International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 06, June-2016

Identification of Damages in Skeletal Structures using Modal Data

Abhishek Kumar Sahu
M.Tech Scholar
Department of Civil Engineering
National Institute of Technology
Agartala, India

Dr. Surajit Das
Assistant Professor
Department of Civil Engineering
National Institute of Technology
Agartala,India

***Abstract*— In the present work the effect of transverse cracks on natural frequencies of a simply supported beam with symmetric overhangs is investigated and an algorithm has been developed for identifying damage in the same. In the numerical example, single and triple cracks are considered in the dynamic analysis. Flexibility matrix of the intact beam and an additional flexibility matrix due to damaged beam is derived and added up to obtain the flexibility matrix of the cracked beam element. Stiffness matrix of cracked beam element is derived by multiplying a transformation matrix with the inverse of the final flexibility matrix of the cracked beam element. The natural frequencies and the corresponding mode shapes of vibration are obtained by solving eigen value problem. It is found that for a simply supported beam with symmetric overhangs, the 1st frequency decreases with an increase in the crack depth, and, it decreases the most at mid span. In case of 2nd and 3rd frequency, these decreases the most at 20% and 80% of the total length from the left support. For triple cracks several important observations are also made.**

Keywords—Damage Detection, Structural Health Monitoring, Modal Analysis, Cracks, Beams.

I. INTRODUCTION

Engineering Structures withstand loads during their service life. Buildings are usually designed on strong column, weak beam concept. So, beams are more vulnerable to cracks. Vibrational measurements are an efficient means of crack detection. Crack leads to reduction in stiffness of beam, thereby reducing its natural frequency. A lot of research work has been done to develop effective methods for crack detection. Pandey and Biswas (1991) have evaluated changes in flexibility matrix in order to locate damage. Pandey et.al. (1990) have used curvature mode shape to detect and locate damage in structure. It is shown that curvature mode shape localizes in the damage region whereas the displacement mode shapes are not localized. Further, MAC and COMAC are not sensitive enough to detect damage in its earlier stage. Morassi and Rovere (1997) have identified localized damage in a multistory steel frame. Vibration tests were performed on a five story steel frame with a notch of fixed position and variable depth. Damage is localized by considering frequencies related to shear type modes only. Rizos et.al. (1989) have used the measurement of flexural vibrations of a cantilever beam with rectangular cross section having transverse surface crack extending uniformly along the width of the beam to locate crack location and crack depth. The method requires amplitude measurements at two positions of the structure only. The application of this method is limited to moderate cracks only. Liang et.al (1992) have developed theoretical relationship between eigen frequency changes, crack location and crack depth of damaged cantilever and simply supported beam. This theory can be more specifically applied to steel frame structures. Chondros and Dimarogonas (1979) have discussed the influence of crack in a welded joint on the dynamic behavior of a structural member. Local flexibility was used to establish relationship between crack depth to the change of natural frequency for the cases of a cantilever beam with a transverse crack at the welded root of the beam and of a beam welded (clamped) at both ends with a transverse crack at one welded end. This method is applicable to members of simple geometry. It is applied to individual members of large structures where member flexibility is larger than flexibility of supporting members. Mostafa Attar (2012) has used an analytical approach to investigate natural frequencies and mode shapes of a stepped beam with an arbitrary number of cracks and general form of boundary conditions. A simple transfer matrix is used to obtain general form of characteristic equation for the cracked beam. It is a function of crack location, crack depth, frequency, boundary conditions, geometrical and physical parameters of the beam. Boltezar et.al. have shown the crack identification procedure for free-free uniform beams in flexural vibrations. Khiem and Toan (2014) have proposed a novel method for calculating the natural frequencies of a multiple cracked beam and detecting unknown number of multiple cracks from measured natural frequencies. An explicit expression for natural frequencies through crack parameters is derived as modification of Rayleigh quotient for multiple cracked beams. Hu and Liang (1993) have developed two damage modeling techniques. First modeling technique involves use of massless, infinitesimal springs to represent discrete cracks and other employs a continuum damage concept. In spring model, castigliano's theorem and perturbation technique are used to derive crack location, extent of crack and eigen frequency changes. In continuum damage model, effective stress concept together with Hamilton's principle are used to derive similar relationship in continuum form. Antonino Morassi (1993) has shown that frequency sensitivity for any beam like structure can be evaluated on the basis of undamaged system by general

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 06, June-2016

perturbation approach. Frequency sensitivity plays a vital role in crack identification. Freund Herrmann concept of using a spring to represent effect of crack on section is taken into account. Frequency sensitivity is proportional to potential energy stored for relevant mode shape at cross section where crack occurs. Ratios of frequency changes of various orders are independent of crack severity but helps in localizing damage.

II. PROPOSED ALGORITHM FOR IDENTIFYING TRANSVERSE CRACKS IN SIMPLY SUPPORTED BEAM WITH OVERHANGS

a) Derivation Of Differential Equation Of Transverse Vibration Of Beam

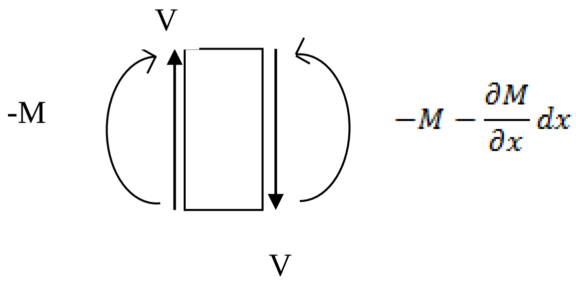

Figure 3.1. Free body diagram of forces acting on beam

Pittman (2004) derived the solution of the differential equation for transverse beam vibration.From the above free body diagram and equation of equilibrium of the vertical forces according to Newton's second law, we have

$$\mu dx \frac{\partial^2 v}{\partial t^2} = V + \frac{\partial V}{\partial x}dx - V \quad \text{or} \quad \mu \frac{\partial^2 v}{\partial t^2} = \frac{\partial V}{\partial x} \qquad (1)$$

The sum of moments about any point of the element yields:

$$Vdx - M + M + \frac{\partial M}{\partial x} = 0 \quad \text{or} \quad \frac{\partial M}{\partial x} = -V \qquad (2)$$

Substituting eq. 2 into eq. 1, we have

$$-\frac{\partial^2 M}{\partial x^2} = \mu \frac{\partial^2 v}{\partial t^2} \qquad (3)$$

It is assumed that vibration occurs in symmetric planes of beam. So, the differential equation of the deflected curve is:

$$EI\frac{\partial^2 v}{\partial x^2} = M \qquad (4)$$

Combining eq. 3 and 4, we have

$$EI\frac{\partial^4 v}{\partial x^4} + \mu \frac{\partial^2 v}{\partial t^2} = 0 \qquad (5)$$

Using the method of separation of variables:

$$v(x,t) = X_v(x)Y_v(t) \qquad (6)$$

Substituting eq.6 in 5 and $\mu = \rho A$, we have,

$$\frac{EIX_v''''(x)}{\rho A X_v(x)} = -\frac{Y_v''(t)}{Y_v(t)} \qquad (7)$$

The term on left side of eq. 7 is dependent only on x and the right side only on t. To be equal to each other, both side must be equal to same constant ω^2. Then the left hand side of eq. 7 can be written as

$$X_v''''(x) - \lambda^4 X_v(x) = 0 \qquad (8)$$

Where, $\lambda^4 = \omega^2 \frac{\rho A}{EI}$

And, ω = Natural frequency in rad/sec.

The characteristic equation of the above differential equation is

$k^4 - \lambda^4 = 0$ or $(\lambda+k)(\lambda-k)(\lambda^2+k^2)=0$

Which gives the eigen values as

$k_1 = -\lambda$, $k_2 = \lambda$, $k_3 = i\lambda$, $k_4 = -i\lambda$
which yields the general solution as
$X(x) = C_1e^{-\lambda x} + C_2e^{\lambda x} + C_3 \cos(\lambda x) + C_4 \sin(\lambda x)$
Or
$X(x) = C_1 \cosh(\lambda x) + C_2 \sinh(\lambda x) + C_3 \cos(\lambda x) + C_4 \sin(\lambda x)$

C_1 , C_2 , C_3,and C_4 are determined from boundary conditions at the ends of a beam.

Since the beam has symmetric overhangs, so we divide it into three sections. Each section will have a separate coordinate system for measuring the distance x with the origin for each section being at the left end of each section.

b) Proposed damage identification algorithm for simply supported beam with overhangs

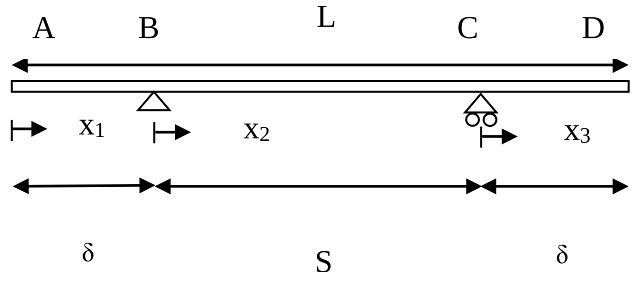

Figure 2. Geometry of simply supported beam with symmetric overhangs and coordinate system.

From the basic vibration theory, the harmonic motion of the beam and its first, second and third derivative for left overhang (i = 1), centre span (i=2) and right overhang(i=3) are as follows

$$\left.\begin{aligned} X_i(x_i) &= A_i\cos(\lambda x_i) + B_i\sin(\lambda x_i) + C_i\cosh(\lambda x_i) + D_i\sinh(\lambda x_i) \\ \frac{d}{dx_i}X_i(x_i) &= -A_i\lambda\sin(\lambda x_i) + B_i\lambda\cos(\lambda x_i) + C_i\lambda\sinh(\lambda x_i) + D_i\lambda\cosh(\lambda x_i) \\ \frac{d^2}{dx_i^2}X_i(x_i) &= -A_i\lambda^2\cos(\lambda x_i) - B_i\lambda^2\sin(\lambda x_i) + C_i\lambda^2\cosh(\lambda x_i) + D_i\lambda^2\sinh(\lambda x_i) \\ \frac{d^3}{dx_i^3}X_i(x_i) &= A_i\lambda^3\sin(\lambda x_i) - B_i\lambda^3\cos(\lambda x_i) + C_i\lambda^3\sinh(\lambda x_i) + D_i\lambda^3\cosh(\lambda x_i) \end{aligned}\right\} \quad (9)$$

General boundary Conditions for crack lying in beam:

For a single crack located anywhere in the left overhang, Centre span and right overhang, the general boundary conditions are as follows

At $x_i = R_c$,

$$X_{iL}(\mathrm{Rc}) = X_{iR}(\mathrm{Rc}) \quad (10)$$

$$X''_{iL}(\mathrm{Rc}) = X''_{iR}(\mathrm{Rc}) \quad (11)$$

$$X'''_{iL}(\mathrm{Rc}) = X'''_{iR}(\mathrm{Rc}) \quad (12)$$

$$X'_{iL}(\mathrm{Rc}) + \left(\frac{EI}{K}\right) X''_{iL}(\mathrm{Rc}) = X'_{iR}(\mathrm{Rc}) \quad (13)$$

Here, R_c is the distance of crack from left. i =1,2,3 for left overhang, Centre span and right overhang respectively. Subscript L and R refers to the left and right side of the crack.

General boundary Conditions for Point A in the beam:

At $x_1 = 0$,

$$\frac{d^2 X_M}{dx_1^2} = 0 \quad (14)$$

$$\frac{d^3 X_M}{dx_1^3} = 0 \quad (15)$$

Here, M = 1L for left overhang and 1 for Centre span and right overhang. Subscript L refers to the left side of the crack.

General boundary Conditions for Point D in the beam:

At $x_3 = \delta$

$$\frac{d^2 X_N}{dx_3^2} = 0 \quad (16)$$

$$\frac{d^3 X_N}{dx_3^3} = 0 \quad (17)$$

Here, N = 3 for left overhang and Centre span and 3R for right overhang. Subscript R refers to the right side of the crack.

General boundary Conditions for Point B in the beam:

At $x_1 = \delta$, $x_2 = 0$,

$$X_M = 0 \quad (18)$$

$$X_N = 0 \quad (19)$$

$$\frac{dX_Q}{dx_1} - \frac{dX_R}{dx_2} = 0 \quad (20)$$

$$\frac{d^2 X_S}{dx_1^2} - \frac{d^2 X_T}{dx_2^2} = 0 \quad (21)$$

Here, M = 1R for left overhang and 1 for centre span and right overhang. N = 2 for left and right overhang and 2L for centre span. Here, Q and S = 1R for left overhang and 1 for centre span and right overhang. Also, R and T = 2 for left and right overhang and 2L for centre span.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 06, June-2016

General boundary Conditions for Point C in the beam:

At $x_2 = S$, $x_3 = 0$,

$$X_M = 0 \tag{22}$$

$$X_N = 0 \tag{23}$$

$$\frac{dX_Q}{dx_2} - \frac{dX_R}{dx_3} = 0 \tag{24}$$

$$\frac{d^2X_S}{dx_2^2} - \frac{d^2X_T}{dx_3^2} = 0 \tag{25}$$

Here, M = 2 for left overhang and right overhang and 2R for centre span. N = 3 for left overhang and centre span and 3L for right overhang. Here, Q and S = 2 for left overhang and right overhang and 2R for centre span. Also, R and T = 3 for left overhang and centre span and 3L for right overhang.

III. Present Finite Element Formulation For Finding Natural Frequencies

A. *Finite Element Formulation*

When crack is induced in a beam, then its flexibility is increased. So, first we calculate the additional flexibility induced in it. Then it is added up with the flexibility matrix of intact beam element. The inverse of the overall flexibility matrix thus obtained is multiplied with the transformation matrix to obtain the required stiffness matrix of the cracked beam element. This stiffness matrix is assembled along with the stiffness matrices of the intact beam element and thereafter the natural frequencies are calculated from the equation $K - \omega^2 M = 0$, where K= Assembled stiffness matrix of the beam, M=Assembled mass matrix and ω= Natural frequency (rad/sec). According to Dimarogonas et.al. (1983) and Tada et.al. (2000) the additional stain energy due to existence of crack can be expressed as

$$\Pi_C = \int_{A_C} G\, dA_C \tag{26}$$

Where, G = the strain energy release rate, and
A_C = the effective cracked area.

$$G = \frac{1}{E'}\left[\left(\sum_{n=1}^{2} K_{In}\right)^2 + \left(\sum_{n=1}^{2} K_{IIn}\right)^2 + k\left(\sum_{n=1}^{2} K_{IIIn}\right)^2 \right] \tag{27}$$

Where, E' = E for plane stress
E' = E/1-ν² for plane strain
k = 1 + ν
ν=Poisson's ratio
E=Young's Modulus of elasticity.

K_I, K_{II} and K_{III} = stress intensity factors for sliding, tearing and opening type cracks respectively. Neglecting effect of axial force and for open cracks above equation can be written as

$$G = \frac{1}{E'}\left[(K_{I1} + K_{I2})^2 + K_{II1}^2 \right] \tag{28}$$

The expressions for stress intensity factors from earlier studies are given by Uttam Kumar Mishra (2014) as follows

$$K_{I1} = \frac{6P_1L_C}{bh^2}\sqrt{\pi\xi}\,F_1\left(\frac{\xi}{h}\right) \tag{29}$$

$$K_{I2} = \frac{6P_2}{bh^2}\sqrt{\pi\xi}\,F_1\left(\frac{\xi}{h}\right) \tag{30}$$

$$K_{II1} = \frac{P_2}{bh}\sqrt{\pi\xi}\,F_{II}\left(\frac{\xi}{h}\right) \tag{31}$$

From definition, the elements of the overall additional flexibility matrix C_{ij} can be

$$C_{ij} = \frac{\partial \delta_i}{\partial P_j} = \frac{\partial^2 \Pi_C}{\partial P_i \partial P_j} \quad , (I, j=1,2) \tag{32}$$

Substituting Eq (29),(30),(31) into Eq (28), then into Eq (26) and Eq (32) subsequently we get,

$$C_{ij} = \frac{b}{E'}\frac{\partial^2}{\partial P_i \partial P_j}\int\left[\left\{\frac{6P_1L_c}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right) + \frac{6P_1}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right)\right\}^2 + \left\{\frac{P_1}{bh}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right)\right\}^2\right]d\xi \tag{33}$$

Substituting i,j (1,2) values, we get

$$C_{11} = \frac{2\pi}{E'b}\left[\frac{36L_C^2}{h^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx + \int_0^{\frac{a}{h}} xF_{II}^2(x)dx\right] \tag{34}$$

$$C_{12} = \frac{72\pi L_c}{E'bh^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx = C_{21} \tag{35}$$

$$C_{22} = \frac{72\pi}{E'bh^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx \tag{36}$$

$$F_I(s) = \sqrt{\frac{\tan\left(\frac{\pi s}{2}\right)}{\left(\frac{\pi s}{2}\right)}}\left[\frac{0.923 + 0.199\left(1 - \sin\left(\frac{\pi s}{2}\right)\right)^4}{\cos\left(\frac{\pi s}{2}\right)}\right] \tag{37}$$

$$F_{II}(s) = \frac{1.122 - 0.561s + 0.085s^2 + 0.180s^3}{\sqrt{1-s}} \tag{38}$$

$$C_{total} = \begin{bmatrix} \frac{L_e^3}{3EI} + C_{11} & \frac{L_e^2}{2EI} + C_{12} \\ \frac{L_e^2}{2EI} + C_{21} & \frac{L_e}{EI} + C_{22} \end{bmatrix} \tag{39}$$

The stiffness matrix K_{crack} of a cracked beam element can be obtained as $K_{crack} = LC_{tot}^{-1}L^T$, Where, L is the transformation matrix for equilibrium condition

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 06, June-2016

$$L = \begin{bmatrix} -1 & 0 \\ -L_e & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (40)$$

Here, equation 1, 2and 3 are coefficients of additional flexibility matrix, a= crack depth, h= total depth of the beam, L_c= distance of crack from right node of beam element. $E' = E/(1-\nu^2)$, where ν= Poisson's ratio, E= modulus of elasticity.

IV. RESULTS BASED ON FORWARD PROCESS FOR FINDING NATURAL FREQUENCIES

Natural Frequencies of simply supported beam with symmetric overhangs having single crack:

Problem Description: A simply supported beam with symmetric overhangs on both ends having various crack depth ratio at various locations is taken. It has following properties and is divided into 4 elements, Length, L = 0.78m, Breadth, b = 0.04m, Height, h = 0.01m, Mass density, ρ = 7860 kg/m^3, Young's Modulus, E = 210 GPa, Number of elements = 4, Supports located at node 2 and node 4, Elemental length = 0.195 m

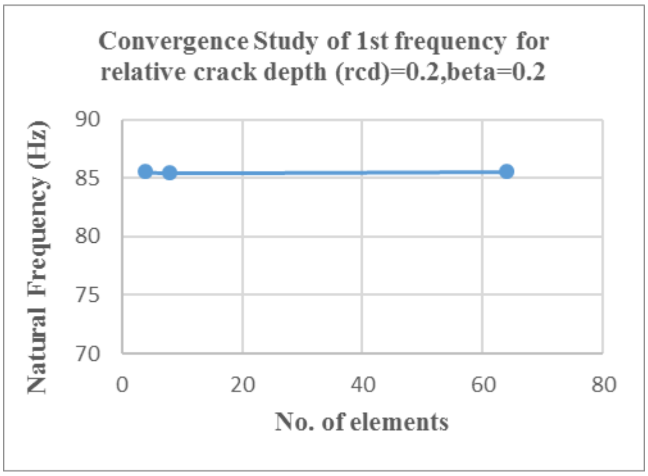

Figure 1. Convergence Study for single crack simply supported beam with symmetric overhangs.

TABLE I. 1ST NATURAL FREQUENCY OF SIMPLY SUPPORTED BEAM WITH SYMMETRIC OVERHANGS

beta	rcd=0.0	rcd=0.2	rcd=0.4	rcd=0.6	rcd=0.8
0.1	85.650656	85.63637	85.58624	85.42792	84.51325
0.2	85.650656	85.55792	85.22798	84.14173	77.0658
0.4	85.650656	85.26477	83.94458	80.10458	64.98897
0.6	85.650656	85.26477	83.94459	80.10458	64.98898
0.8	85.650656	85.55792	85.22798	84.14173	77.06582
0.9	85.650656	85.63637	85.58624	85.42792	84.51326

TABLE II. 2ND NATURAL FREQUENCY OF SIMPLY SUPPORTED BEAM WITH SYMMETRIC OVERHANGS

beta	rcd=0.0	rcd=0.2	rcd=0.4	rcd=0.6	rcd=0.8
0.1	141.98397	141.8973	141.5952	140.6614	135.8193
0.2	141.98397	141.4341	139.5647	134.254	116.4698
0.4	141.98397	141.785	141.1351	139.4817	135.2091
0.6	141.98397	141.785	141.1351	139.4817	135.2091
0.8	141.98397	141.4341	139.5647	134.254	116.4698
0.9	141.98397	141.8973	141.5952	140.6614	135.8193

TABLE III. 3RD NATURAL FREQUENCY OF SIMPLY SUPPORTED BEAM WITH SYMMETRIC OVERHANGS

beta	rcd=0.0	rcd=0.2	rcd=0.4	rcd=0.6	rcd=0.8
0.1	297.13671	296.9138	296.1418	293.8025	282.7459
0.2	297.13671	295.8371	291.6278	281.176	256.1695
0.4	297.13671	296.9124	296.1733	294.243	288.756
0.6	297.13671	296.9124	296.1733	294.2431	288.7561
0.8	297.13671	295.8371	291.6278	281.176	256.1695
0.9	297.13671	296.9138	296.1418	293.8025	282.746

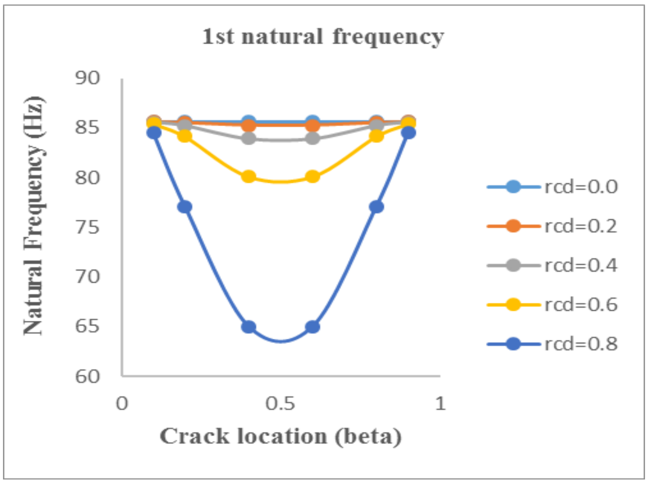

Figure 2. 1st natural frequency of a simply supported beam with symmetric overhangs.

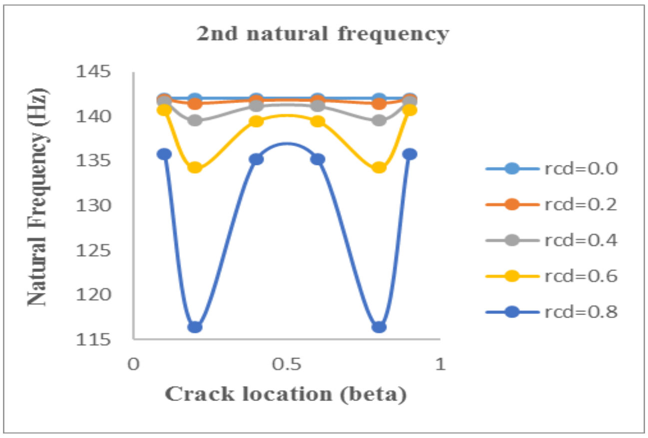

Figure 3. 2nd natural frequency of a simply supported beam with symmetric overhangs.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 06, June-2016

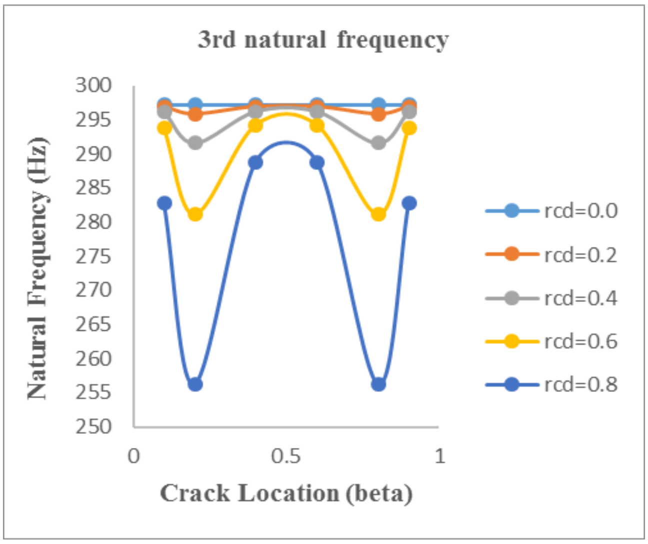

Figure 4. 3rd natural frequency of a simply supported beam with symmetric overhangs.

From the above figure it can be concluded that for a simply supported beam with overhangs 1st frequency, as the crack depth increases, the frequency decreases. It decreases the most at mid span. In case of 2nd frequency, as the crack depth increases the frequency decreases. It decreases the most near the first support and second support i.e at 20% and 80% of the total length. In case of third frequency, also the frequency decreases with increase in crack depth and decreases the most at 20% and 80% of the total length.

Natural Frequencies of simply supported beam with symmetric overhangs having triple crack:

Problem Description: A simply supported beam with symmetric overhangs on both ends having various crack depth ratio at various locations is taken. It has following properties and is divided into 4 elements, Length, L = 0.78m, Breadth, b = 0.04m, Height, h = 0.01m, Mass density, ρ = 7860 kg/m3, Young's Modulus, E = 210 GPa, Number of elements = 4, Supports located at node 2 and node 4, Elemental length = 0.195 m.

TABLE IV. 1ST NATURAL FREQUENCY OF SIMPLY SUPPORTED BEAM WITH SYMMETRIC OVERHANGS HAVING TRIPLE CRACK

Beta / rcd	beta=0.2,0.4,0.6	beta=0.2,0.4,0.8	beta=0.2,0.6,0.8	beta=0.4,0.6,0.8
0.1	85.42541912	85.50185911	85.50185897	85.42541911
0.3	83.71291933	84.35523699	84.35523717	83.71291952
0.5	79.22638862	81.22157178	81.22157319	79.22639125
0.7	66.96330109	71.6677455	71.66774656	66.96330167

TABLE V. 2ND NATURAL FREQUENCY OF SIMPLY SUPPORTED BEAM WITH SYMMETRIC OVERHANGS HAVING TRIPLE CRACK

Beta / rcd	beta=0.2,0.4,0.6	beta=0.2,0.4,0.8	beta=0.2,0.6,0.8	beta=0.4,0.6,0.8
0.1	141.7352322	141.6444224	141.6444223	141.7352321
0.3	139.8257202	139.0683819	139.068376	139.8257211
0.5	134.6595384	132.3569261	132.3569234	134.6595553
0.7	119.1343277	114.082506	114.0825185	119.1343434

TABLE VI. 3RD NATURAL FREQUENCY OF SIMPLY SUPPORTED BEAM WITH SYMMETRIC OVERHANGS HAVING TRIPLE CRACK

Beta / rcd	beta=0.2,0.4,0.6	beta=0.2,0.4,0.8	beta=0.2,0.6,0.8	beta=0.4,0.6,0.8
0.1	296.6757407	296.3914888	296.3914843	296.6757406
0.3	293.1816274	290.7360878	290.7360562	293.1816314
0.5	284.1079446	276.000332	276.0003583	284.107977
0.7	259.4173901	236.0703659	236.0704891	259.4173403

TABLE VII. 4TH NATURAL FREQUENCY OF SIMPLY SUPPORTED BEAM WITH SYMMETRIC OVERHANGS HAVING TRIPLE CRACK

Beta / rcd	beta=0.2,0.4,0.6	beta=0.2,0.4,0.8	beta=0.2,0.6,0.8	beta=0.4,0.6,0.8
0.1	823.5553966	823.3770847	823.3770825	823.5553952
0.3	807.8733341	806.6115086	806.6115075	807.8733313
0.5	767.7336102	765.62396	765.6239695	767.7336251
0.7	665.2350624	671.2620994	671.2620997	665.2349858

TABLE VIII. 5TH NATURAL FREQUENCY OF SIMPLY SUPPORTED BEAM WITH SYMMETRIC OVERHANGS HAVING TRIPLE CRACK

Beta / rcd	beta=0.2,0.4,0.6	beta=0.2,0.4,0.8	beta=0.2,0.6,0.8	beta=0.4,0.6,0.8
0.1	1295.374277	1294.538285	1294.538274	1295.374277
0.3	1289.118019	1282.067365	1282.06724	1289.118014
0.5	1274.636156	1252.50435	1252.504407	1274.636196
0.7	1244.677417	1189.939438	1189.94033	1244.677722

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 06, June-2016

TABLE IX. 6TH NATURAL FREQUENCY OF SIMPLY SUPPORTED BEAM WITH SYMMETRIC OVERHANGS HAVING TRIPLE CRACK

Beta / rcd	beta=0.2,0.4,0.6	beta=0.2,0.4,0.8	beta=0.2,0.6,0.8	beta=0.4,0.6,0.8
0.1	1717.062955	1717.303203	1717.303177	1717.062969
0.3	1710.840064	1712.87886	1712.878668	1710.840161
0.5	1695.381055	1701.879419	1701.879576	1695.381344
0.7	1658.943026	1675.671543	1675.672746	1658.942178

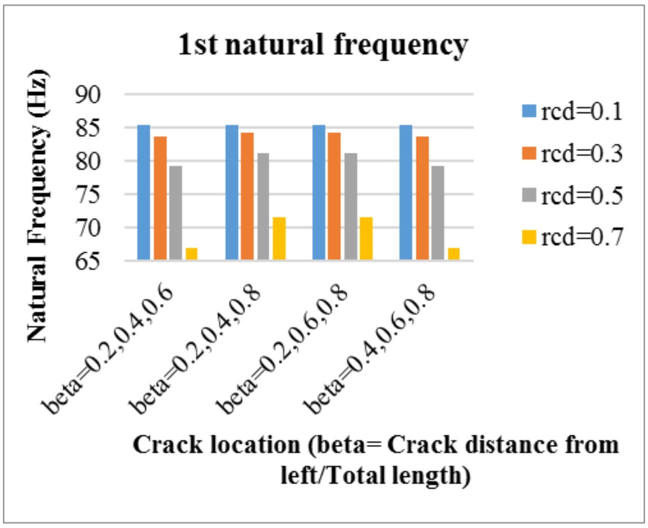

Figure 5. 1st natural frequency of a simply supported beam with symmetric overhangs having triple crack.

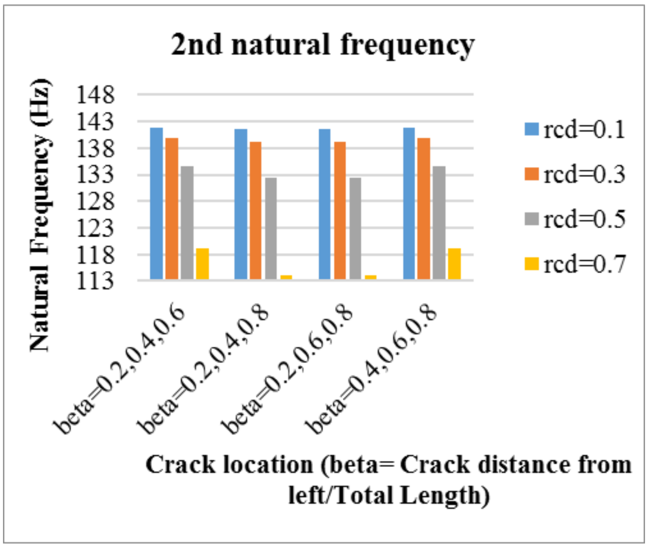

Figure 6. 2nd natural frequency of a simply supported beam with symmetric overhangs having triple crack.

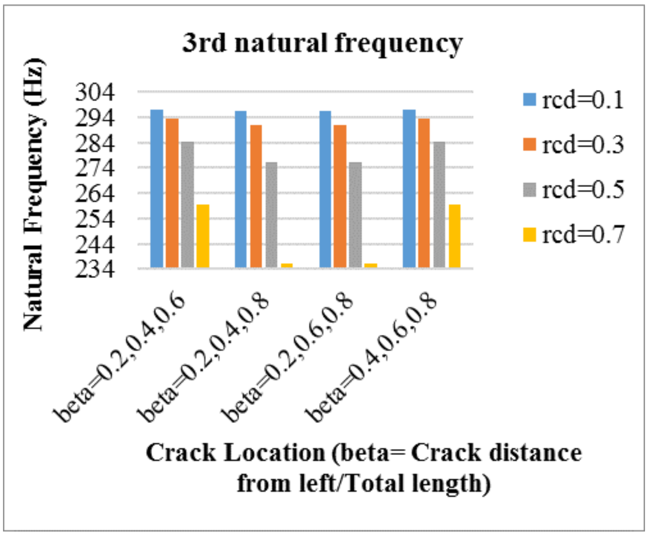

Figure 7. 3rd natural frequency of a simply supported beam with symmetric overhangs having triple crack.

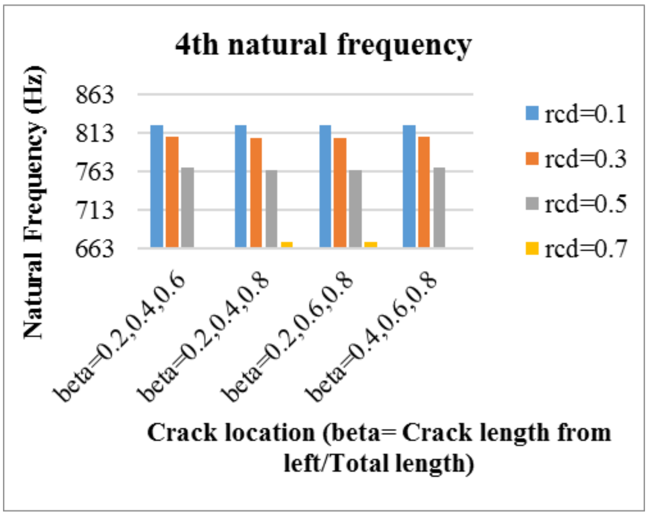

Figure 8. 4th natural frequency of a simply supported beam with symmetric overhangs having triple crack.

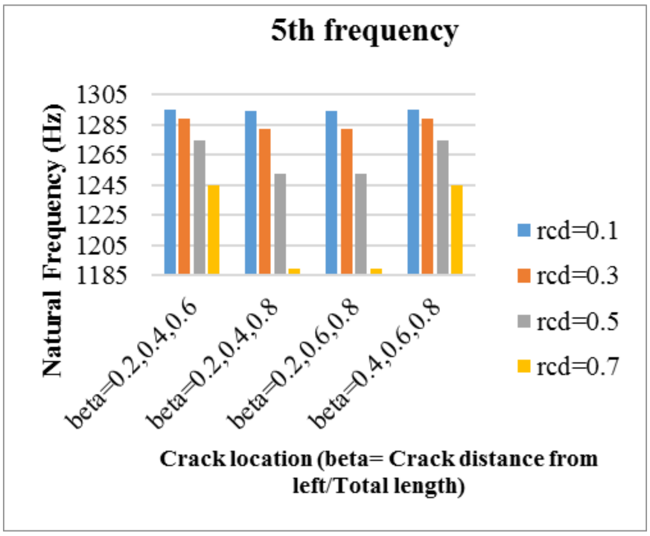

Figure 9. 5th natural frequency of a simply supported beam with symmetric overhangs having triple crack.

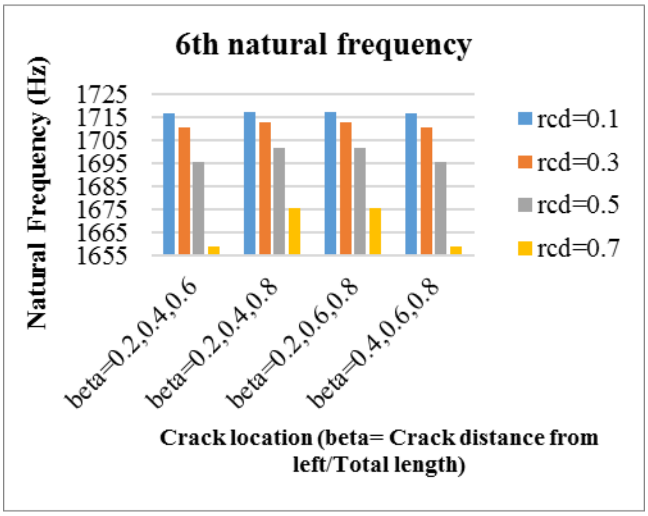

Figure 10. 6th natural frequency of a simply supported beam with symmetric overhangs having triple crack.

Validation of proposed FEM method for triple cracked cantilever beam

Problem Description: A cantilever with crack depth ratio of 0.1 for all the three cracks with cracks located at various locations is taken. It has following properties and is divided into 16 elements

Length, L = 0.5m
Breadth, b = 0.02m
Height, h = 0.02m
Mass density, ρ = 7860 kg/m^3.
Young's Modulus, E = 210 GPa
Poisons ratio =0.3

V. Results Based On Inverse Process For Identifying Cracks From First Three Natural Frequencies

Problem Description: A simply supported beam with symmetric overhangs having cracks located at various locations with various depths is taken. It has following properties. Cracks are identified for single cracked simply supported beam with symmetric overhangs by finding determinant of matrix formed by applying boundary conditions.

Length, L = 10m
Breadth, b = 0.2m
Height, h = 0.6m
Mass density, ρ = 2350 kg/m^3.
Young's Modulus, E = 2.8*10^10 N/m^2.
I=0.0036 m^4.

TABLE XII. Predicted Crack Location For Single Crack Located At Different Locations In Centre Span Of Simply Supported Beam With Symmetric Overhangs.

CENTRE SPAN					
Sl.no.	1st Natural frequency (ω_1) in rad/sec	2nd Natural frequency (ω_2) in rad/sec	3rd Natural frequency (ω_3) in rad/sec	Actual Crack Location, in metres	Predicted Crack Location, in metres
1	126.58	206.27	442.6	0.5	0.5
2	124.092	216.165	436.3544	3	3
3	125.494	210.566	450.17	4	4
4	122.317	198.99	437.47	4.5	4.5

TABLE X. First Three Natural Frequencies Of A Triple Cracked Cantilever Beam

Case	Crack Location			Method	Natural Frequency		
	X_1/L	X_2/L	X_3/L		ω_1	ω_2	ω_3
1	0.2	0.4	0.6	M. Attar	416.9159	2612.213	7324.21
				Present FEM	417.16	2613.9	7328.4
2	0.2	0.4	0.8	M. Attar	417.0864	2620.455	7318.811
				Present FEM	415.07	2618.8	7342.7
3	0.2	0.6	0.8	M. Attar	417.6464	2617.786	7315.833
				Present FEM	415.58	2616.4	7339.6
4	0.4	0.6	0.8	M. Attar	418.7517	2610.361	7311.243
				Present FEM	416.57	2609.7	7335.9

TABLE XI. 4TH ,5TH AND 6TH Natural Frequency Of A Triple Cracked Cantilever Beam

Case	Crack Location			Method	Natural Frequency		
	X_1/L	X_2/L	X_3/L		ω_4	ω_5	ω_6
1	0.2	0.4	0.6	M. Attar	14357.28	23592.02	35604.06
				Present FEM	14366.24	23634.15	35634.05
2	0.2	0.4	0.8	M. Attar	14301.02	23602.31	35574
				Present FEM	14373.25	23688.01	35549.85
3	0.2	0.6	0.8	M. Attar	14301.53	23603.48	35574.12
				Present FEM	14374.84	23688.36	35548.87
4	0.4	0.6	0.8	M. Attar	14338.46	23577.32	35598.16
				Present FEM	14408.62	23669.04	35568.63

TABLE XIII. PREDICTED CRACK LOCATION FOR SINGLE CRACK LOCATED AT DIFFERENT LOCATIONS IN LEFT AND RIGHT OVERHANG OF SIMPLY SUPPORTED BEAM WITH SYMMETRIC OVERHANGS.

LEFT OVERHANG					
Sl.no.	1st Natural frequency (ω_1) in rad/sec	2nd Natural frequency (ω_2) in rad/sec	3rd Natural frequency (ω_3) in rad/sec	Actual Crack Location, in metres	Predicted Crack Location, in metres
1	130.785	215.276	445.77	1	1
2	125.65	194.23	411.37	2.3	2.3
3	130.43	213.27	441.39	1.5	1.5
RIGHT OVERHANG					
1	131.04	216.84	450.23	2	2.1

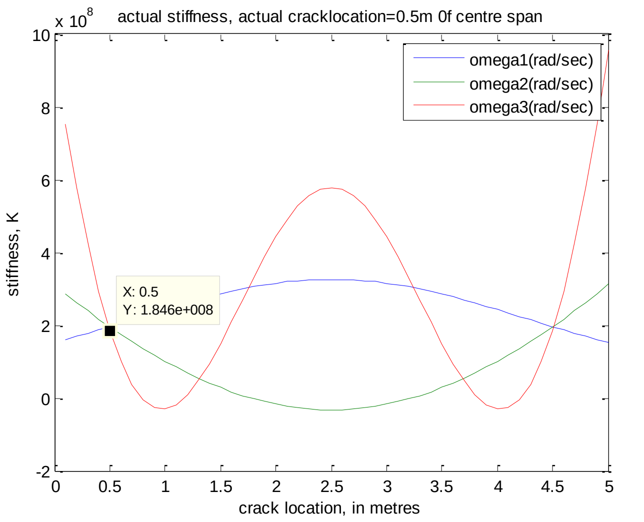

Figure 11: Location of crack in a single cracked simply supported beam with symmetric overhangs having actual crack location = 0.5m of centre span.

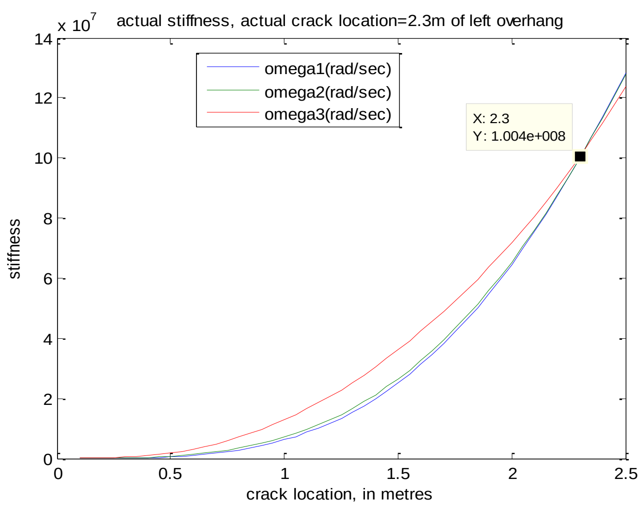

Figure 12: Location of crack in a single cracked simply supported beam with symmetric overhangs having actual crack location = 2.3m of left overhang.

Discussion:

A new algorithm has been proposed to identify cracks in a simply supported beam with symmetric overhangs. A 16×16 matrix is formed from the harmonic equation of the beam after applying the boundary conditions. The determinant of the above matrix is equated to zero and therefore graph is plotted between the two unknowns, stiffness (K) and crack location for the first three natural frequencies (rad/sec). The intersection of the curves for corresponding first three natural frequencies gives the crack location and corresponding stiffness.

VI. CONCLUSIONS

Based on the results obtained for simply supported beam having single and multiple cracks following conclusions can be drawn

- Frequencies decreases with increase in crack depth.
- Frequencies are same for symmetric location of cracks.
- For single crack 1st frequency decreases the most at midspan whereas for 2nd and 3rd frequencies it decreases the most near the supports.
- For a simply supported beam with overhangs 1st frequency, as the crack depth increases, the frequency decreases. It decreases the most at mid span. In case of 2nd frequency, as the crack depth increases the frequency decreases. It decreases the most near the first support and second support i.e at 20% and 80% of the total length. In case of third frequency, also the frequency decreases with increase in crack depth and decreases the most at 20% and 80% of the total length.
- For the case of simply supported beam with symmetric overhangs having triple cracks, as the crack depth increases, frequency decreases. Frequencies are same for the crack location combination of 0.2L-0.4L-0.6L and 0.4L-0.6L-0.8L. It is also same for the crack location combination of 0.2L-0.4L-0.8L and 0.2L-0.6L-0.8L. A significant difference among the first six frequencies is that the 1st ,4th and 6th frequencies increases as the crack location shifts towards the midspan whereas it is vice versa for the 2nd ,3rd and 5th one.
- An algorithm has been developed to identify cracks in simply supported beam with symmetric overhangs. The algorithm can also identify cracks located in simply supported beam with different overhanging lengths.

REFERENCES

[1]. A. Morassi and N. Rovere , "Localizing a notch in a steel frame from frequency measurements", Journal of engineering mechanics 422, 1997.

[2]. A.K. Pandey, M. Biswas and M. M. Samman , "Damage detection from changes in curvature mode shapes", Journal of sound and vibration 145(2), 321-332,1991.

[3]. A.K. Sahu , S. Das, "Comparison of transfer mass matrix method with finite element method for modal analysis of beams", International journal of engineering research and technology, ISSN : 2278-0181, Vol. 5 Issue 03, March-2016, pg No- 98-101.

[4]. Antonino Morassi, "Crack-Induced changes in eigen parameters of beam structures", Journal of engineering mechanics, vol. 119, No.9 page No. 4178, 1993.

[5]. Jialou Hu and Robert Y. Liang , "An integrated approach to detection of cracks using vibration characteristics", Journal of franklin institute, 1993.

[6]. L.T Khiem and L.K. Toan , "A novel method for crack detection in beam-like structures by measurements of natural frequencies", Journal of sound and vibration 333(2014) 4084-4103, 2014.

[7]. M. Boltezar, B. Strancar and A. Kuhelj. "Identification of transverse crack location in flexural vibrations of free-free beams", Journal of sound and vibration 211, 729-734, 1998.

[8]. Mostafa Attar, "A transfer matrix method for free vibration analysis and crack identification of stepped beams with multiple edge cracks and different boundary conditions", International journal of mechanical sciences 57(2012), 19-33, 2012.

[9]. P.F. Rizos, N. Aspragathos and A.D. Dimarogonas, "Identification of crack location and magnitude in a cantilever beam from the vibration modes", Journal of sound and vibration 138(3), 381-388, 1990.

[10]. Pittman. M , "Calculation of the natural frequency of steel and wooden glued laminated beams of similar load bearing capacities", *Project for Math 222-Differential Equations, Spring 2004 FVCC*,2004.

[11]. Robert Y. Liang, Jialou Hu and Fred Choy , "Theoretical study of crack-induced eigenfrequency changes on beam structures", Journal of engineering mechanics, Vol.188, No.2, 1992.

[12]. T.G Chondros and A.D. Dimarogonas, "Identification of cracks in welded joints of complex structures", Journal of sound and vibration 69(4), 531-538,1980.