Hybrid Beam-Forming Algorithms for Planar Adaptive Antenna Array

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Abstract—Adaptive smart antenna arrays are used for cellular communication. This paper presents a comparative study of beamforming techniques for adaptive smart antenna using hybrid algorithms like Simple Matrix Inversion with Recursive Least Square (SMI-RLS) and Least Mean Square with Recursive Least Square (LMS-RLS) algorithm. The results are compared on the basis of null depth and error plot for different signal –to-noise (SNR) values, ranging from high to low values of SNR for heavily faded case.

Index Terms—Adaptive antenna; beamforming; LMS algorithm; RLS algorithm; SMI algorithm

I. INTRODUCTION

Smart antenna system consists of several antenna elements, whose signal is processed adaptively in order to exploit the spatial domain of the mobile radio channel. The smart antenna technology can significantly improve wireless system performance. It improves wireless system performance by increasing signal quality, network capacity and coverage area. Smart antenna system can automatically change the directionality of its radiation patterns to its signal environment. Smart antennas are antenna arrays with smart signal processing algorithms to identify spatial signal signature such as the Direction of Arrival (DoA) of the signal and use it to calculate beam forming vector, to track and locate the antenna beam on the mobile targets. By producing only radiation beam along the direction of arrival (DoA) of signal, appreciable power saving can be achieved using smart antenna [1-2]. Smart antenna techniques are used in acoustic signal processing, track and scan RADAR, radio astronomy and radio telescopes and mostly in cellular systems.

Signals are processed adaptively in order to exploit the spatial domain of the mobile radio channel. Usually the signals received at the different antennas are multiplied with complex weights and then adaptively weights are summed up. Basically, there are two types of smart antennas, viz., switched beam smart antenna and adaptive smart antenna. In switched beam smart antenna, antenna system has several fixed beam patterns and according to detected condition most appropriate beam is used for communication. Whereas, in adaptive smart antenna, beam can be steered in any direction according to DoA estimation and at the same time null can be generated in the direction of the interferer (Fig. 1). Smart antenna estimates direction of arrival of incoming signals and the direction of interfering signals. Then using beamforming algorithm, antenna beam is generated toward the desired

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direction and null is generated toward the direction of interferer. There are various types of algorithms for beamforming, having their advantages and disadvantages [3-8].



Fig. 1. Main beam toward desired user and null toward interferer

In the proposed work hybrid algorithms are used in an adaptive array system of 8×8 antenna elements and the algorithms are compared with respect to their convergence speed, stability and with different SNR. It is observed that for noisy environment performance of LMS-RLS algorithm is better as compared to SMI-RLS algorithm.

II. HYBRID BEAMFORMING ALGORITHMS

LMS algorithm is a stochastic and a steepest descent method, where iterative procedure is used making successive corrections to the weight vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error. Here LMS Algorithm is used for a uniform linear array with N isotropic elements, which forms the integral part of the adaptive beamforming system. Adaptive algorithm is used to minimize the error e(n)between desired signal d(n) and array output y(n), as e(n) = d(n) - y(n) (1)

Output of adaptive beamformer, at time n, is given by a linear combination of the data at the N antenna elements, can be expressed as:

$$y(n) = w^H x(n) \tag{2}$$

$$w = [w_1, w_2 \dots, w_N]^H$$
(3)

Where, H denotes Hermitian (complex conjugate) transpose. The weight vector w is a complex vectors. Signal received by multiple antenna elements is

$$x(n) = [x_1(n), x_2(n), \dots, x_N(n)$$
(4)

LMS algorithm updates the weight vectors according to the following equation [6, 9-10]

$$w(n+1) = w(n) + \mu x(n)e^{*}(n)$$
(5)

Where, μ is the step size parameter and e(n) is the error between output and the desired signal.

Recursive least square (RLS) algorithm is a deterministic algorithm, where weight vectors are updated recursively according to the equation [11]

 $\overline{w}(k) = \overline{w}(k-1) + \overline{g}(k)[d^*(k) - \overline{x}^H(k)\overline{w}(k-1)] \quad (6)$ Where, $\overline{g}(k) = \widehat{R}_{xx}^{-1}(k)\overline{x}(k)$ is gain vector, \widehat{R}_{xx} is correlation matrix and given by $\widehat{R}_{xx}(k) = \propto \widehat{R}_{xx}(k-1) + \overline{x}(k)\overline{x}^H(k) \quad (7)$

and ' α ' is forgetting factor or exponential weighting factor. ' α ' is a positive constant, $0 \le \alpha \le 1$.

Sample matrix inversion algorithm is discontinuous adaptive algorithms. Sample matrix is a Time average estimate of the array correlation matrix using k-time samples. It's used in discontinuous transmission, however it requires the number of interferers and their positions remain constant during the duration of the block acquisition. SMI has faster conversion rate since it employs direct inversion of the covariance matrix. One of the drawbacks of the LMS adaptive scheme is that the algorithm must go through many iterations before satisfactory convergence is achieved. SMI algorithm uses a block adaptive approach which would give a better performance than a continuous approach. The SMI has a faster convergence rate because it exploits the direct inversion of the covariance matrix. Sample matrix is a time average estimate of the array correlation matrix using K-time samples. The sample matrix is defined as the time average estimate of the array correlation, which uses N samples, and if the random process is ergodic in correlation, then time average estimate is equal to the real correlation matrix [12-14]:

$$R_{xx} \approx \frac{1}{N} \sum_{n=1}^{N} x(n) x^{H}(n)$$
(8)

$$\mathbf{r} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{d}^{*}(n) \mathbf{x}(n)$$
(9)

The matrix $x_N(n)$ is defined as the n-th block of vectors x ranges over N -data snapshots:

$$x_{N}(n) = \begin{bmatrix} x_{1}(1+nN)x_{1}(2+nN) & \cdots & x_{1}(N+nN) \\ \vdots & \ddots & \vdots \\ x_{N}(1+nN)x_{N}(2+nN) & \cdots & x_{N}(N+nN) \end{bmatrix}$$
(10)

where, n represents the block number, and N is the block length. So, R_{xx} can be given by

$$R_{xx}(n) = \frac{1}{N} x_N(n) x_N^H(n)$$
(11)

If the desired signal is

$$d(n) = [d(1 + nK)d(2 + nK)d(3 + nK) \dots \dots d(N + nK) \quad (12)$$

 $r = \frac{1}{N}d^*(n)x_N(n)$

Then,

(13)

The sample matrix inversion weights of the n th block can be computed as

$$w_{SMI}(n) = R_{xx}^{-1}(n)r(n) = [x_N(n)x_N^H(n)]^{-1}d^*(n)x_N(n)$$
(14)

Flow chart for SMI-RLS algorithm is shown in Fig.2.



Fig. 2. Flow chart of hybrid algorithm used for beamforming

Similar procedure is followed for LMS-RLS algorithm. Both the algorithms are applied for two dimensional weights updating for two dimensional planar array.

III. ADAPTIVE BEAM-FORMING USING HYBRID ALGORITHMS

For planar antenna array (Fig. 3) inter-element spacing is $d_x=d_y=d'$. Antennas are excited with progressive phase shift of ' α '. The array factor for M×N element planar antenna array is given by [12, 14]

$$AF = \sum_{m=1}^{M} \sum_{n=1}^{N} w_{MN} e^{j \left[(m-1)(\frac{2\pi d_x}{\lambda} \sin\theta \cos\varphi + (n-1)(\frac{2\pi d_y}{\lambda} \sin\theta \sin\varphi) \right]}$$
(15)



Fig. 3. Planar antenna array

Where to generate the main beam at wavelength λ toward the desired beam direction θ_0 from the broadside direction, the progressive phase shift is

$$\alpha = \frac{-2\pi d}{\lambda} \cos\theta_0 \tag{16}$$

Normalized array factor is $AF_{norm} = \frac{AF}{AF_{max}}$ (17)

In Eq. (17), AF_{max} is the maximum value of array factor AF, given by Eq. (15).

Hybrid algorithms are programmed for 8×8 antenna elements and Eq. (15) are used as cost function in the computation. Element spacing d = 0.5 λ , number of elements in the uniformed planar array, 8x8, desired direction for main beam 30⁰, desired direction for null 60⁰, block length (K) is 1000. Beams are generated using different hybrid algorithms and the results are plotted in Fig. 4-Fig. 7. In Fig. 4 and Fig. 5, SNR=20dB and SNR=10dB which are the cases of antenna beam generation under good signal condition. In Fig. 6 and Fig. 7, SNR=0dB and SNR=-5dB respectively, which are the cases of antenna beam generation under poor signal condition.



Fig. 4. Array factor for 8x8 elements planar array using hybrid algorithms for K=1000, step size=0.02, forgetting factor=0.9 with SNR=20dB

LMS-RLS algorithm produces deepest null as compared to SMI-RLS. Similar beam patterns are obtained for different forgetting factor using SMI-RLS algorithm, but null depth increases with decreasing block length.



Fig. 5. Array factor for 8x8 elements planar array using hybrid algorithms for K=1000, step size=0.02, forgetting factor=0.9 with SNR=10dB



Fig. 6. Array factor for 8x8 elements planar array using hybrid algorithms for K=1000, step size=0.02, forgetting factor=0.9 with SNR=0dB



Fig. 7. Array factor for 8x8 elements planar array using hybrid algorithms for K=1000, step size=0.002, forgetting factor=0.9 with SNR = - 5dB

Square error graphs for SMI-RLS and LMS-RLS algorithm, applied to planar array, are shown in Fig. 8 and Fig. 9 respectively. These are presented for low SNR of 0dB.



Fig. 8. Mean square error plot for 8x8 elements linear array using SMI-RLS algorithm for μ =0.02 and α =0.9 with SNR=0dB



Fig. 9. Mean square error plot for 8x8 planar array using LMS-RLS algorithm for K=1000 and α =0.9 with SNR=0dB

The convergence speed is similar for both the hybridalgorithms. But LMS-RLS algorithm exhibits low square error. The parameters, like, maximum side lobe level (SLL_{max}), first-null beamwidth (FNBW), direction of user and direction of interferer of SMI-RLS and LMS-RLS algorithms in adaptive beam forming are compared in Table I. Here, the desired direction of main beam is 30° and desired direction of null is 60° .

 TABLE I.
 PERFORMANCES OF SMI-RLS AND LMS-RLS

 ALGORITHMS IN ADAPTIVE BEAMFORMING

Parameters	Algorithm	SNR 20dB	SNR 10dB	SNR 0dB	SNR -5dB
SLLmax	SMI-RLS	-26dB	-25dB	-25dB	-24dB
	LMS-RLS	-27dB	-28dB	-27dB	-20dB
FNBW	SMI-RLS	47^{0}	44^{0}	44^{0}	41 ⁰
	LMS-RLS	45 ⁰	44 ⁰	47 ⁰	43 ⁰
Direction of	SMI-RLS	30^{0}	30^{0}	30^{0}	30 ⁰
Main Beam	LMS-RLS	30^{0}	30^{0}	30^{0}	30^{0}
Direction of	SMI-RLS	60^{0}	60^{0}	57^{0}	51 ⁰
Null	LMS-RLS	60^{0}	60°	55 ⁰	53°

IV.CONCLUSION

In noisy environment with low SNR, performance of beam generation using LMS-RLS algorithm is better as compared to SMI-RLS algorithm. The error increases with the decrease in SNR value for both the algorithms. But square error is less for LMS-RLS algorithm compared to SMI-RLS algorithm. As mentioned in Table I, the direction of null deviates from desired direction of 60^o and side lobe level goes up as SNR decreases.

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