Pr\right)}}{2}\right)$$

$$a_{59}=-\frac{2B_1B_2\beta_1 Ec\Pr}{\phi_2\sqrt{\phi_1 K}\left[\left(\beta_1+\frac{1}{\sqrt{\phi_1 K}}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(\beta_1+\frac{1}{\sqrt{\phi_1 K}}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{60}=\frac{2B_1 a_{40}\beta_1 Ec\Pr}{\phi_2\sqrt{\phi_1 K}\left[\left(-\beta_1+\frac{1}{\sqrt{\phi_1 K}}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(-\beta_1+\frac{1}{\sqrt{\phi_1 K}}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{61}=\frac{2B_2B_7\beta_1 Ec\Pr}{\phi_2\sqrt{\phi_1 K}\left[\left(\beta_1-\frac{1}{\sqrt{\phi_1 K}}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(\beta_1-\frac{1}{\sqrt{\phi_1 K}}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{62}=-\frac{2B_2 a_{40}\beta_1 Ec\Pr}{\phi_2\sqrt{\phi_1 K}\left[\left(\beta_1+\frac{1}{\sqrt{\phi_1 K}}\right)^2-\frac{\lambda\Pr}{\phi_2}\left(\beta_1+\frac{1}{\sqrt{\phi_1 K}}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{63}=\frac{B_3B_9 a_{53}\Pr}{\phi_2\left[\left(\pi+a_{53}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(\pi+a_{53}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{64}=\frac{B_3B_{10} a_{54}}{\phi_2\left[\left(\pi+a_{54}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(\pi+a_{54}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{65}=\frac{2B_3 a_{55}\Pr}{\phi_2\sqrt{\phi_1 K}\left[\left(\pi+\frac{2}{\sqrt{\phi_1 K}}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(\pi+\frac{1}{\sqrt{\phi_1 K}}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{66}=-\frac{2B_3 a_{55}\Pr}{\phi_2\sqrt{\phi_1 K}\left[\left(\pi-\frac{2}{\sqrt{\phi_1 K}}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(\pi-\frac{1}{\sqrt{\phi_1 K}}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{67}=\frac{B_4B_9 a_{53}\Pr}{\phi_2\left[\left(-\pi+a_{53}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(-\pi+a_{53}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{68}=\frac{B_4B_{10} a_{54}\Pr}{\phi_2\left[\left(-\pi+a_{54}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(-\pi+a_{54}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{69}=\frac{2B_4 a_{55}\Pr}{\phi_2\sqrt{\phi_1 K}\left[\left(-\pi+\frac{2}{\sqrt{\phi_1 K}}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(-\pi+\frac{2}{\sqrt{\phi_1 K}}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{70}=-\frac{2B_4 a_{56}\Pr}{\phi_2\sqrt{\phi_1 K}\left[\left(\pi+\frac{2}{\sqrt{\phi_1 K}}\right)^2-\frac{\lambda\Pr}{\phi_2}\left(\pi+\frac{2}{\sqrt{\phi_1 K}}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{71}=\frac{B_5 B_9 a_{53}\Pr}{\phi_2\left[\left(\beta_1+a_{53}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(\beta_1+a_{53}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{72}=\frac{B_5 B_{10} a_{54}\Pr}{\phi_2\left[\left(\beta_1+a_{54}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(\beta_1+a_{54}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{73}=\frac{2B_5 a_{55}\Pr}{\phi_2\sqrt{\phi_1 K}\left[\left(\beta_1+\frac{2}{\sqrt{\phi_1 K}}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(\beta_1+\frac{2}{\sqrt{\phi_1 K}}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{74}=-\frac{2B_5 a_{56}\Pr}{\phi_2\sqrt{\phi_1 K}\left[\left(\beta_1-\frac{2}{\sqrt{\phi_1 K}}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(\beta_1-\frac{2}{\sqrt{\phi_1 K}}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{75}=\frac{B_6 B_9 a_{53}\Pr}{\phi_2\left[\left(-\beta_1+a_{53}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(-\beta_1+a_{53}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{76}=\frac{B_6 B_{10} a_{54}\Pr}{\phi_2\left[\left(-\beta_1+a_{54}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(-\beta_1+a_{54}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{77}=\frac{2B_6 a_{56}\Pr}{\phi_2\sqrt{\phi_1 K}\left[\left(-\beta_1+\frac{2}{\sqrt{\phi_1 K}}\right)^2+\frac{\lambda\Pr}{\phi_2}\left(-\beta_1+\frac{2}{\sqrt{\phi_1 K}}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{78}=-\frac{2B_6 a_{56}\Pr}{\phi_2\sqrt{\phi_1 K}\left[\left(\beta_1+\frac{2}{\sqrt{\phi_1 K}}\right)^2-\frac{\lambda\Pr}{\phi_2}\left(\beta_1+\frac{2}{\sqrt{\phi_1 K}}\right)-\pi^2+\frac{S\Pr}{\phi_2}\right]}$$

$$a_{79}=1-a_{55}e^{-\frac{0.4}{\sqrt{\phi_1 K}}}-a_{56}e^{-\frac{0.4}{\sqrt{\phi_1 K}}}-\frac{2B_1 B_2 Ec}{\phi_1 KS}$$

$$a_{80}=-\frac{2\lambda^3 Ec\Pr A_1^2}{\left(4\lambda^2-2\lambda^2\Pr+S\Pr\right)}+\frac{2\phi_2 a_{55}}{\sqrt{\phi_1 K}}-\frac{2\phi_2 a_{56}}{\sqrt{\phi_1 K}}$$

$$a_{81}=\frac{\lambda^2 Ec\Pr A_1^2}{\left(4\lambda^2-2\lambda^2\Pr+S\Pr\right)}+a_{55}+a_{56}+\frac{2B_1 B_2 Ec}{\phi_1 KS}$$

$$a_{83}=a_{80}+\phi_2 a_{54}a_{79}e^{0.2a_{54}}$$

$$a_{84}=e^{0.2\left(a_{54}-a_{53}\right)}-1$$

$$a_{85}=a_{81}+a_{79}e^{0.2a_{54}}$$

$$a_{86}=\left(a_{59}+a_{60}+\ldots+a_{78}\right)-\left(a_{50}+a_{51}+a_{52}\right)$$

$$a_{87}=a_{50}\left(\lambda+r_1\right)+a_{51}\left(2\lambda+\pi\right)+a_{52}\left(2\lambda+r_1\right)+\phi_2\left[\left(a_{59}+a_{60}+\ldots+a_{80}\right)\left(\beta_1+\frac{2}{\sqrt{\phi_1 K}}\right)\right]$$

$$a_{88}=-\left[a_{59}e^{-0.2\left(\beta_1+\frac{1}{\sqrt{\phi_1 K}}\right)}+a_{60}e^{-0.2\left(-\beta_1+\frac{1}{\sqrt{\phi_1 K}}\right)}+a_{61}e^{-0.2\left(\beta_1-\frac{1}{\sqrt{\phi_1 K}}\right)}+a_{62}e^{0.2\left(\beta_1+\frac{1}{\sqrt{\phi_1 K}}\right)}+a_{78}e^{0.2\left(\beta_1+\frac{2}{\sqrt{\phi_1 K}}\right)}\right]$$

$$a_{89}=\phi_2\left(a_{57}-e^{0.2\left(a_{58}-a_{57}\right)}a_{58}\right)$$

$$a_{90}=\left[a_{87}+\phi_2 a_{58}a_{88}e^{0.2a_{58}}\right]$$