International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 08, August-2016

Implementation of a Self Tuning Regulator and Optimal Design of Advanced Control Strategies with Full order Observer for Stepper Motors

R. Narendran
Department of Instrumentation Engineering,
Madras Institute of Technology Campus, Anna University,
Chennai - 600044, India

V Anand
Department of Instrumentation Engineering,
Madras Institute of Technology Campus, Anna University,
Chennai - 600044, India

***Abstract*—This paper deals with robust control of speed and position profiles in Permanent Magnet Stepper Motor (PMSM) using high performance control strategies. At first, a Self Tuning Regulator (STR) namely Minimum Variance Control is presented by minimizing an objective function. Thereupon, a Lyapunov based Controller is employed in order to eliminate the rigorous recursive least square identification procedure carried out in the aforementioned scheme. Also, a full order augmented observer is proposed to estimate the unknown states of rotor angular speed in PMSM. In order to get the best tracking results with the elimination of oscillations and for modulation of load torque, a nonlinear torque modulator is developed. Additionally control of phase current vectors along with global stability using Lyapunov candidate is ensured. Finally, all the proposed schemes are evaluated and compared by simulation results and International Standards of Performance Indices.**

Keywords—Permanent Magnet Stepper Motor; Controller; Lyapunov; Minimum Variance; Torque modulator

I. INTRODUCTION

Nonlinear Control of systems [1] is one area of research, which has been studied extensively for the past 20 years. The Permanent Magnet Stepper Motor (PMSM) [2, 3] is a kind of stepper motor whose position, speed and current control are of significant use in many fields like biomedical instrumentation, robotics, and satellite positioning and even in defence. Permanent Magnet Stepper Motor (PMSM) works on the principle of electromagnetism. It is an electromechanical device that consists of a rotor which is a permanent magnet and a stator which is essentially an electromagnet. Here the rotor gets magnetized and moves in a revolving magnetic field when the stator is energized by excitation. This excitation could be microstepping, full or half stepping. Several feedback control methods have been studied in order to improve the current tracking performance of microstepping [4]. Thus the rotor is made to rotate by the variation of voltage between the windings. Also the PMSM can be rotated at different step angles like 1.8 or 3.6 degrees and so on. A technique of exact feedback linearization using full state feedback was proposed with extensions to the partial state feedback in [5, 6, 13]. Direct Quadrature (DQ) transformation is also employed in order to make the system dynamics linear [7], thereby bringing about an improvement in the position tracking performance. To guarantee asymptotic tracking and stability, a nonlinear adaptive controller is proposed in [8]. Also various robust control schemes for machine systems especially the PMSM were discussed in [9-11], while ensuring the stability of the system dynamics. A servo compensator based controller was proposed in [9] to improve transient response. This adaptive controller thus proposed has unknown parameters and so it can adapt to unfavourable conditions. The angular position tracking was developed using sliding mode control algorithm in [21] and with these measurements, some robust control schemes were presented in [12, 15]. Simple field weakening methods for position control of Permanent Magnet Stepper Motor (PMSM) were explored in [16] and in [14, 20], they were combined with backstepping control. The minimum variance controller for the optimal tracking of position and speed in PMSM has also been elaborated in [17-19]. Control of Motor currents reduces the motor model in to a second order model as in [23, 24] and the idea of passivity is ventured in [25].

This paper has been split into 4 parts. The first part being the discussion about the Mathematical model of Permanent Magnet Stepper Motor (PMSM) and the Direct Quadrature (DQ) transformation involved in it and the second part explains about the various control strategies implemented in this paper. The third part then outlines the simulation results, the different motor parameters used along with the comparison of the controllers using performance indices. The fourth and final part finally presents the concluding remarks and the references used. For nonlinear system like PMSM which has a continuously changing process variable i.e. position and speed vector, effective control action can be executed by transforming its equations and variables into DQ frame and then applying the required control scheme depending upon its application.

II. MATHEMATICAL MODEL OF PMSM

In this paper we have considered a two phase Permanent Magnet Stepper Motor. The Mathematical dynamics of two phase PMSM model [1], [2], [4], [22] is

$$\dot{\theta} = \omega$$
$$\dot{\omega} = \frac{1}{J}\left(-K_m i_a \cos(\tfrac{\pi}{2} - N_r\theta)\right) + K_m i_b \sin(\tfrac{\pi}{2} + N_r\theta) - B\omega - \tau_F) \quad (1)$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 08, August-2016

$$i_a' = \frac{1}{L}\left(v_a - R_p\, i_a + K_m \omega \cos\left(\frac{\pi}{2} - N_r\theta\right)\right)$$
$$i_b' = \frac{1}{L}\left(v_b - R_p\, i_b + K_m \omega \cos\left(\frac{\pi}{2} - N_r\theta\right)\right)$$

where v_a, v_b are voltages (V) and i_a, i_b are current (A) vectors in phases A and B respectively. Also ω is rotor speed (rad/sec), R_p is the resistance (Ω) of the phase winding, θ is the rotor position (rad), K_m is the motor torque constant (N.m/A) and N_r is the number of rotor teeth. Also L is the inductance of the phase winding (H), J is the moment of inertia of the motor (kg.m2) while B is a viscous friction coefficient (N.m.s/rad). Here τ_F is the friction torque (N.m). The DQ transformed voltage and current vectors [4] are given as follows:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} \sin(\frac{\pi}{2} + N_r\theta) & \cos(\frac{\pi}{2} - N_r\theta) \\ -\cos(\frac{\pi}{2} - N_r\theta) & \sin(\frac{\pi}{2} + N_r\theta) \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix}$$

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \sin(\frac{\pi}{2} + N_r\theta) & \cos(\frac{\pi}{2} - N_r\theta) \\ -\cos(\frac{\pi}{2} - N_r\theta) & \sin(\frac{\pi}{2} + N_r\theta) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} \qquad (2)$$

In (2), i_d and i_q are the direct and quadrature currents respectively while $v_d$and $v_q$are direct and quadrature voltages respectively. Finally by applying DQ transformation [12] technique to (1), we get (3)

$$i_d' = \frac{1}{L}(v_d - R_p i_d + N_r L\omega i_q)$$
$$i_q' = \frac{1}{L}(v_q - R_p i_q + N_r L\omega i_d - K_m\omega) \qquad (3)$$
$$\dot\omega = \frac{1}{J}(K_m i_q - B\omega - \tau_F)$$
$$\dot\theta = \omega$$

III. ADVANCED CONTROL STRATEGIES

A. Minimum Variance Control

One of the prominent adaptive control schemes which are inherently self-tuning in nature is the Minimum variance controller. In our paper, the Minimum Variance Controller is clubbed with conventional PID controller to reduce the additional errors in it. The PID controller has a controller output where the error input is magnified K_p times, accumulated K_i times and the future values are incorporated K_d times. This controller takes into account the Recursive Least Square Estimator Algorithm in its parameter identification and DQ transformation. Consider the following Controlled Auto Regressive Moving Average model with Exogenous input (CARMAX):

$$P(q^{-1}) * y(k) = R(q^{-1})q^{-m}U(k) + S(q^{-1})e(k) \qquad (4)$$

where P, R and S are polynomials functions of the forward shift operator ‘q’ with m delay determined by the recursive least square estimator. The performance of the above the system at the discrete time k is given below:

$$I = E\{[y(k+m) - \dot y(k+m)]^2\} \qquad (5)$$

The Minimum Variance Controller has the objective function which it tries to minimize in order to take the control action. In (5) I is the objective function. The minimization of the mean square error between the predicted output and the desired value $\dot y(k+d)$ is done in the above equation after simplifying (5) where U (k) is the input. If the desired set point is zero, then $\dot y(k+d) = 0$.

$$\therefore I = E\{[y^2(k+m)^2]^2\} \qquad (6)$$

The process output at time $(k+m)$ is as follows:

$$y(k+m) = T(q^{-1})/S(q^{-1})Y(k) + B(q^{-1})/S(q^{-1})R(q^{-1}) + N$$

where $N = B(q^{-1})\, e(k+m)$ and T and B are unique polynomials defined as:

$$T(q^{-1}) = t_0 + t_1q^{-1} + t_2q^{-2} + \cdots t_{d-1}q^{-(d-1)}$$
$$B(q^{-1}) = 1 + b_1q^{-1} + b_2q^{-2} + \cdots b_{d-1}q^{-(d-1)}$$

If $B(q^{-1})P(q^{-1}) + q^{-d}T(q^{-1}) = S(q^{-1})$, then equation (7) becomes,

$$I = E\{[(T/S)y(k) + (RB/G)u(k) + Be(k+m)]^2\} \qquad (7)$$

The aim is to minimize equation (7). Control variable $U(k)$ depends upon the two previous values of y and u ach. Here the control strategy proposed is given by:

$$U(K) = -\left(T(q^{-1})/R(q^{-1}) + B(q^{-1})\right)y(k)$$
$$A(k) = L(k-1) + N \qquad (8)$$
$$A(k-1) = \theta(1) * C(k)$$
$$N = \sum_{i=1}^{n-1} \theta(i+1)C(k-1) - \theta(b+i+1)U(k-i)$$

Finally,$U(k) = A(k)/\theta(k)$ where ‘b’ represents the elements in the ‘θ’ vector and C is the positional errors in measurement. The Minimum Variance structure is shown in Fig.1. The controller had considerable amount of error in Fig.2 and an overshoot is also present in its rotor position trajectory as in Fig.3.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 08, August-2016

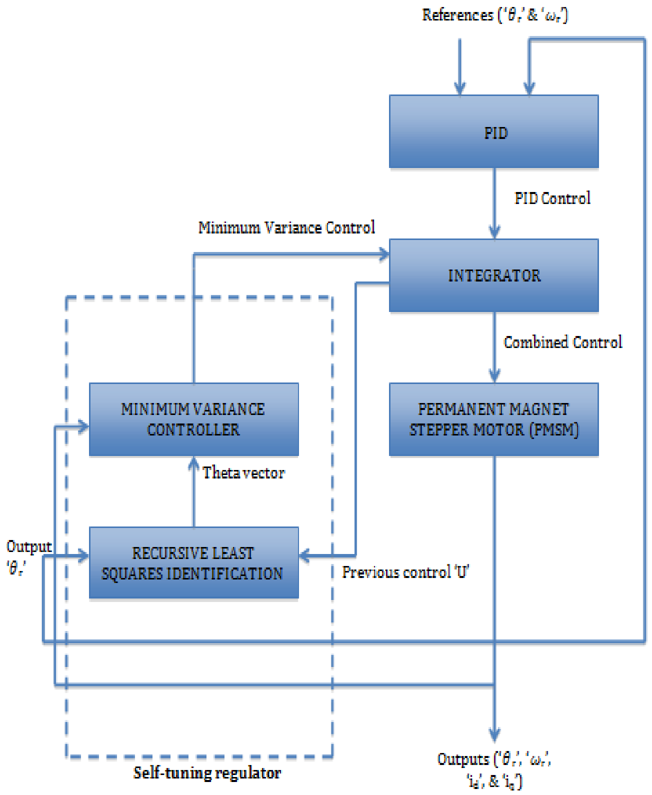

Fig.1: Minimum Variance Controller Scheme

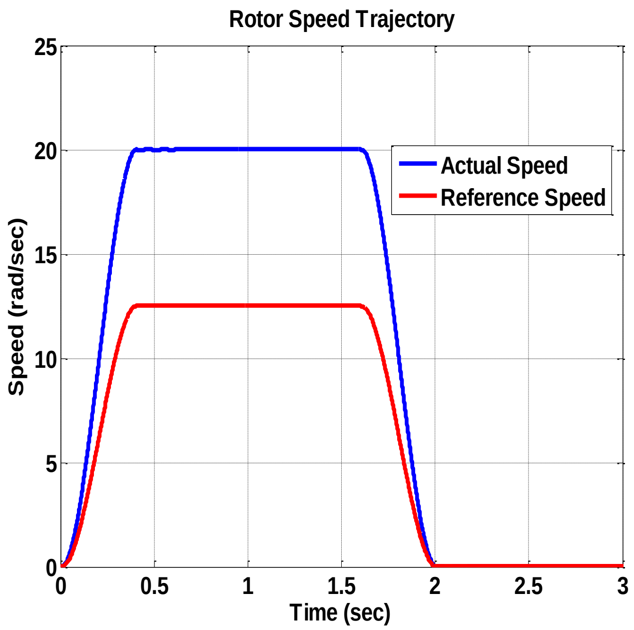

Fig.2: Tracking of Rotor Speed Trajectory

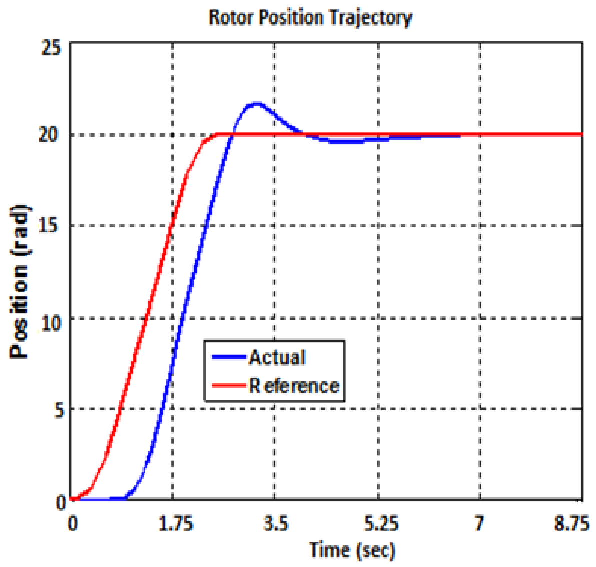

Fig.3: Tracking of Rotor Position Trajectory

B. Lyapunov Function

An additional recursive least square estimator algorithm which is required in the previous controller makes it quite complex in nature. Also the control action is effective only if more iteration is done and the sampling size is very small. Thus a comparatively simpler controller is needed which leads us for the use of a Lyapunov based controller.

There are three common modes of excitation in PMSM. They are full step, half step and microstepping. Microstepping is preferred over the previous two for its smooth tracking trajectory. Since the stepper motors move step by step, their smoothness is not in fluid rotation. Hence, an input in which the step size is very small is preferred. This is the reason for the microstepping excitation which is essentially a series of step inputs sent immediately one after the other within one complete cycle. It can have 8 or 16 or more steps within a single cycle. If the number of step sizes is increased, then it results in a sinusoidal wave. Since there are two phases, the inputs are follow the order in which one wave is 900 phase shifted from one another. Hence, the inputs can be designed as follows:

$$V_a = V_{max}\sin\left(\frac{\pi}{2} + N_r\theta\right)$$

$$V_b = V_{max}\cos\left(\frac{\pi}{2} - N_r\theta\right)$$

where V_{max} is the maximum amplitude of the microstepping voltage. To achieve the desired position θ^d the inputs are modified as follows:

$$V_a{}^d = V_{max}\sin\left(\frac{\pi}{2} + N_r\theta^d\right) \qquad (9)$$

$$V_b{}^d = V_{max}\cos\left(\frac{\pi}{2} - N_r\theta^d\right)$$

Where $V_a{}^d$ and $V_b{}^d$ are the desired input voltages in phases A and B respectively.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 08, August-2016

C. Controller Design

A Lyapunov Function is a scalar function V(y) defined on a domain P which is continuous and positive definite if its first order partial derivative V'(y) < 0. It is then asymptotically stable and converges to zero at infinity. From the previous section, it can be inferred that $i_a(t)$ and $i_b(t)$ converges to $\frac{V_a{}^d}{R_p}$ and $\frac{V_b{}^d}{R_p}$ respectively. Hence,

$$i_a{}^d = \frac{V_{max}}{R_p}\sin(\frac{\pi}{2} - N_r\theta^d)$$
$$i_b{}^d = \frac{V_{max}}{R_p}\cos(\frac{\pi}{2} - N_r\theta^d)$$
$$i_a{}^d = \frac{V_a{}^d}{R_p} \qquad (10)$$
$$i_b{}^d = \frac{V_b{}^d}{R_p}$$

The errors in current is given as

$$e_a = i_a{}^d - i_a$$
$$e_b = i_b{}^d - i_b$$

By Lyapunov's method, a suitable Lyapunov candidate function considering errors in both the phases would be

$$V_1 = \frac{1}{2}e_a{}^2 + \frac{1}{2}e_b{}^2 \qquad (11)$$

The time derivative of the Lyapunov function is given as:

$$\dot{V}_1 = \frac{dV(e_a, e_b)}{dt}$$
$$= \frac{1}{2}.2.e_a.\dot{e_a} + \frac{1}{2}.2.e_b.\dot{e_b}$$
$$= e_a\left(\dot{i}_a^d - \dot{i}_a\right) + e_b\left(\dot{i}_b^d - \dot{i}_b\right)$$
$$= e_a\left(\dot{i}_a^d - \frac{\left[v_a - R_p i_a + K_m\omega\cos\left(\frac{\pi}{2} - N_r\theta\right)\right]}{L}\right) + e_b\left(\dot{i}_b^d - \frac{\left[v_b - R_p i_b - K_m\omega\sin\left(\frac{\pi}{2} + N_r\theta\right)\right]}{L}\right)$$

The function V_1 is asymptotically stable if $\dot{V}_1(\dot{e_a}, e_b) < 0,\ \ e_a, e_b \neq 0$ i.e., $\dot{V}_1(\dot{e_a}, e_b)$ is a negative definite function. Here the control inputs v_a and v_b are designed in such way that $\dot{V}_1(\dot{e_a}, e_b)$ becomes negative definite.

$$v_a = \left(R_p i_a - K_m\omega\cos\left(\frac{\pi}{2} - N_r\theta\right)\right) + L(\dot{i}_a^d + \rho e_a)$$
$$(12)$$
$$v_b = \left(R_p i_b + K_m\omega\sin\left(\frac{\pi}{2} - N_r\theta\right)\right) + L(\dot{i}_b^d + \rho e_b)$$

Here ρ is any controller gain. The equation thus becomes,

$$\dot{V}_1 = -\rho e_a{}^2 - \rho e_b{}^2$$

Here both $e_a{}^2$ and $e_b{}^2$ are positive. If ρ is chosen as a positive value, $\dot{V}_1$ is always less than zero and the errors converge to zero as $t \to \infty$. The tracking performances for speed and position in Lyapunov controller is shown below in Fig.4 and Fig.5 respectively.

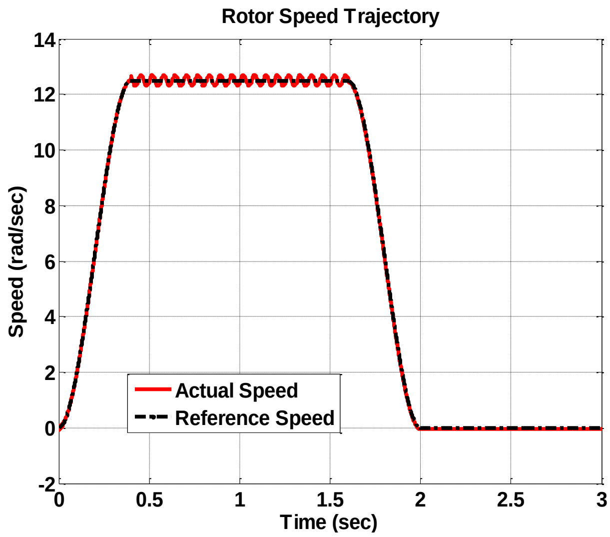

Fig.4: Tracking of Rotor Speed Trajectory by Lyapunov Controller

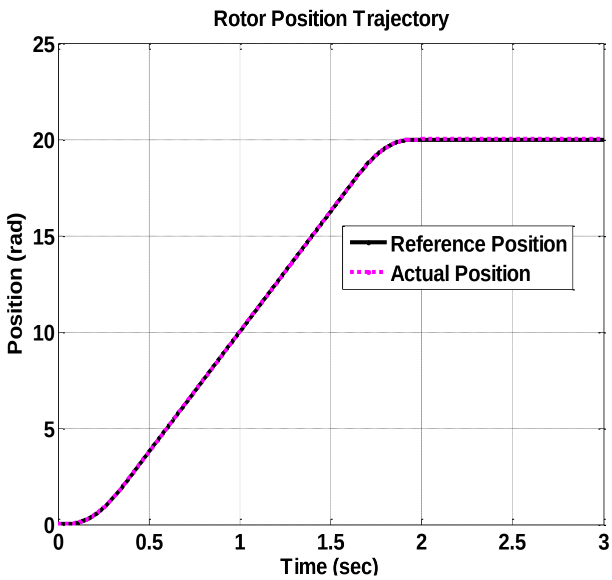

Fig.5: Tracking of Rotor Position Trajectory by Lyapunov Controller

D. Observer Design

The design of an augmented observer is made in those cases where all the states are not observable in the controller. Though the controller presented in the previous section is stable, it may not be fully observable due to various reasons like pole zero cancellation etc. Suppose if we are known the rotor angular position with the help of an optical encoder, we can estimate the other unknown states. The estimator dynamics can be written as follows:

$$\dot{\hat{\theta}} = \hat{\omega} + g_1(\theta - \hat{\theta})$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 08, August-2016

$$\dot{\hat{\omega}} = \left[-K_m\hat{i_a}\cos\left(\frac{\pi}{2}-N_r\theta\right)+K_m\hat{i_b}\sin\left(\frac{\pi}{2}-N_r\theta\right)-B\hat{\omega}\right]/J+g_2(\theta-\hat{\theta}) \quad (13)$$

$$\dot{\hat{i_a}} = \frac{\left[v_a-R_p\hat{i_a}+K_m\hat{\omega}\cos\left(\frac{\pi}{2}-N_r\theta\right)\right]}{L}+g_3(\theta-\hat{\theta})$$

$$\dot{\hat{i_b}} = \frac{\left[v_b-R_p\hat{i_b}-K_m\hat{\omega}\sin\left(\frac{\pi}{2}-N_r\theta\right)\right]}{L}+g_4(\theta-\hat{\theta})$$

where g1, g2, g3 and g4 are the observer gains. The error between the actual and the estimated values is thus given by,

$$\breve{\theta} = \theta-\hat{\theta}$$
$$\breve{\omega} = \omega-\hat{\omega} \quad (14)$$
$$\breve{i_a} = i_a-\hat{i_a}$$
$$\breve{i_b} = i_b-\hat{i_b}$$

The estimator equations can be written in terms of estimator error dynamics as follows:

$$\dot{\breve{\theta}} = \hat{\omega}-g_1\breve{\theta}$$
$$\dot{\breve{\omega}} = \left[-K_m\hat{i_a}\cos\left(\frac{\pi}{2}-N_r\theta\right)+K_m\hat{i_b}\sin\left(\frac{\pi}{2}-N_r\theta\right)-B\hat{\omega}\right]/J-g_2\breve{\theta}$$
$$\dot{\breve{i_a}} = \frac{\left[v_a-R_p\hat{i_a}+K_m\hat{\omega}\cos\left(\frac{\pi}{2}-N_r\theta\right)\right]}{L}-g_3\breve{\theta} \quad (15)$$
$$\dot{\breve{i_b}} = \frac{\left[v_b-R_p\hat{i_b}-K_m\hat{\omega}\sin\left(\frac{\pi}{2}-N_r\theta\right)\right]}{L}-g_4\breve{\theta}$$

To prove the stability of the above dynamics, consider the Lyapunov candidate function,

$$V_2 = \frac{1}{2}\breve{\theta}^2+\frac{1}{2}\breve{\omega}^2+\frac{1}{2}\breve{i_a}^2+\frac{1}{2}\breve{i_b}^2 \quad (16)$$

According to Lyapunov's direct method, the error converges to zero if $\dot{V}_2 < 0$

i.e. $\breve{\theta}.\dot{\breve{\theta}}+\breve{\omega}.\dot{\breve{\omega}}+\breve{i_a}.\dot{\breve{i_a}}+\breve{i_b}.\dot{\breve{i_b}} < 0$

Or,

$$-g_1\breve{\theta}^2-\frac{B}{J}\breve{\omega}^2-\frac{K_p}{L}{i_a}^2-\frac{K_p}{L}{i_b}^2+\breve{\theta}(\breve{\omega}-g_2\breve{\omega}-g_3\breve{i_a}-g_4\breve{i_b}) < 0$$

Hence, the gains are chosen so as to satisfy the condition for negative definite; so g1 can be chosen as a positive value, g2 as 1, g3 and g4 as zero. When these values are substituted, we get

$$-g_1\breve{\theta}^2-\frac{B}{J}\breve{\omega}^2-\frac{K_p}{L}\breve{i_a}^2-\frac{K_p}{L}\breve{i_b}^2 < 0 \quad (17)$$

This confirms that the error converges to zero. Hence, the control inputs V_a, V_b are modified again to suit the Lyapunov controller augmented with the observer.

$$v_a = \left(R_p i_a-K_m\omega\cos\left(\frac{\pi}{2}-N_r\theta\right)\right)+L\left(i_a^d+\rho e_a\right)+\left(-R_p\breve{i_a}+K_m\breve{\omega}\cos\left(\frac{\pi}{2}-N_r\theta\right)+\rho L\breve{i_a}\right) \quad (18)$$

$$v_b = \left(R_p i_b+K_m\omega\sin\left(\frac{\pi}{2}-N_r\theta\right)\right)+L\left(i_b^d+\rho e_b\right)+\left(-R_p\breve{i_b}-K_m\breve{\omega}\sin\left(\frac{\pi}{2}-N_r\theta\right)+\rho L\breve{i_b}\right)$$

The above equation guarantees that the Lyapunov candidate functions are negative definite and that the error converges to zero. The estimation of rotor angular position is given in Fig.6.

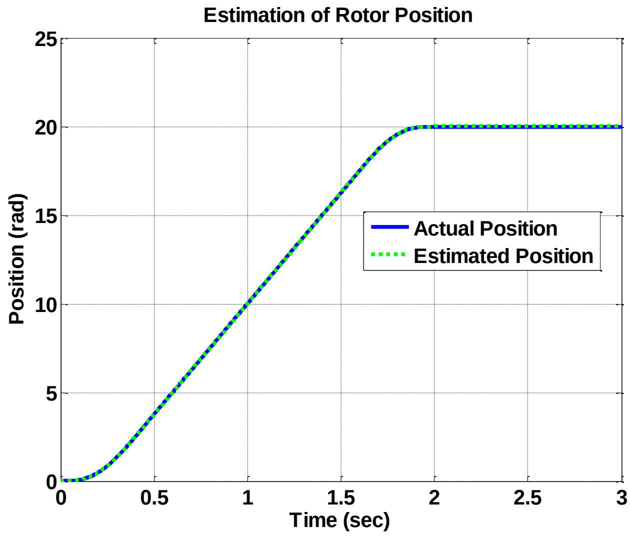

Fig.6: Estimation of Rotor Position by Lyapunov based Observer

E. Nonlinear Torque Modulation

Nonlinear Torque modulation controller is proposed to eliminate the disadvantages present while using conventional Micro stepping for the purpose of position tracking in Permanent Magnet Stepper Motors. The disadvantage is due to the presence of a large error at non zero velocity time periods and that the torque is not subjected to change. This being the case, a nonlinear torque modulating control scheme along with the use of a Nonlinear Current Tracking Controller is articulated to improve the tracking performance. Here, the control scheme makes use of Field Oriented Control (FOC) method to produce desired profiles for current and position tracking. This method maintains zero direct current for generating higher power efficiency with high torque value. The PMSM equation is modified in a way in which the input is considered as τ.

$$\dot{\theta} = \omega$$
$$\dot{\omega} = 1/J(\tau-B\omega-\tau_F) \quad (19)$$

where τ is the input which is given as:

$$\tau = -K_m i_a\cos\left(\frac{\pi}{2}-N_r\theta\right)+K_m i_b\sin\left(\frac{\pi}{2}+N_r\theta\right) \quad (20)$$

Now the tracking error of the system is defined by

$$e_\omega = \omega^d-\omega$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 08, August-2016

$$e_\theta = \theta^d - \theta$$
$$\dot{e}_\theta = \omega^d - \omega$$
$$\dot{e}_\omega = \dot{\omega}^* - (1/J)(\tau - B\omega - \tau_F)$$

where θ^d and ω^d are the desired position and speed respectively and ω* is made to adjust with the actual and desired speed.

$$\omega^* = \omega^d + K_1(\theta^d - \theta) \qquad (21)$$

Whenever the actual position is greater than the desired position, ω^* becomes lesser than ω^d so that the actual position becomes equal to the desired position. On the other hand, if actual position is lesser than the desired position, ω^* becomes greater than ω^d so that the position is converged to the desired value. Now considering the errors both in position and velocity, the first Lyapunov candidate is given as:

$$V_1 = \frac{1}{2}e_1^2 + \frac{1}{2}e_1^2$$
$$\dot{V}_1 = e_1(\omega^d - \omega) + e_2(J\dot{\omega}^* - \tau + B\omega + \tau_l)$$
$$\tau = K_2(\omega^* - \omega) + (\theta^d - \theta + B\omega + J\dot{\omega}^* + \tau_l) \qquad (22)$$
$$\therefore \dot{V}_1 = K_1 e_1^2 - K_2 e_2^2 < 0$$

where τ is the control input. Therefore, origin of the tracking error dynamics is exponentially stable. The actual input to the system is not the torque but the phase currents. So in order to generate the above lemma, we propose a new scheme which is as follows:

$$i_a^d = -\tau/K_m \cos(\tfrac{\pi}{2} - N_r\theta) \qquad (23)$$
$$i_b^d = \tau/K_m \sin(\tfrac{\pi}{2} - N_r\theta)$$

The fundamental electrical dynamics of the system is given by

$$i_a = 1/L\int v_a - R_p i_a - K_m\omega\cos(\frac{\pi}{2} + N_r\theta)$$
$$i_b = 1/L\int v_b - R_p i_b - K_m\omega\sin(\frac{\pi}{2} + N_r\theta)$$
$$e_a = i_a^d - i_a$$
$$e_b = i_b^d - i_b$$

where e_a and e_b are tracking errors. For defining the values for the voltages and currents in both the phases, we consider a second Lyapunov candidate considering the errors present in currents in both the phases.

$$V_2 = \frac{1}{2}e_a^2 + \frac{1}{2}e_b^2$$
$$\dot{V}_2 = e_a(\dot{i}_a^d - \dot{i}_a) + e_b(\dot{i}_b^d - \dot{i}_b) \qquad (24)$$
$$= e_a\left(\dot{i}_a^d - \tfrac{1}{L}\left[v_a - R_p i_a + K_m\omega\cos(\tfrac{\pi}{2} - N_r\theta)\right]\right) + e_b\left(\dot{i}_b^d - \tfrac{1}{L}\left[v_b - R_p i_b - K_m\omega\sin(\tfrac{\pi}{2} + N_r\theta)\right]\right)$$

The voltages in both the phases are chosen as:

$$v_a = R_p i_a + K_m\omega\cos(\frac{\pi}{2} + N_r\theta) + L(\dot{i}_a^d + k_3 e_a)$$
$$v_b = R_p i_b + K_m\omega\sin(\tfrac{\pi}{2} + N_r\theta) + L(\dot{i}_a^d + k_3 e_a) \qquad (25)$$
$$\therefore \dot{V}_2 = -K_3(e_a^2 + e_b^2) < 0$$

Since $\dot{V}_2$ is lesser than zero, we can say that it is asymptotically stable and the error converge to zero when the control input is taken as in (25). The advantage of this method is that there is no necessity for us to take the Direct-Quadrature (DQ) Transformation for various parameters. The rotor speed and position profiles of nonlinear torque modulator along with their references are shown in Fig.7 and Fig.8 respectively. The tracking of current and voltage vectors in both phases are as given below in Figs.9-12 respectively.

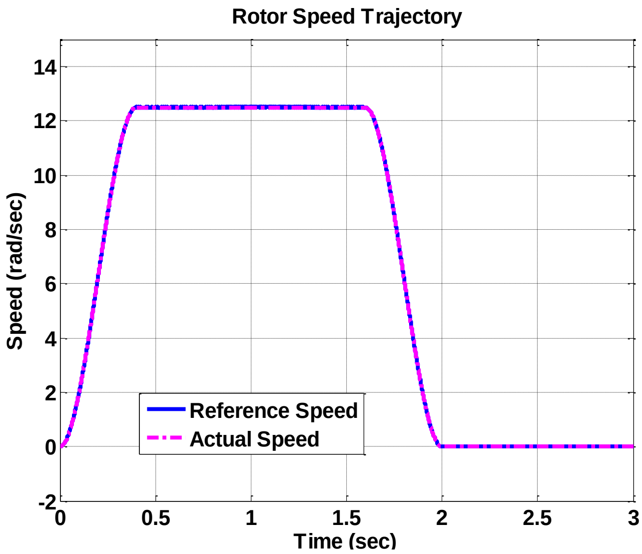

Fig.7: Tracking of Rotor Speed Trajectory

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 08, August-2016

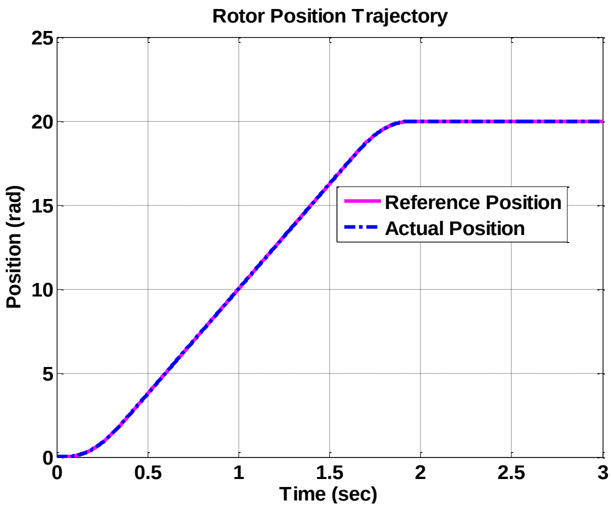

Fig.8: Tracking of Rotor Position Trajectory

IV. SIMULATION RESULTS

The MATLAB software was used to simulate the above results. We found that the position tracking in Minimum Variance controller with reference profile happened only after 5.25 seconds indicating that it had a lesser performance. This sluggish action of the controller can be ascribed to the short span of simulation time where the recursive least square estimator needed more number of samples for proper estimation of parameter. The Lyapunov based controller had better tracking performance than the Minimum Variance Control but it had oscillations present in its speed profile. An augmented observer is then proposed where the unknown states are estimated. The Nonlinear Torque Modulator output is later presented where there is complete tracking of both rotor speed and rotor position. The simulation results of all the controllers clearly shows that the Torque modulator had the best tracking results than the other two.

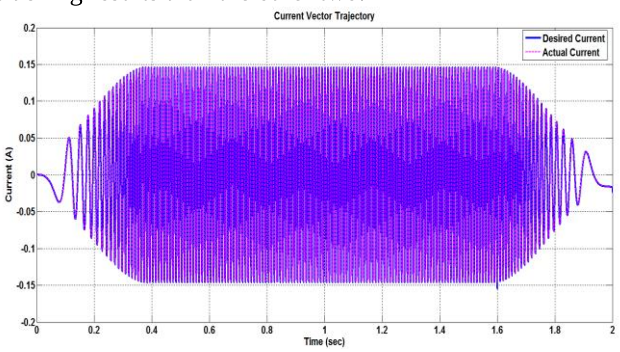

Fig.9: Tracking of Current Vectors in Phase A

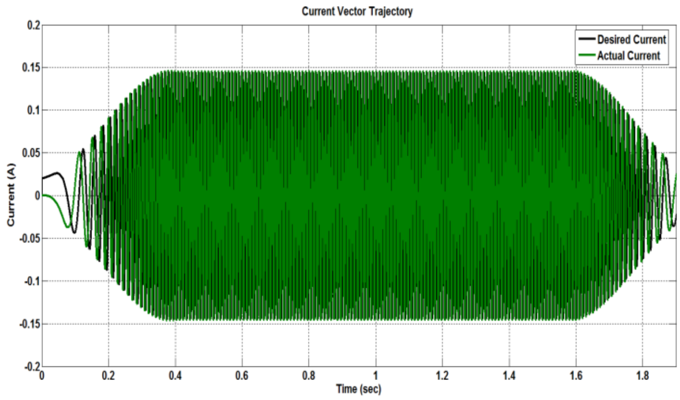

Fig.10: Tracking of Current Vectors in Phase A

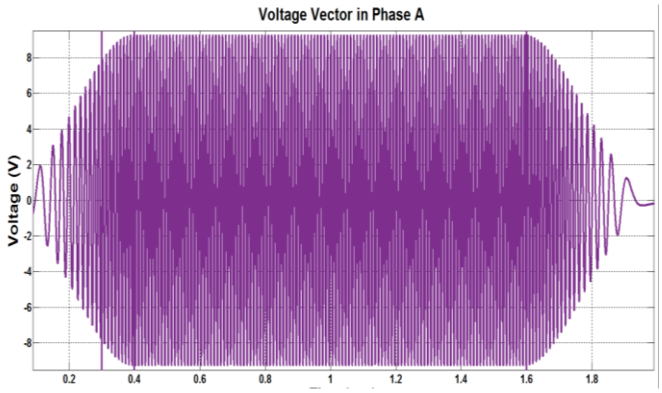

Fig.11: Voltage Vector in Phase A

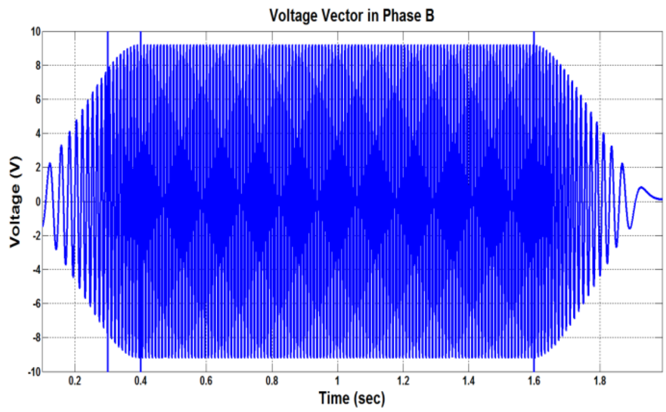

Fig.12: Voltage Vector in Phase B

International Standards of rating the controllers namely ISE, IAE and ITAE are shown in Table I where the values of Torque Modulator were the least when compared to the other two controllers. For instance, the ISE value for Torque modulation controller is almost hundred times lesser than that of Lyapunov based control and almost thousand times lesser than Minimum Variance control. This distinctly shows that the Torque modulator had the best performance followed by the Lyapunov based controller and then by the Minimum Variance Controller. Also, The motor parameters which are taken in our paper are shown in Table II.

TABLE I
COMPARISON OF THE THREE CONTROLLERS

Performance Indices	Minimum Variance	Lyapunov	Torque Modulation
ISE	5.3571	0.0554	0.0037
IAE	7.9410	0.2761	0.0947
ITAE	2.3832	0.0883	0.0092

TABLE II
MOTOR PARAMETERS

Parameter	Value
Km	0.51 N.m/A
Rap	14.8Ω
J	8 x 10-5 kg.m2
F	0.006 Names/rad
Nr	50
L	40 my
Vim	24

V. CONCLUSION

To sum up, this paper elaborates the control of position and speed trajectories in Permanent Magnet Stepper Motor using different methods. The efficiency of all the controllers are critically analysed and their performances are compared based on simulation results and international standards of indices. The Minimum Variance control had mediocre results in both position and speed profiles which were outperformed by the Lyapunov based controller. Furthermore, we have also presented a full order augmented observer so as to bring about the estimation of unknown states of rotor angular velocity. The Nonlinear Torque Modulator is proposed so that the developed torque in the PMSM can be modulated without the usage of Direct Quadrature (DQ) Transformation. This is the predominant merit of this controller used in our paper. Servo and Regulatory operations are also carried out here to ensure its robustness. We have also shown the tracking of current vectors in both phases with its desired value. Finally, a Lyapunov Candidate is used to ensure the stability of both the Torque Modulator and Lyapunov based control with zero error convergence.

REFERENCES

[1] P. Acarnley, Stepping Motors: a Guide to Modern Theory and Practice, 4th ed. London: Institution of Electrical Engineers, 1982.

[2] Takashi Kenjo, and Akira Sugawara, “Stepping Motors and Their Microprocessor Controls”, Clarendon Press, Oxford, London, 1994.0

[3] M. KristicI. Kanellakopoulous, and P. Kokotovic, Nonlinear and Adaptive Control Design. New York: Wiley, 1995

[4] M. Bodson, J. Chiasson, R. Novotnak, and R. Rekowski, “High performance nonlinear feedback control of a permanent magnet stepper motor,” IEEE Trans. Contr. Syst. Technol., vol. 1, no. 1, pp. 5-14, Mar. 1993.

[5] R. C. Speagle, J. Hu, D. M. Dawson, and Z. Qu, "Robust tracking control of a permanent magnet stepper motor without current measurements," in IEEE Conference on Control Applications, (Vancouver, Canada), pp. 153-158, Sep. 1993.

[6] H.Nijmeijer and A.J.van der Schaft, Nonlinear Dynamical Control Systems. New York: Springer-Verlag, 1990

[7] M. Zribi, J. Chiasson, “Position control of a PM stepper motor by exact linearization,” IEEE Trans. Autom. Control, vol 36, no. 5, pp. 620-625, May 1991.

[8] R. Marino, S. Peresada, and P. Tomei, “Nonlinear adaptive control of permanent magnet step motors,” Automatica, vol. 31, no. 11, pp. 1595-1604, 1995.

[9] S. Seshagiri, “Position control of permanent magnet stepper motors using conditional servocompensators,” IET Contr. Theory Applications, vol. 3, no. 9, pp. 1196-1208, 2008.

[10] J. Krener, Approximate linearization by state feedback and coordinate change, Systems & Control Letters, vol.5, 1984, pp. 181-185.

[11] Paul C. Krause, Analysis of Electric Machinery, McGraw-Hill, New York, 1986.

[12] P.Krishnamoorthy, F.Khorrami, "Robust control of permanent magnet stepper motors using only rotor position and velocity", IEEE conference on Decision and Control, Orlando, USA, Dec-2001.

[13] Bellini, C. Concari, G. Franceschini, and A. Toscani, “Mixedmode PWM for high-performance stepping motors,” IEEE Trans. Ind.Electron., vol. 54, no. 6, pp. 3167-3177, Dec. 2007.

[14] J. Krener, On the equivalence of control systems and the linearization of nonlinear systems, SIAM Journal on Control, 1973, pp. 670-676.

[15] Khorrami, P. Krishnamurthy, and H. Melkote, Modelling and Adaptive Nonlinear Control of electric Motors. Heidelberg: Springer Verlag, 2003.

[16] A.Isidori, Nonlinear Control Systems. New York: Springer Verlag, 2nd ed., 1989.

[17] K.J.Astrom and B. Wittenmark, “On Self tuning regulators. “ Automatica, Vol. P, 1973, pp.185-199.

[18] D.W.Clarke, “Self-tuning Control of non minimum phase systems.” Automatica, Vol.20, 1984, pp.501- 517.

[19] D.W. Clarke et al, Adaptive methods for control system design, Self-tuning Control, M Gupta, editor, 1986, pp.195-202.

[20] Darren M. Dawson, Jun Hu, Timothy C. Burg, “Nonlinear Control of Electric Machinery”, Marcel Dekker, Inc., New York, 1998.

[21] M.Defoort, F.Nollet, T.Floquet and W.Perruquetti, "A third-order sliding-mode controller for a stepper motor", IEEE Transactions on Industrial Electronics, Vol.56, No.9, pp.3337-3346, Sep.2009.

[22] Athani V.V., "Stepper Motors: Fundamentals, Applications and Design", New Age International Publishers, Reprint-2005.

[23] J. Tai, L. Antognini, P. Gandel, and N. Veignat, “Damping a two-phase step motor with velocity coils,” in Proc. Incremental Motion Contr. Syst.Devices Symp., Champaign, IL, 1985, pp. 305–309.

[24] D. Regnier, “Damping circuit and rotor encoder cut disc magnet step motor overshoot, settling time, resonances,” in Proc. Power Conv. Intell. Motion (PCIM), Apr. 1988, pp. 64–69.

[25] C.I.Byrnes, A.Isidori, and J.C.Williems, “Passivity, Feedback equivalence, and the global stabilization of minimum phase nonlinear systems”, IEEE trans. Automatic. Contr., vol. 36, no.11, pp. 1228-1240, Nov. 1991.