Heat Transfer on Flow Past a Linearly Vertical Accelerated Plate With Constant Temperature and Variable Mass Diffusion

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Abstract-An analysis of flow past a linearly accelerated infinite vertical plate is offered in the presence of variable mass diffusion with constant temperature. The temperature of the plate is raised to Tw and species concentration level closer to the plate rises linearly with respect to the time. The non-dimensional governing equations are solved by Laplace-transform methods. The effects of concentration , temperature and velocity are calculated for different parameters like thermal Schmidt number , Prandtl number , Grashof number, mass Grashof number and time. It shows that the velocity increases due to increasing values of thermal Grashof or mass Grashof number. It is also observed that the velocity increases with decreasing values of the Schmidt number.

Keywords: accelerated, constant temperature, isothermal, vertical plate, heat transfer, mass diffusion.

I. INTRODUCTION

The effects of heat and mass transfer plays an important role are spacecraft design, solar energy collectors, filtration processes, nuclear reactors the drying of porous materials in textile industries and the saturation of porous materials by chemicals, design of chemical processing equipment and pollution of the environment. Effects of mass transfer on flow past a uniformly accelerated vertical plate was studied by Soundalgekar[1]. The above problem

was extended to include heat and mass transfer effects subjected to variable suction or injection by Kafousias and Raptis[2]. It is proposed to study the effects of on flow past a linearly accelerated isothermal infinite vertical plate in the presence of variable mass diffusion with constant temperature. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions in terms of error complementary function and also exponential form.

II. MATHEMATICAL FORMULATION

The flow of a incompressible fluid past a linearly vertical accelerated infinite plate with constant temperature and variable mass diffusion has been considered. The ***x*** -axis is taken in the vertical direction along the plate and the *y*-axis is considered in horizondal direction . At time $t' \le 0$, the plate and fluid are at the same temperature T_∞. The plate is linearly accelerated at time $t' \le 0$, with a velocity $u = u_0 t'$ in its own plane and the temperature of the plate is raised to T_w and the mass is diffused to the fluid from the plate with respect to time *t*. Then by Boussinesq's approximation the governing equations of unsteady flow are as follows:

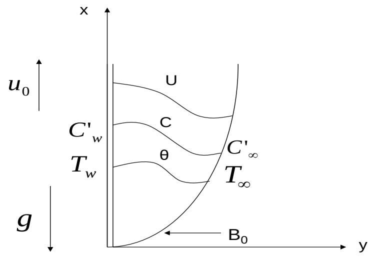

Figure: Physical model of the problem

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$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2}$$
$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} \qquad (1)$$
$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2}$$

The initial and boundary conditions are as follows:

$$\begin{array}{llllll} & u = 0, & T = T_\infty, & C' = C'_\infty & forall & y, t' \le 0 \\ t' > 0: & u = u_0 t', & T = T_w, & C' = C'_\infty + (C'_w - C'_\infty)\,A\,t' & at & y = 0 \\ & u \to 0 & T \to T_\infty, & C' \to C'_\infty & as & y \to \infty \end{array} \qquad (2)$$

where $A = \left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}}$

The dimensionless quantities are defined as:

$$U=\frac{u}{(\nu u_0)^{\frac{1}{3}}},\ t=t'\left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}},\ Y=y\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}},$$

$$Pr=\frac{\mu C_p}{k},\ Sc=\frac{\nu}{D}$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\ Gr=\frac{g\beta(T_w-T_\infty)}{u_0}, \qquad (3)$$

$$Gc=\frac{g\beta^*(C'_w-C'_\infty)}{u_0},\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty}$$

in equations (1) to (3) leads to

$$\frac{\partial U}{\partial t} = Gr\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2}$$
$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2}$$
$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \qquad (4)$$

The initial and boundary conditions in non-dimensional form are

$$\begin{aligned} & U=0,\ \theta=0,\ C=0 \quad \text{for all } Y, t\le 0 \\ t>0:\ & U=t,\ \theta=a,\ C=t \quad \text{at } Y=0 \\ & U\to 0,\ \theta\to 0,\ C\to 0 \quad \text{as } Y\to\infty \end{aligned} \qquad (5)$$

III. METHOD OF SOLUTION

The non-dimensional governing equations (4) subject to the conditions (5) are solved by Laplace method and we get the following solutions

$$\theta = a\, erfc(\eta\sqrt{Pr}) \qquad (6)$$

$$C=t\left[(1+2\eta^2 Sc)\, erfc(\eta\sqrt{Sc})-\frac{2\eta}{\sqrt{\pi}}\sqrt{Sc}\exp(-\eta^2 Sc)\right] \qquad (7)$$

$$\begin{aligned} U=\ & t\left[(1+2\eta^2)\, erfc(\eta)-\frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] \\ & +\frac{Grt}{Pr-1}\left[(1+2\eta^2)\, erfc(\eta)-\frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)-(1+2\eta^2 Pr) erfc(\eta\sqrt{Pr})+\frac{2\eta\sqrt{Pr}}{\sqrt{\pi}}\exp(-\eta^2 Pr)\right] \\ & +\frac{Gc\,t^2}{6(Sc-1)}\left[\begin{aligned}&(3+12\eta^2+4\eta^4) erfc(\eta)-\frac{\eta}{\sqrt{\pi}}(10+4\eta^2)\exp(-\eta^2)-(3+12\eta^2 sc+4\eta^4(sc)^2) erfc(\eta\sqrt{sc})\\ &+\frac{\eta\sqrt{sc}}{\sqrt{\pi}}(10+4\eta^2 sc)\exp(-\eta^2 sc)\end{aligned}\right] \end{aligned} \qquad (8)$$

where, $\eta=\dfrac{Y}{2\sqrt{t}}$

IV. RESULTS

In sequence to get a physical problem, numerical computations are carried out for different parameters *Gr*, *Gc*, *Sc*, *Pr* and *t* upon the nature of the flow and transport. The value of the Schmidt number *Sc* is taken to be 0.6 which corresponds to water-vapor. Also, the values of Prandtl number *Pr* are chosen such that they represent air (Pr =0.71) and water (Pr =7.0). The values of the concentration ,velocity and temperature are calculated for different parameters like Prandtl number, Schmidt number ,thermal Grashof(Gr), mass Grashof number(Gc) and time. The velocity profiles for different $Sc=0.6, Pr=0.71$ ($t=0.2, 0.4, 0.6), Gr=Gc=5$ at t = 0.2 are studied and presented in fig 1. It is observed that the velocity increases with increasing values of *t*. Fig 2. demonstrates the effects of different thermal Grashof number (*Gr* =2,5) and mass Grashof number (*Gc*=2,5) on the velocity at time *t* = 0.2. It was observed that the velocity increases with increasing of the thermal Grashof or mass Grashof number.

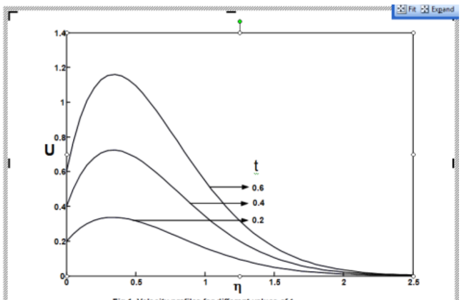

Fig 1. Velocity profiles for different values of t.

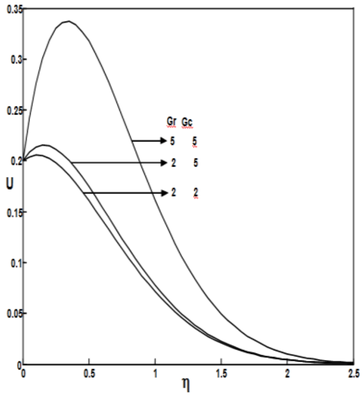

Fig 2. Velocity profiles for different values of Gr and Gc

Fig 3 represents the result of concentration at time *t*=0.2 for varies Schmidt number (*Sc*=0.16, 0.3,0.5,2.01). The result of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It was observed that the concentration increases with decreasing values of the Schmidt number.

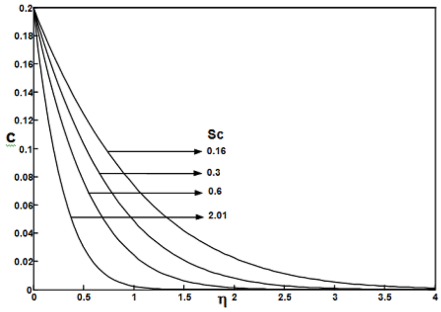

Fig 3. Concentration profiles for different values of Sc.

The result of velocity for varies values of the Schmidt number (*Sc*=0.16, 0.3,0.6,2.01) ,*Gr*=*Gc*=5 and time *t*=0.2 are shown in fig 4. The trend shows that the velocity increases with decreasing Schmidt number . It was observed that the variation of the velocity with the magnitude of the Schmidt number.

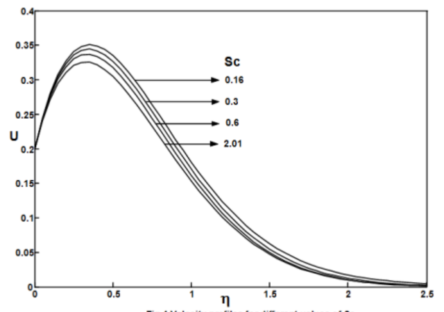

Fig 4 Velocity profiles for different values of Sc.

The temperature profiles are calculated for water and air from Equation (6) and these are shown in Fig 5. at time t = 0.2. The result of the Prandtl number plays an important role in temperature field. It was observed that the temperature increases with decreasing Prandtl number. This shows that the heat transfer is more in air than in water.

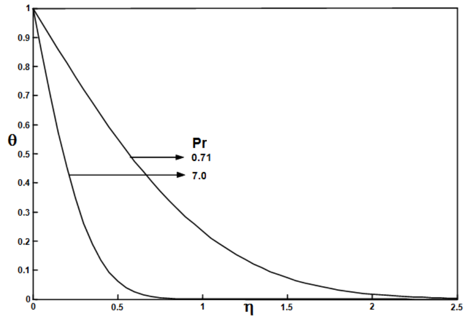

Fig 5 Temperature profiles for different values of Pr.

A	constant	Sc	Schmidt number
B_0	external magnetic field	Sh	Sherwood number
C'	species concentration in the fluid	T_∞	temperature of the fluid far away from the plate
C'_w	concentration of the plate	T_w	temperature of the plate
C'_∞	concentration of the fluid far away from the plate	T'_w	temperature on the wall
C	dimensionless concentration	T, T'	temperature of the fluid near the plate
C_p	specific heat at constant pressure	t'	time
D	mass diffusion coefficient	t	dimensionless time
Gc	mass Grashof number	u	velocity of the fluid in the x-direction
Gr	thermal Grashof number		
g	accelerated due to gravity		
j''	mass flux per unit area at the plate	u_0	velocity of the plate
k	thermal conductivity	U	dimensionless velocity
M	magnetic field parameter	x	spatial coordinate along the plate
Nu	Nusselt number	y	coordinate axis normal to the plate
Pr	Prandtl number	Y	dimensionless coordinate axis normal to the plate
q	heat flux per unit area at the plate		

Subscripts

ω	conditions at the wall
∞	conditions in the free stream

V. CONCLUSIONS

The effects of Heat and Mass transfer on flow past a linearly accelerated infinite vertical plate in the presence of variable mass diffusion have been studied. The non-dimensional governing equations are solved by the usual Laplace-method. The result of concentration ,velocity and temperature and for various parameters like thermal Grashof (Gr), mass Grashof number(Gc), Schmidt number(Sc), Prandtl number(Pr) and time (t) are studied graphically. The study concludes that the velocity increases with increasing values of thermal Grashof number(Gr), mass Grashof number(Gc) and time (t). But the velocity increases with decreasing Schmidt number (Sc). The wall concentration increases with decreasing values of the Schmidt number.

NOMENCLATURE

Greek symbols

α	thermal diffusivity	ρ	density of the fluid
β	volumetric coefficient of thermal expansion	τ	dimensionless skin-friction
$\beta*$	volumetric coefficient of expansion with concentration	θ	dimensionless temperature
μ	cofficient of viscosity	η	similarity parameter
σ	electric conductivity	erfc	complementary error function
υ	kinematic viscosity		

VI. REFERENCES

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