Image Denoising Using Complex Double Density Dual Tree Wavelet Transform

Miss Sandeepa Dewangan
Department of Electronics & Telecommunication Engineering, Shri Shankaracharya Technical Campus, Bhilai, India.
Address: Lakhe Nagar Near Kanji House, In front of Divya Jyoti Vidhya Mandir, Before Sai Mandir, Raipur (C.G.) Pin-492001, INDIA

Mr. Devanand Bhonsle
Faculty of Electrical & Electronics Engineering Department, Shri Shankaracharya Technical Campus, Bhilai, India.

Keywords: *Complex Wavelet Transform (CWT), Double Density Wavelet Transform, Dual-Tree Complex Wavelet Transform, Complex Double Density Dual Tree Wavelet Transform, Filter Bank, Shift Invariance.*

***Abstract:* -** We have implemented the image denoising method which is hybridization of double density discrete wavelet transform and dual tree complex wavelet transform. This hybrid model possesses the properties of both the double density discrete wavelet transform and the dual tree complex wavelet transform. The design procedure for the complex double density dual tree wavelet transform is based on two distinct scaling functions and four distinct wavelets where the two wavelets are offset from one another by one half and where the two wavelets form an approximate Hilbert transform pair & on the flat-delay filter, spectral factorization and paraunitary filter bank completion. The solutions have vanishing moments and compact support. The resulting wavelets are much smoother than the dual tree wavelets and double density wavelets. The complex double density dual tree wavelet transform based image denoising method is analyzed and the experimental results demonstrate that the complex double density dual tree wavelet transform outperforms for applications like image denoising and enhancement.

Keywords: *Complex Wavelet Transform (CWT), Double Density Wavelet Transform (DDWT), Dual Tree Complex Wavelet Transform (DTCWT), Complex Double Density Dual Tree Wavelet Transform (CDDDTWT), Filter Bank, Shift Invariance.*

I. INTRODUCTION

Wavelet techniques are successfully applied to various problems in signal and image processing. Data compression, motion estimation, texture synthesis, segmentation, classification and denoising are only some examples. Denoising of images is an important task in image processing and analysis and it plays a significant role in modern applications in different fields, including medical imaging and preprocessing for computer vision. The goal of denoising is to remove that noise. The Discrete Wavelet Transform (DWT) of image signals produces a non-redundant image representation, which provides better spatial and spectral localization of image formation. But, it has lack of shift invariance, lack of symmetry of the mother wavelet and poor directional selectivity. However, a major problem of the common decimated discrete wavelet transform is its lack of shift invariance. This means that on shifts of the input signal, the wavelet coefficients vary substantially. The DWT suffers from the following two problems: -

• **Lack of shift invariance: -** this results from the down sampling operation at each level. When the input signal is shifted slightly, the amplitude of the wavelet coefficients varies so much.

• **Lack of directional selectivity: -** as the DWT filters are real and separable the DWT cannot distinguish between the opposing diagonal directions.

These disadvantages can be diminished using a complex wavelet transform. Complex wavelets have not been used widely in image processing due to the difficulty in designing complex filters which satisfy a perfect reconstruction property. To overcome this problem, Kingsbury proposed a dual-tree implementation of the CWT (DT-CWT), which uses two trees of real filters to generate the real and imaginary parts of the wavelet coefficients separately. The DT-CWT is an alternative to the basic DWT, the outputs of each tree are down sampled by summing the outputs of the two trees during reconstruction and the aliased components of the signal are suppressed and approximate shift invariance is achieved. The DT-CWT has improved shift-invariance and directional selectivity than the separable DWT. Another type of complex wavelet transform is double density wavelet transform. The three-channel filter bank develops the double density DWT. The double density DWT is less shift-sensitive than an orthonormal wavelet basis and has fewer rectangular artifacts.

Thus, we proposed the complex double density dual tree wavelet transform which is an over complete discrete wavelet transform designed to simultaneously possess the properties of the double density discrete wavelet transform and the dual tree complex wavelet transform. The double density DWT and the dual tree CWT are similar in several respects such as they are both overcomplete by a factor of two, they are both nearly shift-invariant and they are both based on FIR perfect reconstruction filter banks, but they are quite different from one another in other important respects. Both wavelet transforms can outperform the critically sampled DWT for several signal processing applications, but they do so for different reasons. It is therefore natural to investigate the possibility of a single wavelet transform that has the characteristics of both the double density DWT and the dual tree complex wavelet transform.

II. DUAL TREE COMPLEX WAVELET TRANSFORM

The Dual Tree Complex Wavelet Transform (DT-CWT) has been developed to incorporate the good properties of the Fourier transform in the wavelet transform. As the name implies, two wavelet trees are used, one generating the real part of the complex wavelet coefficients real tree and the other generating the imaginary part of the complex wavelet coefficients imaginary tree.

The Dual-Tree Complex Wavelet Transform (DT-CWT) provides the following properties:-

1. Shift Invariance: -

DT-DWT has approximate shift-invariance or in other words, improved time-shift sensitivity in comparison with standard DWT. The reconstructed details at various levels and approximation at the last level have almost uniform shifts for the time-shifted unit step functions. The property of shift invariance makes the DT-DWT well suited for applications such as Motion estimation and Image fusion at various resolution levels.

2. Good Selectivity and Directionality: -

DT-DWT gives better directional selectivity in 2-D with Gabour like filters (also true for higher dimensionality m-D). Standard DWT offers the feature selectivity in only 3 directions with poor selectivity for diagonal features, where as DT-DWT has 12 directional wavelets (6 for each of real and imaginary trees) oriented at angles of in 2-D. The improved directionality with more orientations suggests the advantage of DT-DWT in a vide rage of directional image processing applications e.g. texture analysis.

3. Phase Information: -

Local phase extraction is possible through analytic interpretation of two parallel trees of DT-DWT. The phase of any given sub-band at a given level can be computed with its corresponding real and imaginary coefficients.

4. Perfect Reconstruction (PR): -

The DT-DWT structure follows PR conditions; hence, the original signal can be reconstructed from the transform domain complex wavelet coefficients.

5. Limited Redundancy: -

DT-DWT has redundancy of 2:1 (2^m:1) for 1-D (m-D) independent of scales (levels) of iteration. Though DT-DWT structure is expensive than standard DWT, it is significantly less expensive than WP, or non-decimated DWT for the same advantage of reduced shift-sensitivity. Moreover, DT-DWT has other advantages such as improved directionality and phase information to compensate for its limited redundancy over standard DWT.

The Dual-Tree CWT comprises of two parallel wavelet filter bank trees that contain carefully designed filters of different delays that minimize the aliasing effects due to down sampling. The Dual-Tree CWT of a signal x(n) is implemented using two critically-sampled DWTs in parallel on the same data. The transform is two times expansive because for an N-point signal it gives 2N DWT coefficients. If the filters in the upper and lower DWTs are the same, then no advantage is gained. So the filters are designed in a specific way such that the sub-band signals of the upper DWT can be interpreted as the real part of a complex wavelet transform and sub-band signals of the lower DWT can be interpreted as the imaginary part. When designed in this way the DT CDWT is nearly shift invariant, in contrast to the classic DWT.

The filter bank structure of Dual Tree CWT that it resembles the filter bank structure of standard DWT with twice the complexity. It can be seen as two standard DWT trees operating in parallel. One tree is called as a real tree and other is called as an imaginary tree. Sometimes the real tree will be referred to as tree-a and the imaginary tree as tree-b.

The design of the filters is particularly important for the transform to occur correctly and the necessary characteristics are: -

- The low-pass filters in the two trees must differ by half a sample period.
- Reconstruction filters are the reverse of analysis.

- All filters are from the same orthonormal set.
- Tree (a) filters are the reverse of tree (b) filters.
- Both trees have the same frequency response.

The orthogonal two-channel filter banks with analysis low-pass filter given by the *z*-transform $H_0(z)$, analysis high-pass filter $H_1(z)$ and with synthesis filters $G_0(z)$ and $G_1(z)$ is shown in Figure1.

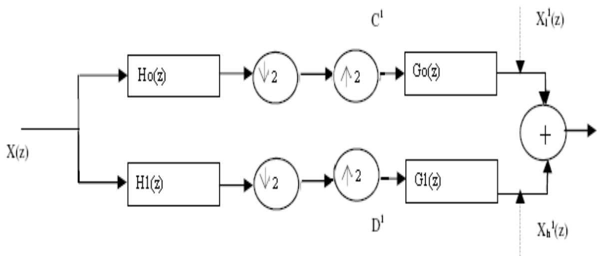


Fig 1: DWT Filter Bank

For an input signal X(z), the analysis part of the filter bank followed by upsampling produces the low-pass and the high-pass coefficients respectively and decomposes the input signal into a low frequency part $X_l^1(z)$ and a high frequency part $X_h^1(z)$, the output signal is the sum of these two components.

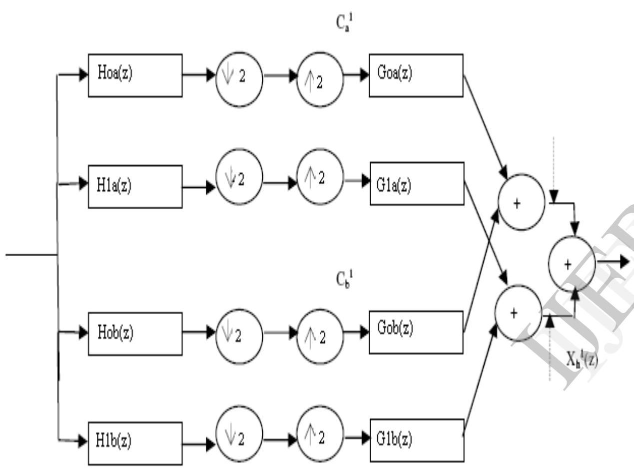
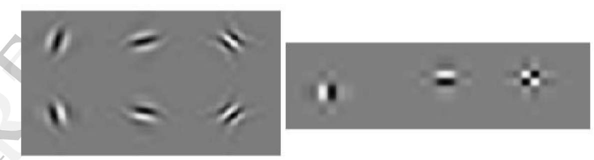


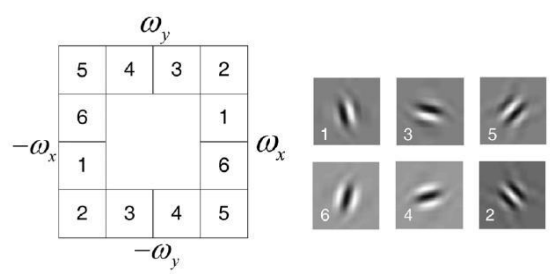
Fig 2: One level complex dual tree.

One possibility to obtain a shift invariant decomposition can be achieved by the addition of a filter bank with shifted analysis filters and synthesis filters and subsequently taking the average of the low-pass and the high-pass branches of both filter banks as shown in Figure 2.

DT-DWT is developed to overcome two main drawbacks of DWT: shift variance and poor directional selectivity. With carefully designed filter banks, DT-DWT mainly has following advantages: approximate shift invariance, directional selectivity, limited redundancy and similar computation efficiency as DWT. Either the real part or the imaginary part of DT-DWT yields perfect reconstruction and thus can be employed as a stand-alone transform. We use magnitude of sub-bands to calculate feature vector. The implementation of DT-DWT is very straight forward. An input image is decomposed by two sets of filter banks, (H^0a, H^1a) and (H^0b, H^1b) separately, filtering the image horizontally and then vertically just as conventional 2D DWT does. Then eight sub bands are obtained: LL*a*, *HLa*, *LHa*, *HHa* , *LLb* , *HLb* , *LHb* and *HHb* . Each high-pass sub-band from one filter bank is combined with the corresponding sub-band from the other filter bank by simple linear operations: averaging or differencing. The size of each sub-band is the same as that of 2D DWT at the same level. But there are six high pass sub-bands instead of three high pass sub-bands at each level. The two low pass sub-bands, *LLb* and *LLa*, are iteratively decomposed up to a desired level within each branch. The basic functions of 2D DT-DWT and 2D DWT are shown in Fig. 3a and Fig. 3b respectively. Each DT-DWT basis function is oriented at a certain direction, including ±75º, ± 15º, and ±45º. However, the basis function of HH sub-band of 2D DWT mixes directions of ±45º together.



(a) (b)



(c)

Fig 3: (a) Six basis functions of 2D DT-DWT (real part) at level 3 and (b) three basis functions of 2D separable DWT at the same level.(c) frequency tiling of 2-D DT-DWT and six wavelets of 2-D DT-DWT.

The Dual Tree CWT uses two trees of real filters to generate the real and imaginary parts of the wavelet coefficients separately. The outputs of each tree are down sampled by summing the outputs of the two trees during reconstruction, the aliased components of the signal can be suppressed and approximate shift invariance can be achieved.

Moreover, the Dual Tree Complex DWT can be used to implement 2D wavelet transforms where each

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 5, May - 2013

wavelet is oriented, which is especially useful for image processing. The Dual-Tree Complex DWT outperforms the critically-sampled DWT for applications like image denoising and enhancement.

III. DOUBLE DENSITY WAVELET TRANSFORM:

The Double Density discrete wavelet transform is based on a single scaling function and two distinct wavelets where the two wavelets are designed to be offset from one another by one half, the integer translates of one wavelet fall midway between the integer translates of the other wavelet. In this way, the Double Density DWT approximates the continuous wavelet transform (having more wavelets than necessary gives a closer spacing between adjacent wavelets within the same scale). The Double Density DWT is two-times expansive regardless of the number of scales implemented potentially much less than the Undecimated DWT. The Double Density DWT has twice as many coefficients as the critically sampled DWT.

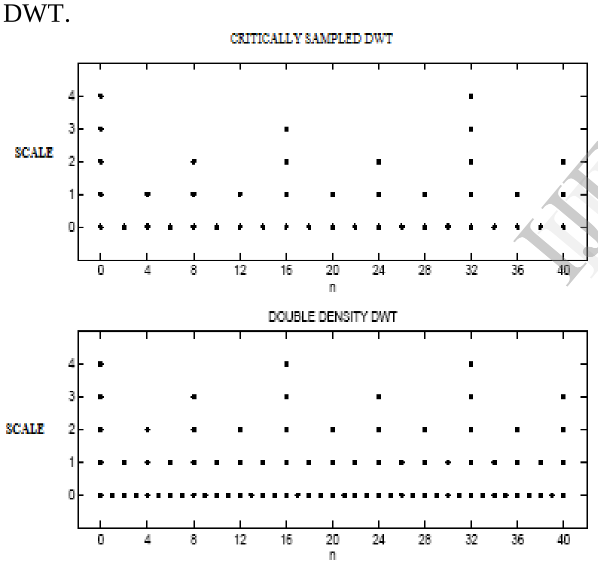


Fig 4: Idealized time-frequency localization diagrams. The double density DWT gives a denser sampling of the time-frequency plane than the critically sample DWT.

The number of points shown in the figure 4 indicates the redundancy incurred by each of the transforms. The Double Density DWT is redundant by a factor of two regardless of the number of scales used. Having a closer spacing between adjacent wavelets within the same scale makes the Double Density DWT less shift-sensitive than the critically sampled DWT while keeping the redundancy much lower than that of the Undecimated DWT.

The new version of DWT known as Double Density DWT has the following important additional proprieties:

- It employs one scaling function and two distinct wavelets which are designed to be offset from one another by one half.
- The double density DWT is over complete by a factor of two.
- It is nearly shift-invariant where complex wavelets with real and imaginary parts approximating Hilbert pairs are proposed for denoising signal.

To develop the Double Density DWT we begin by selecting an appropriate filter bank structure. The filter bank matches the strategy for sampling the time-frequency plane in the second panel. This resembles the usual two-channel filter bank used in implementing the critically sampled DWT; however, the down-sampler and up-sampler in the high-pass channel have been deleted. This is called an oversampled filter bank because the total rate of the sub-band signals is exceeds the input rate by 3/2, we have called the corresponding transform the Double Density DWT. This is because, when the filter bank is iterated a single time on its low pass branch (h_0), the total oversampling rate will be 7/4. For a three-stage filter bank, the oversampling rate will be 15/8. The Double Density DWT is then implemented by recursively applying this filter bank on the low-pass sub-band signal, the total oversampling rate increases towards two.

To construct a Double Density DWT with FIR filters we will use the oversampled filter bank which is the basic design of the Double-Density DWT. The filter $h_0(n)$ will be a low-pass (scaling) filter, while $h_1(n)$ and $h_2(n)$ will both be high-pass (wavelet) filters.

The three-channel filter bank which we use to develop the Double Density DWT corresponds to a wavelet frame based on a single scaling function $\Phi(t)$ and two distinct wavelets $\Psi_1(t)$ and $\Psi_2(t)$.

$$\phi(t) = \sqrt{2}\sum_n h_0(n)\,\phi(2t-n)$$

$$\psi_i(t) = \sqrt{2}\sum_n h_i(n)\,\phi(2t-n), \quad i = 1, 2.$$

The scaling function $\Phi(t)$ and the wavelets $\Psi_1(t)$, $\Psi_2(t)$ are defined through these equations by the low-pass (scaling) filter $h_0(n)$ and the two high-pass (wavelet) filters $h_1(n)$ and $h_2(n)$.

To design the filters $hi(n)$ we can follow Daubechies' program: look for filters $hi(n)$ of minimal length under the constraint that the scaling function and wavelets satisfy certain polynomial properties. Similar to the critically sampled case, the properties we ask $\Phi(t)$ to satisfy can be translated into conditions on $hi(n)$. However, in the oversampled case under consideration, more degrees of freedom are available. Accordingly, it is possible to obtain wavelets that are much smoother.

The analysis filter bank consists of three analysis filters-one low pass filter denoted by $h_0(-n)$ and two distinct high pass filters denoted by $h_1(-n)$ and $h_2(-n)$. As the input signal $x(n)$ travels through the system, the analysis filter bank decomposes it into three sub-bands, each of which is then down-sampled by 2. From this process we obtain the signals $c(n)$, $d_1(n)$, and $d_2(n)$, which represent the low frequency (or coarse) sub-band, and the two high frequency (or detail) sub-bands respectively.

The synthesis filter bank consists of three synthesis filters-one low pass filter denoted by $h_0(n)$ and two distinct high pass filters denoted by $h_1(n)$ and $h_2(n)$ which are essentially the inverse of the analysis filters. As the three sub-band signals travel through the system, they are up-sampled by two filtered and then combined to form the output signal $y(n)$.

We can obtain the minimal-length low-pass filter $h_0(n)$ satisfying the perfect reconstruction conditions and the constraints. As in Daubechies' construction, the filter $h_0(n)$ can be obtained through the spectral factorization of a suitably designed symmetric filter. Once the low-pass filter $h_0(n)$ is obtained, the two (non-unique) wavelet filters $h_1(n)$ and $h_2(n)$ can be obtained using a poly phase formulation. Then the three-channel filter bank of Figure 5 can be redrawn as the filter bank of figure 5.

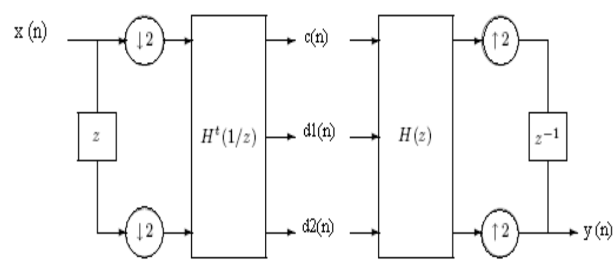


Fig 5: The Oversampled filter bank in Poly phase form.

One of the main concerns in filter bank design is to ensure the perfect reconstruction condition. That is, to design $h_0(n)$, $h_1(n)$, and $h_2(n)$ such that $y(n)=x(n)$.

A separable 2D Double Density DWT can be obtained by alternating between rows and columns as is usually done for 2D separable DWTs. The corresponding filter bank, illustrated in figure 6, is iterated on the low pass branch (the first branch). While the 1D Double-Density DWT is redundant by a factor of 2, the corresponding 2D version is redundant by a factor of 8/3, not by 2 or 4 as one might initially expect. In the oversampled filter bank for the 2D case, the 1D oversampled filter bank is iterated on the rows and then on the columns. This gives rise to 9 2D branches. One of the branches is a 2D low pass scaling filter, while the other 8 make up the 8 2D wavelet filters.

Like the Dual Tree DWT of Kingsbury, the overcomplete DWT described above is less shift-sensitive than an orthonormal wavelet basis and in the 2D case has fewer rectangular artifacts.

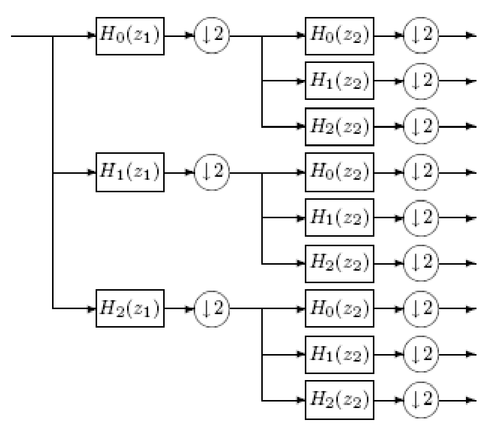


Fig 6: An Oversampled filter bank for 2D signals.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 5, May - 2013

IV. COMPLEX DOUBLE DENSITY DUAL TREE WAVELET TRANSFORM:

We proposed the Double-Density Dual-Tree discrete wavelet transform which is an overcomplete discrete wavelet transform designed to simultaneously possess the properties of the Double-Density discrete wavelet transform and the Dual-Tree CWT. The Double-Density DWT and the Dual-Tree CWT are similar in several respects (they are both overcomplete by a factor of two, they are both nearly shift-invariant, and they are both based on FIR perfect reconstruction filter banks), but they are quite different from one another in other important respects. Both wavelet transforms can outperform the critically sampled DWT for several signal processing applications, but they do so for different reasons. It is therefore natural to investigate the possibility of a single wavelet transform that has the characteristics of both the Double-Density DWT and Dual-Tree Complex DWT. This is the motivation for the development of the Double-Density Dual-Tree DWT described in this thesis.

The Double-Density Dual-Tree DWT proposed which is designed to simultaneously possess the properties of the Double-Density DWT and the Dual-Tree DWT is based on two distinct scaling functions and four distinct wavelets where the two wavelets are offset from one another by one half and where the two wavelets form an approximate Hilbert transform pair. One pair of the four wavelets are designed to be offset from the other pair of wavelets so that the integer translates of one wavelet pair fall midway between the integer translates of the other pair. Simultaneously, one pair of wavelets are designed to be approximate Hilbert transforms of the other pair of wavelets so that two complex (approximately analytic) wavelets can be formed. Therefore, they can be used to implement complex and directional wavelet transforms. The design procedure for the Double-Density Dual-Tree CWT is based on the flat-delay filter, spectral factorization and paraunitary filter bank completion. The solutions have vanishing moments and compact support. The resulting wavelets are much smoother than the Dual-Tree wavelets and unlike the Double-Density wavelets; they form approximate Hilbert transform pairs. The design procedure also includes a parameter L that determines the degree to which the approximation is satisfied.

The Double-Density Dual-Tree DWT proposed in this thesis is based on concatenating two oversampled DWTs. The filter bank structure corresponding to the Double-Density Dual-Tree DWT consists of two oversampled iterated filter banks operating in parallel similar to the Dual-Tree DWT. The oversampled filter bank is illustrated in figure 7.

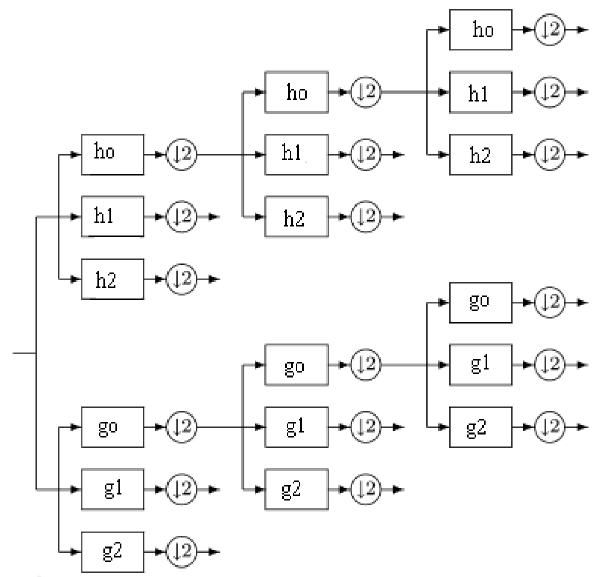


Fig 7: Iterated filter bank for the Double-Density Dual-Tree DWT.

The iterated oversampled filter bank corresponding to the implementation of the Double-Density Dual-Tree is illustrated in figure 8. We will denote the filters the first filter bank by $h_i(n)$ and the filters in the second filter bank by $g_i(n)$, for i=0,1,2 [28].

Note that in each of the filter banks to be considered, the synthesis filters are the time-reversed versions of the analysis filters. The goal will be to design the six FIR filters so that they do the following:-

- They satisfy the perfect reconstruction property.
- The wavelets form two (approximate) Hilbert transform pairs.
- The wavelets have specified vanishing moments.
- The filters are of short support.

Using the infinite product formula, it was shown that for two orthogonal wavelets to form a Hilbert transform pair, the scaling filters should be offset by a half sample. In that, a design problem was formulated for the minimal length scaling filters such that the wavelets each have a specified number of vanishing moments (K) and the half-sample delay approximation is flat at w=0 with specified degree (L).

The low pass filters $h_0(n)$, $g_0(n)$ fully determine the two orthogonal wavelet bases. We look for pairs of orthonormal wavelets where the low pass scaling filters have the form [28]

$$h_0(n) = f(n) * d(n)$$
$$g_0(n) = f(n) * d(L-n)$$

where d(n) the filter will be chosen to achieve the (approximate) half-sample delay. The first step of the design procedure will be to determine the appropriate filter to achieve the desired relationship between $h_0(n)$ and $g_0(n)$ [28]. In terms of the transfer functions, we have

$$H_0(z) = F(z)\, D(z)$$
$$G_0(z) = F(z)\, z^{-L} D(1/z)$$

$H_0(z)$ and $G_0(z)$ have the common divisor F(z), so we can write

$$G_0(z) = H_0(z) \frac{z^{-L} D(1/z)}{D(z)}$$

The design procedure depends on the design of an All pass filter with approximately constant fractional delay. Several authors have addressed the design of All Pass systems that approximate a fractional delay. The following formula for the maximally flat delay All Pass filter is adapted from Thiran's formula for the maximally flat delay all pole filter [28]. We can recognize that the transfer functions. The maximally flat approximation to a delay of τ samples is given by

$$A(z) = \frac{z^{-L} D(1/z)}{D(z)}$$

$$D(z) = 1 + \sum_{n=1}^{L} d(n)\, z^{-n}$$

with

$$d(n) = (-1)^n \binom{L}{n} \frac{(\tau - L)_n}{(\tau + 1)_n}$$

is an all pass system A(ω)=1. Therefore,

$$|G_0(w)| = |H_0(w)|, \quad |G1(w)| = |H1(w)|$$

and

$$|\Psi_{g(w)}| = |\Psi_{h(w)}|$$

If the All Pass system A(z) is an approximate half sample delay [28]

$$A(w) \approx e^{-jw/2} \quad \text{around } w = 0$$

or equivalently

$$A(z) \approx z^{-1/2} \quad \text{around } z = 1$$

If the $H_0(\omega)$ and $G_0(\omega)$ are low pass CQF filters with

$$G_0(w) = H_0(w)\, e^{-j(w/2)} \quad \text{for } |w| < \pi$$

then the corresponding wavelets are the Hilbert pair

$$\psi_g(t) = H\{\psi_h(t)\}$$

Equivalently, digital filter $g_0(n)$ is an half sample delay version of $h_0(n)$

$$g_0(n) = h_0(n - 1/2)$$

Using longer filters, we have obtained solutions that have both good smoothness and good half-sample delay properties. Note that $h_0(n)$ and $g_0(n)$ do not need to have linear phase in order for ψ_h and ψ_g to make a Hilbert transform pair, although it may be desirable for other reasons depending on the application [28].

This paper highlighted wavelet based enhancement of gray scale digital images corrupted by additive Gaussian noise. The Complex Double Density Dual Tree discrete wavelet transform outperforms in comparison with others wavelet transform in the highly corrupted images. In terms of image enhancement, the Double-Density complex wavelet transform performed much better at suppressing noise over the Double-Density discrete wavelet transform. However, to improve the performance further it is necessary to use a different threshold for each sub-band because for this transform the wavelets associated with different sub-bands have different norms. The simulation results indicate that the Complex Double Density Dual Tree discrete wavelet transform performances better than others wavelet transform.

V. EXPERIMENTAL RESULTS:

The properties of CWTs shift invariance and directionality are used in many areas of image processing like denoising, feature extraction, object segmentation and image classification. Here we shall consider the denoising example to analyze the effectiveness of the DDDT-CWT, different thresholds points and noise variance were selected from 0 to 45 and from 25 to 75 on Lena & Peppers images with the distance of 5 respectively. But optimal thresholds points were giving the minimum square error from the original image and less noise variance values were giving the more PSNR value, showing a great effectiveness in removing the noise compared to the classical discrete wavelet transform.

Lena images are corrupted with different noise variance ranging from 25 to 70. The first test involved reduction of Gaussian noise applied in an additive form on the ‘Lena’ . From the PSNR point of view, the results obtained are presented in table 1 and 2. Table 1 shows the PSNR value of the output image for ‘Lena’ image using Dual Tree complex wavelet transform, Double Density wavelet transform & Complex Double Density Dual Tree wavelet transform approaches.

Methods	Dual Tree	Double Density	Double Density Dual Tree
Noise Variance	**PSNR Value**	**PSNR Value**	**PSNR Value**
25	37.6	38.7	45.2
30	37.4	38.3	44.8
35	37.2	37.9	44
40	36.9	37.8	43.6
45	36.6	37.7	43.1
50	35.9	36.8	42.4
55	35.2	36	41.5
60	34.2	34.9	40.6
65	32.9	33.5	39.5
70	31.4	31.9	38.3

Table 1: PSNR Values on ‘Lena’ image.

From the RMS Error point of view, the results btained with different threshold points ranging from 0 to 45 are presented in table 2 and 3. Table 3 & 4 shows the RMS Vs. Threshold value of the output image for ‘Lena’ image with noise variance 45 & 55 using Dual Tree complex wavelet transform, Double Density wavelet transform & Complex Double Density Dual Tree wavelet transform approaches respectively.

Methods	Dual Tree	Double Density	Double Density Dual Tree
Threshold Point	**RMS Error**	**RMS Error**	**RMS Error**
0	91.2	93.7	83.4
5	87.9	91.2	79.8
10	85.1	89.3	76.6
15	83.3	87.4	74.8
20	82.1	86	73.3
25	81.2	85.1	72.4
30	80.5	84.2	71.8
35	80.2	83.8	71.2
40	79.6	82.4	70.9
45	79.3	81.2	70.7

Table 2: RMS Error Vs. Threshold Point with Noise Variance=45 for "Lena" Image

Methods	Dual Tree	Double Density	Double Density Dual Tree
Threshold Point	**RMS Error**	**RMS Error**	**RMS Error**
0	95	97.7	87.8
5	91	95	83.2
10	87.8	92.4	79.6
15	85.1	90.3	76.7
20	83.4	88.5	74.7
25	81.9	87.1	73.4
30	81.1	85.8	72.5
35	80.4	84.9	71.7
40	79.9	84.3	71.3
45	79.3	83.9	70.7

Table 3: RMS Error Vs. Threshold Point with Noise Variance=55 for "Lena" Image

Peak Signal to Noise Ratio at Noise Variance from 25 to 75 are obtained from Dual Tree complex wavelet transform, Double Density & Double Density Dual Tree complex wavelet transform methods for ‘Lena’ images are plotted and shown in the figure 8.

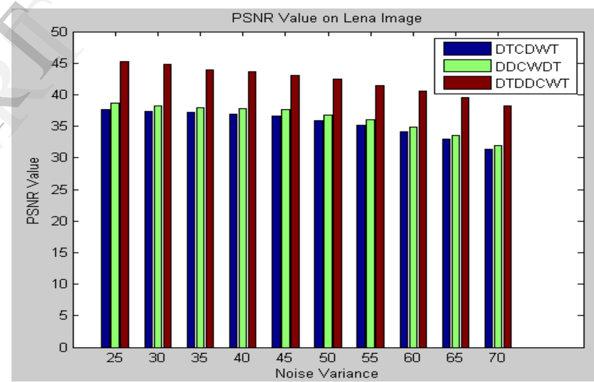


Fig 8: PSNR value of ‘Lena’ image

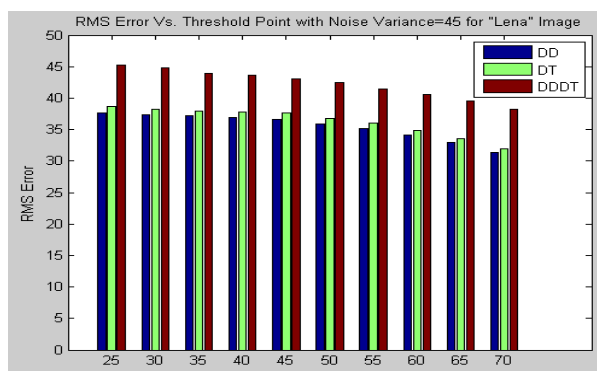


Fig 9: RMS error vs. Threshold point with noise variance=45 for ‘Lena’ Image

Root Mean Squared error at various threshold points from 0 to 45 are obtained from Dual Tree complex wavelet transform, Double Density & Double Density Dual Tree complex wavelet transform methods at Noise variance = 45 & 55 for 'Lena' images are plotted in Bar Chart & its graph are shown in the figure 9, 10, 11 & 12 respectively.

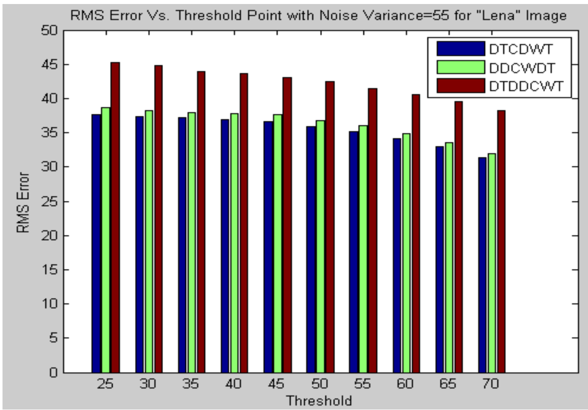


Fig 10: RMS error vs. Threshold point with noise variance=55 for 'Lena' Image

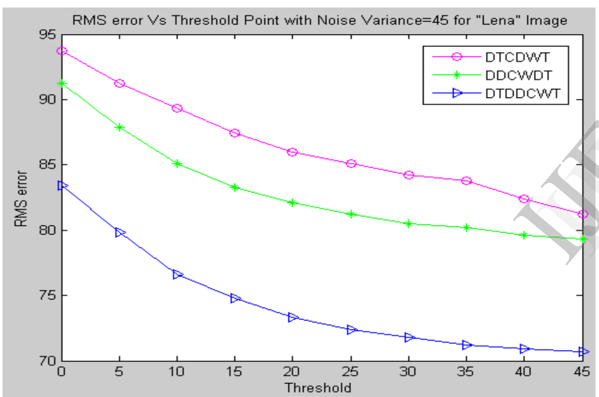


Fig 11: RMS error vs. Threshold point with noise variance=45 for 'Lena' Image

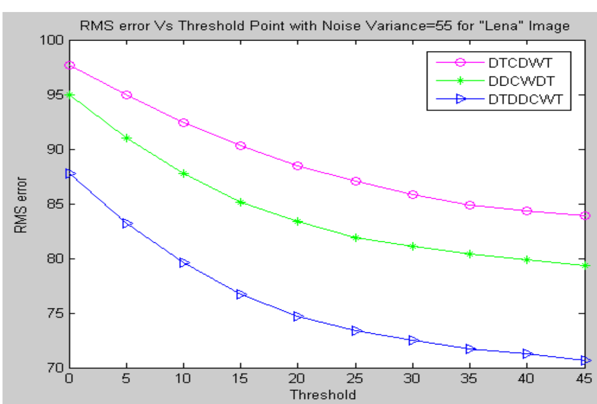


Fig 12: RMS error vs. Threshold point with noise variance=55 for 'Lena' Image


Fig13a: Original Image Fig13b: Noisy Image


Fig13c:Output from DTCWT Fig13d: Output from DDCWT

Fig 13e: Output from DDDTCWT

Fig 13: Output Images of 'Lena' when Noise variance=45

VI. CONCLUSIONS

Discrete wavelet transform has been used in digital image denoising more frequently due to its excellent spatial localization and multi-resolution characteristics which are similar to the theoretical models of the human visual system. Denoising using discrete wavelet transform produces superb results because Wavelet Transform has compaction property of having only a small number of large coefficients and large number of small coefficients. But discrete wavelet transform functions insufficiently in some of signal processing tasks due to strong shift dependence, lack of directional selectivity, aliasing and oscillations of the coefficients. To solve these problems, complex discrete wavelet transform algorithms have been proposed to represent an input signal by the magnitude and phase where the magnitude is shift invariant and the phase offset encodes the shift.

The complex wavelets are a useful tool and it has wider implications of the research. The DT-CWT due to its approximate shift invariance and improved directional selectivity outperforms in a wide range of applications. For denoising application, the experimental results for the complex wavelet transform methods display an improvement in accuracy over the standard discrete wavelet methods. For most of the applications the CWT also gives better results than the discrete wavelet transform. The complex wavelet models were found to be particularly good for image denoising.

Image Denoising using complex wavelet transform has been reviewed. Advantages, applications and limitations of popular standard DWT and its extensions are realized. Complex wavelet transform, a powerful extension to real valued DWT is thoroughly investigated, with the motivation to reduce the major limitations of standard DWT. The CWTs with their certain signal processing applications are collectively and comprehensively analyzed and its extensions in certain signal processing applications.

In this paper, we proposed a hybridizing model of Double Density Dual Tree CWT for denoising estimation and significantly improve the performance of the noisy image. Simulation and experimental results demonstrated on Lena image and found that the Complex Double Density Dual Tree wavelet transform outperforms a number of other existing wavelet transform techniques and it is particularly effective for the very highly corrupted images. Individual software codes are developed for simulation of Image Denoising using CWTs. Promising results are obtained using individual implementation of existing algorithms incorporating novel ideas into well-established frameworks.

These CWTs has some possible future research directions for some additional applications tasks and suggests such as segmentations, Texture synthesis, imaging radar such as Synthetic Aperture Radar etc. and there is also investigating the application of the DT-CWT to 3D data sets such as from medical imaging techniques. For these data sets analogous segmentation techniques can be used to identify different biological structures. A single complex wavelet transform could be used for detecting both texture and the object edges. Large complex wavelet coefficients correspond to edges in an image and a simple edge detector can be built by simply detecting such large coefficients.

VII. REFERENCES

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