Impact Force Analysis Of Valve Train Clearance Mechanism

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ABSTRACT

Impact force analysis of a valve-train clearance mechanism is done using finite element technique. The physical system which is selected for the analysis is a cam-driven-valve train mechanism with valve seat, available with present day automobiles with variety of components made of different materials. This method is sufficiently general to permit applications to other machine elements, such as gears, bearings etc or a combination thereof. Use of the method involved establishing an elastic model, specifying clearances, calculating overall stiffness matrix, overall mass matrix, overall damping matrices and determine the form of the equation of motion for each configuration for different clearances of the valve train mechanism and calculated

INTRODUCTION

The finite element techniques for structural analysis have been applied to the prediction of the elastic behavior of a rotating mechanism while it is being acted upon by its own inertial forces as well as arbitrary external forces[1]. In order to analyze a mechanism rather than a structure, it is necessary to consider two additional problems a mechanism will undergo rigid body motion, and it will usually experience large changes in geometry. The changes in geometry is handled easily with a relative algorithm, however accounting for the rigid body motion requires some skill, because the stiffness matrix for the mechanism is singular and the flexibility matrix, therefore, does not exist. One major assumption, which is usually applied to both mechanisms and structures, is that they are linear system and hence, are subject to methods of superposition. Even the large changes in geometry found in the mechanism are handled linearly in a piecewise fashion. Because all machinery is non-linear to some degree and it must contain

the impact force between pushrod and rocker arm. A computer program is developed to solve the various equations of overall stiffness matrix, overall mass matrix, overall damping matrices and few results, such as Impact forces and stresses which acting on the machine components for different speeds. Also this quantifies the maximum safe running speeds of mechanisms for different configurations depending upon the selection of materials and their mechanical properties.

Keywords: Impact force, Joint clearance, safe running speed, valve seat, finite element technique

clearances if it is to move, consider the classical approach to the design of a camdriven mechanism is to treat the mechanism as a system of rigid bodies[2]. Earlier analysis of cams briefly considered the problem of elasticity, but they dealt more with trying to optimize different cam profiles for minimum acceleration and jerk[3,4]. One early and notable contribution was that of Barken[5], who considered the valve train as an elastic system. However, he took the valve train and model it as a system of lumped masses and springs and then reduced it to an equivalent single degree of freedom spring-mass system, before solving the equation of motion. Johnson [6,7] analyzed the motion of cam mechanisms with one and two degrees of freedom and then with n-degrees of freedom. He used lumped masses and springs for his models, but discussed the desirability of using distributed parameters for the valve train model.

The problem of impact is discussed by Barkan[8] for a single degree of freedom cam

and follower system. However, he considers impact due to initial clearance only, that is, once the follower contacts the cam, it never leaves it. He also uses a coefficient of restitution, which is adequate for his system, because he is only interested in what happens after impact. As will be shown, this coefficient is of questionable value for the analysis of high speed, continuously operating machinery. Impact forces are also discussed by Johnson [9].

Dubowsky and Freudenstein[10,11] treated the problem of clearances in mechanisms, but concern is primarily their with the development of model of a joint rather than the treatment of an entire system. Briefly study of the literature shows the need for the analysis of machinery components particularly with clearances, hence, the present investigation is carried out on the impact force analysis of valve-train clearance mechanism using finite element technique. In the present analysis components of valve train mechanism are considered as bar and beam elements.



Figure 1 Cam-driven valve train mechanism with harmonic cam







Figure 3 Elements

ANALYSIS OF VALVE TRAIN MECHANISM

Analysis of valve train mechanism with harmonic cam as shown in fig.1. The system was modeled as collection of prismatic bar and beam elements as shown in fig.2 and fig.3. The mass and stiffness properties can be derived for each of the elements as shown in fig.3. In this analysis the standard properties from automobile valve train system are taken. Those properties are listed in table 1.

		Modulus of	Density	Dimensions		
Element	Material	Elasticity (E) N/mm ²	kg/m ³	Length (mm)	Diameter (mm)	
Push rod	Mild steel	202086	7720	228.6	7.62	
Left portion of the rocker arm	Cast Iron	89369	7250	25.4	15.24	
Right portion of the rocker arm	Cast Iron	89369	7250	38.1	15.24	
Valve	Hard Chromium	199143	7530	76.2	7.62	

Table 1: Standard dimensions

For valve spring, the values for mass and stiffness are 0.06 kg and 42.9 N/mm respectively.

Calculation of overall stiffness matrix

[k] =

 \Rightarrow

The overall stiffness matrix obtained by transferring stiffness matrices of all the elements from their local coordinate system to reference coordinate system and superposing all stiffness matrices. The overall stiffness properties for the valve train system can be written as

$$[k] = \sum_{i=1}^{5} [T]_{i}^{T} [k]_{i} [T]_{i}$$

Where $[T]_i$ = coordinate transformation matrix



Calculation of Overall Mass Matrix

Mass properties for the valve train system can be written as

$$[m] = \sum_{i=1}^{5} [\mathbf{T}]_{i}^{T} [m]_{i} [\mathbf{T}]_{i}$$

$$\Rightarrow [m] = \frac{p_{A}AL_{1}}{3} - \frac{p_{A}AL_{1}}{6} = 0 = 0 = 0 = 0 = 0$$

$$\frac{p_{A}AL_{1}}{3} + \frac{13p_{2}A_{2}L_{2}}{35} - \frac{11p_{2}A_{2}L_{2}^{2}}{210} = 0 = 0 = 0 = 0$$

$$\frac{p_{2}A_{2}L_{2}^{3}}{105} = 0 = 0 = 0 = 0$$

$$\frac{p_{2}A_{2}L_{2}^{3}}{105} = 0 = 0 = 0 = 0$$

$$\frac{p_{2}A_{2}L_{2}^{3}}{105} = 0 = 0 = 0$$

$$\frac{p_{2}A_{2}L_{2}^{3}}{105} = \frac{-p_{2}A_{2}L_{2}^{3}}{100} = 0$$

[D]

Calculation of overall Damping Matrix

To accomplish this damping for the system, a viscous damping ratio $[\xi_i]$ for each of the seven elastic mode shapes of the system, was selected. These mode shapes are calculated as to take the value of natural coordinate is unity at node 1, and zero at the other node. The system damping matrix[C] was then computed. Knowing matrices [m] and [k], the eigen values $[\emptyset]$, representing the elastic mode shapes of the system are computed from generalized Jacobi method, defining a set of orthogonal coordinates $\{\eta\}$ as

 $\{q\} = [\emptyset] \{\eta\}$

Diagonal stiffness and mass matrices are computed as

$$[K] = [\emptyset]^{T} [k] [\emptyset]$$

 $[M] = [\emptyset]^{T} [m] [\emptyset]$

Orthogonal damping matrix is,

$$[D] = [\emptyset]^{T} [C] [\emptyset]$$

$$= \begin{bmatrix} 2\xi_1 \ \omega_1 \ M_1 & 0 & 0 & \dots \\ 0 & 2\xi_2 \ \omega_2 \ M_2 & 0 & \dots \\ 0 & 0 & 2\xi_3 \ \omega_3 \ M_3 & \dots \\ - & - & - & \dots \end{bmatrix}$$

Then, Damping Matrix [C] = ($[\emptyset]^T)^{-1} [D]$ [$\emptyset]^{-1}$

However, in practice, the inverse of the model matrix is a large computational effort. Instead, taking advantage of orthogonally properties of the mode shapes, one can deduce the following expression for the system damping matrix, namely,

$$[C] = [M] \left(\sum_{i=1}^{N} \frac{2 \, \xi_n \, \omega_n}{M_n} \, \{\phi\}_n \, \{\phi\}_n^T \right) [M]$$

It is now assumed that $\xi_j = 0.05$ for the fundamental, or lowest, frequency mode shape and $\xi_j = 0.01$ for all the higher frequency mode shapes. This lowest frequency mode shape is less than the next higher frequency by a factor of about thirty and as might be expected, it closely resembled the rigid body motion of the valve train as the valve spring was submerge

softest spring in the system.

When the valve train mechanism has clearance

in between pushrod and Rocker- arm, then overall stiffness matrix is

[/	k] =						
$\frac{E_1 A_1}{L_1}$	$\frac{E_1 A_1}{L_1}$	0	0	0	0	0	0
$\frac{E_1 A_1}{L_1}$	$\frac{E_1 A_1}{L_1}$	0	0	0	0	0	0
0	0	$\frac{4E_2I_2}{L_2}$	0	0	$\frac{2E_2I_2}{L_2}$	0	$\frac{6E_2I_2}{L^22}$
0	0	0	$\frac{\frac{12E_2I_3}{L_3^3} + \frac{E_3A_4}{L_4} + k_s}{L_4}$	$\frac{-6E_2I_3}{L_3^2}$	$\frac{-6E_2I_3}{L_3^2}$	$\frac{E_3 A_4}{L_4}$	0
0	0	0	$\frac{-6E_2I_3}{L_3^2}$	$\frac{4E_2I_3}{L_3}$	$\frac{2E_2I_3}{L_3}$	0	0
0	0	$\frac{\frac{2E_2I_2}{L_2}}{L_2}$	$\frac{-6E_2I_3}{L_3^2}$	$\frac{2E_2I_3}{L_3}$	$\frac{4E_2I_2}{L_2} + \frac{4E_2I_3}{L_3}$	0	$\frac{6E_2I_2}{L^22}$
0	0	0	$\frac{E_3 A_4}{L_4}$	0	0	$\frac{E_3 A_4}{L_4}$	0
0	0	$\frac{\frac{6E_2I_2}{L^2_2}}{L^2_2}$	0	0	$\frac{6E_2I_2}{L^2_2}$	0	$\frac{12E_2I_2}{L^32}$
				\mathbf{X}			

When the valve train mechanism has clearance in between pushrod and Rocker- arm, then

overall

is

matrix

mass

[m] =

$\rho_1 A_1 L_1$	$\frac{-\rho_1 A_1 L_1}{\epsilon}$	0	0	0	0	0	
$-\rho_1 A_1 L_1$	$\rho_1 A_1 L_1$	0	0	0	0	0	0
6	3	0	0	0	0	0	0
0	0	$\frac{\rho_2 A_2 L_2^3}{105}$	0	0	$\frac{-\rho_2 A_2 L_2^3}{140}$	0	$\frac{1 1 \rho_2 A_2 L^2_2}{2 1 0}$
0	0	$0^{\frac{1}{2}}$	$\frac{3\rho_2 A_3 L_3}{35} + \frac{\rho_3 A_4 L_4}{3} + \frac{M_s}{3}$	$\frac{-1 l \rho_2 A_3 L^2_3}{2 1 0}$	$\frac{13\rho_2A_3L^2{}_3}{420}$	$\frac{-\rho_{3}A_{4}L_{4}}{6}$	0
0	0	0	$\frac{-1 1 \rho_2 A_3 L^2{}_3}{2 1 0}$	$\frac{\rho_2 A_3 L_3^3}{105}$	$\frac{-\rho_2 A_3 L_3^3}{140}$	0	0
0	0	$\frac{-\rho_{2}A_{2}L^{3}}{140}$	$\frac{13\rho_2 A_3 L^2_3}{420}$	$\frac{-\rho_2 A_3 L^3{}_3}{140}$	$\frac{\rho_2 A_2 L_2^3}{105} + \frac{\rho_2 A_3 L_3^3}{105}$	0	$\frac{-13\rho_2 A_2 L_2^2}{420}$
0	0	0	$\frac{-\rho_{_3}A_{_4}L_{_4}}{6}$	0	0	$\frac{\rho_{3}A_{4}L_{4}}{3}$	0
0	0	$\frac{1 l \rho_2 A_2 L^2_2}{2 1 0}$	0	0	$\frac{-13\rho_2 A_2 L^2_2}{420}$	0	$\frac{13\rho_2A_2L_2}{35}$
L							

Calculation of Impact force between pushrod and Rocker-arm

The system will have only two clearances i.e the clearance at cam and valve seat, clearance between pushrod and rocker arm, the velocity of both the pushrod and rocker arm will have the same, but unknown, velocity at the point of impact. This problem can be solved by using the principle of conservation of momentum just before and just after impact.

The equation relating the momentum at $t=t_n$ just before impact to the momentum at $t=t_{n+1}$, just after impact can be written in the matrix form as shown.

 $[m] \{ q \}^{n} + [F] \{ \Delta t \} = [m] \{ q \}^{n+1}$

In fig.3 assume that q_2 defines the motion of upper end of the pushrod, and that a new coordinate q_8 defines the linear motion of the rocker arm formerly defined by q_2 as shown in fig 4. When there is contact, q_8 does not exist. Now assume that the pushrod and rocker arm

System momentum just before impact is

are about to impact. q_2 and q_8 will have different values just before impact, but will be identical just after the impact.



Fig..4 Elements

 $[m]{q}^{n} =$ System momentum just before impact (is a known quantity)

Let
$$\{G\}^n = [m]\{q\}^n$$

	System momentum just before impact is												
	$G_{1^{\circ}}$	n	$\frac{\rho_{\rm I}A_{\rm I}L_{\rm I}}{3}$	$\frac{-\rho_{\rm i}A_{\rm i}L_{\rm i}}{6}$	0	0	0	0	0	0		(dq ₁ /dt)	n
	G_2		$\frac{-\rho_{1}A_{1}L_{1}}{6}$	$\frac{\rho_1 A_1 L_1}{3}$	0	0	0	0	0	0		dq ₂ /dt	
	G ₃		0	0	$\frac{\rho_2 A_2 L_2^3}{105}$	0	0	$\frac{-\rho_2 A_2 L_2^3}{140}$	0	$\frac{1 l \rho_2 A_2 L^2_2}{2 1 0}$		dq ₃ /dt	
ļ	G_4	=	0	0	0	$\frac{13\rho_2A_3L_3}{35} + \frac{\rho_3A_4L_4}{3} + \frac{M_s}{3}$	$\frac{-1 l \rho_2 A_3 L^2_3}{2 1 0}$	$\frac{13\rho_2 A_3 L^{2_3}}{420}$	$\frac{-\rho_{3}A_{4}L_{4}}{6}$	0		dq4/dt	
	G_5	$\left \right $	0	0	0	$\frac{-1\wp_2 A_3 L^2{}_3}{210}$	$\frac{\rho_2 A_3 L_3^3}{105}$	$\frac{-\rho_2 A_3 L_3^3}{140}$	0	0	K	dq ₅ /dt	$\left.\right\rangle$
	G_6		0	0	$\frac{-\rho_2 A_2 L_2^3}{140}$	$\frac{13\rho_2 A_3 L^2_3}{420}$	$\frac{-\rho_2 A_3 L^{3_3}}{140}$	$\frac{\rho_2 A_2 L_2^3}{105} + \frac{\rho_2 A_2}{1}$	$\frac{A_3L_3}{05}$ 0 -	$\frac{-13\rho_2A_2L^2_2}{420}$		dq ₆ /dt	
	G ₇		0	0	0	$\frac{-\rho_{_3}A_{_4}L_{_4}}{6}$	0	0	$\frac{\rho_{3}A_{4}L_{4}}{3}$	0		dq ₇ /dt	
	G ₈		0	0	$\frac{1 l \rho_2 A_2 L^2_2}{210}$	0	0	$\frac{-13\rho_2 A_2 L_2^2}{420}$	0	$\frac{13\rho_2A_2L_2}{35}$		dq ₈ /dt	
	\ /										1	\ /	

Since
$$F_2 = -F_8$$

 $\{F\}\Delta t = \begin{cases} 0 \\ \Delta t \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\Delta t \end{cases} F_2$

and, since $dq_2/dt = dq_8/dt$. The system momentum just after impact is

Here
$$\begin{cases} dq_1/dt \\ dq_2/dt \\ dq_3/dt \\ dq_4/dt \\ dq_5/dt \\ dq_6/dt \\ dq_7/dt \end{cases}$$
 and F₂ are unknown quantities

$$\{G\}^{n} = [m] \{dq/dt\}^{n+1} - [F] \{\Delta t\}$$

 \Rightarrow

$$\left\{ \begin{array}{c} G_{1} \\ G_{2} \\ G_{3} \\ G_{4} \\ G_{5} \\ G_{6} \\ G_{7} \\ G_{8} \end{array} \right\}^{n} \left[\begin{array}{c} \frac{\rho_{A}A_{L}}{3} & \frac{-\rho_{A}A_{L}}{6} & 0 & 0 & 0 & 0 & 0 \\ \frac{-\rho_{A}A_{L}}{6} & \frac{\rho_{A}A_{L}}{3} & 0 & 0 & 0 & 0 & 0 & -\Delta t \\ \frac{-\rho_{A}A_{L}}{6} & \frac{\rho_{A}A_{L}}{3} & 0 & 0 & 0 & 0 & -\Delta t \\ 0 & \frac{11\rho_{A}A_{L}^{1/2}}{210} & \frac{\rho_{2}A_{2}L^{2}}{105} & 0 & 0 & \frac{-\rho_{A}A_{L}^{1/2}}{140} & 0 & 0 \\ \frac{13\rho_{A}A_{L}}{35} + \frac{\rho_{A}A_{L}}{3} + \frac{M_{*}}{3} & \frac{-1\rho_{A}A_{L}^{1/2}}{210} & \frac{13\rho_{A}A_{L}^{1/2}}{420} & \frac{-\rho_{A}A_{L}}{6} & 0 \\ \frac{10}{6} & 0 & 0 & \frac{-11\rho_{A}A_{L}}{35} + \frac{\rho_{A}A_{L}}{35} + \frac{\rho_{A}A_{L}}{3} + \frac{M_{*}}{3} & \frac{-1\rho_{A}A_{L}^{1/2}}{105} & \frac{13\rho_{A}A_{L}^{1/2}}{420} & \frac{-\rho_{A}A_{L}A_{*}}{6} & 0 \\ \frac{13\rho_{A}A_{L}^{1/2}}{420} & \frac{-\rho_{A}A_{L}^{1/2}}{140} & \frac{13\rho_{A}A_{L}^{1/2}}{140} & \frac{\rho_{A}A_{L}^{1/2}}{105} + \frac{\rho_{A}A_{L}}{105} & 0 & 0 \\ \frac{13\rho_{A}A_{L}^{1/2}}{35} & \frac{-\rho_{A}A_{L}^{1/2}}{140} & \frac{13\rho_{A}A_{L}^{1/2}}{105} & \frac{\rho_{A}A_{L}^{1/2}}{105} & 0 & 0 \\ \frac{13\rho_{A}A_{L}^{1/2}}{35} & \frac{11\rho_{A}A_{L}^{1/2}}{140} & \frac{13\rho_{A}A_{L}^{1/2}}{10} & 0 & 0 & \frac{-13\rho_{A}A_{L}^{1/2}}{3} & 0 \\ 0 & \frac{13\rho_{A}A_{L}}{35} & \frac{11\rho_{A}A_{L}^{1/2}}{210} & 0 & 0 & \frac{-13\rho_{A}A_{L}^{1/2}}{420} & 0 & \Delta t \end{array} \right]$$

 $\{G\}^n = [A] \{S\}^{n+1}$

Where $\{G\}^n$ = System momentum just before impact.(is a known quantity)

$$\Rightarrow \{S\}^{n+1} = [A]^{-1} \{G\}^n$$

thus impact force F(2) between pushrod and rocker arm can be determined.

DISCUSSION OF EXPERIMENTAL RESULTS.

The system considered in the present investigation is first shown with contact occurring only at the cam. The valve seat has been moved up so that the valve can't come in contact with it. Fig 5 shown the pushrod motion superimposed with respect to cam rotation. As indicated fig 5, the pushrod leaves the cam at about 243° and impacts at about 342° . Fig.6 shows that F(1), the force on the pushrod, has become zero, and hence separation has occurred. As it is expected, the impact creates vibrations in the valve train, but these are damped out in about one-half cycle, has a result the impact is very mild. It is known that the yield stress of mild steel is 225.6 N/mm^2 and it is considered that the factor of safety for this material is three. So, working stress of mild steel is 75.2 N/mm^2 . The maximum stress on pushrod, when cam is run at 10192 rpm, is 75.06 N/mm^2 . So, for the valve train mechanism without valve seat, the maximum possible running speed of cam must be equal to or less than 10192 rpm, otherwise the pushrod may behave inelasticity or it may fail. Consider valve train mechanism with valve seat, the valve seat was moved into position so that the valve would impact it. Fig.7 shows that the valve seat snatches the pushrod from the cam when the valve impacts the seat. The forces on the valve and pushrod are shown in fig.8 and fig. 9. It is known that the yield stress of hard chromium is 637.6 N/mm² and it is considered that the factor of safety for this material is 15, So working stress of hard chromium is 42.5 N/mm². The maximum stress on pushrod, as well as on valve, when cam is running at 1224 rpm is 63.4 N/mm² and 41.1 N/mm², respectively. It

is known that the ultimate stress of cast iron is 99.55 N/mm^2 and it is considered that the factor of safety for this material is 20. So, working stress of cast iron becomes 5 N/mm^2 . If a clearance is allowed to occur at a point with in the valve train, such as between the pushrod and rocker arm, then the maximum stress which act on the rocker arm is 1.25 N/mm^2 . So, for the valve train mechanism, with valve seat the maximum possible running speed is limited to 1224 rpm so as to avoid plastic behaviour or failure of the pushrod, the rocker arm or the valve.







1224 rpm, with valve seat



CONCLUSIONS

Impact force and Impact Stress analysis of valve-train clearance mechanism is done using finite element technique. This analysis is carried out on a cam-driven-valve train mechanism with valve seat, available with present day automobiles with variety of components made of different materials. This method is sufficiently general to permit application on other machine elements, such as gears, bearings etc., or a combination thereof. Use of this method is involved, establishing an elastic model, specifying the clearances and determining the form of equations of motion for each configuration. Also this quantifies the maximum safe running speeds of mechanisms for different configurations depending upon the materials used. For the present cam driven valve train mechanism with valve seat, the maximum allowable speed is found to be 1224 rpm. А computer program is written

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exclusively for this purpose to solve various equations of the valve-train mechanism, for different configurations. However, further work is required to obtain more precise model, so as to increase the precision, the two dimensional or three dimensional elements may be used for the finite element modeling and also the effect of buckling and the effect of temperatures may be included.

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