IMPLEMENTATION OF MMSE EQUALIZER USING VARIANTS OF LMS

Ch.Sridevi
M.Tech Student, SVP College of Engineering, Visakhapatnam, AP, India

K. Anusha
Assistant Professor, SVP College of Engineering, Visakhapatnam, AP, India

Abstract

Channel equalization is one of the applications of adaptive filtering. This paper includes an efficient approach to compare the MMSE equalizers with variants of LMS algorithm for the removal of noise in the corrupted signals. The main aim of designing the transmitter and receiver having a channel in communication systems is to minimize the corruption of the input signals. Here equalized output of the model is compared for LMS algorithm, NLMS, Complex NLMS and NLMSSIGN algorithm. It has been observed the better BER vs. SNR plot considerably.

1. Introduction

In statistics and signal processing, a minimum mean square error (MMSE) equalizer describes the approach which minimizes the mean square error (MSE), which is a common measure of equalizer quality. To reduce the complexity caused by matrix inversion of ideal MMSE equalizer, we propose an adaptive MMSE equalizer with different algorithms. A parameter μ is a positive real-valued constant which controls the size of the incremental correction applied to the equalizer coefficient vector. This MMSE equalizer is driven with transmitted symbols along with Gaussian noise in a Rayleigh fading channel. We need proper algorithms to reduce the ISI (Inter symbol Interference) as in the literature [1].

1.1 Adaptive Filters

An adaptive filter is a filter that self-adjusts its transfer function according to an optimization algorithm driven by an error signal. Because of the complexity of the optimization algorithms, most adaptive filters are digital filters. By way of contrast, a non-adaptive filter has a static transfer function. Adaptive filters are required for some applications because some parameters of the desired processing operation (for instance, the locations of reflective surfaces in a reverberant space) are not known in advance. The adaptive filter uses feedback in the form of an error signal to refine its transfer function to match the changing parameters is taken from literature [2].

1.2 Rayleigh fading channel model

Rayleigh fading is a reasonable model when there are many objects in the environment that scatter the radio signal before it arrives at the receiver. This is studied from the literature[3]. The central limit theorem holds that, if there is sufficiently much scatter, the channel impulse response will be well-modelled as a Gaussian process irrespective of the distribution of the individual components. If there is no dominant component to the scatter, then such a process will have zero mean and phase evenly distributed between 0 and 2π radians. The envelope of the channel response will therefore be Rayleigh distributed. Calling this random variable R, it will have a probability density function.

$$p_R(r) = \frac{2r}{\Omega} e^{-r^2/\Omega}, \ r \geq 0 \qquad (1)$$

where $\Omega = E(r^2)$

1.3 Adaptive Filter Performance in channel equalization

Obviously the key aim of the adaptive filter is to minimize the error signal e(k). The success of this minimization will clearly depend on the nature of the input signals, the length of the adaptive filter, and the adaptive algorithm used. To improve the bandwidth of a channel we can attempt to equalize a communication channel. If the telephone channel is a (stationary) communication channel with a continuous time impulse response, then when symbols are transmitted the impulse response will cause a symbol to spread over many time intervals, thus introducing inter symbol interference (ISI). The aim of a data equalizer is to remove this ISI .Compared to simple channel equalization, it should be noted that a data equalizer only requires equalizing the channel at the symbol sampling instants rather than over all time. Hence the problem can be posed with data symbols as inputs, rather than the raw stochastic data taken from literature [4].

In general for channels where the impulse response changes slowly, a decision directed adaptive data equalizer is used, whereby a slicer is used to produce a retraining signal. It is also worth noting that for many data transmission systems, the data is complex, and hence a complex adaptive algorithm is required. Hence channel equalization response is shown in Figure .1

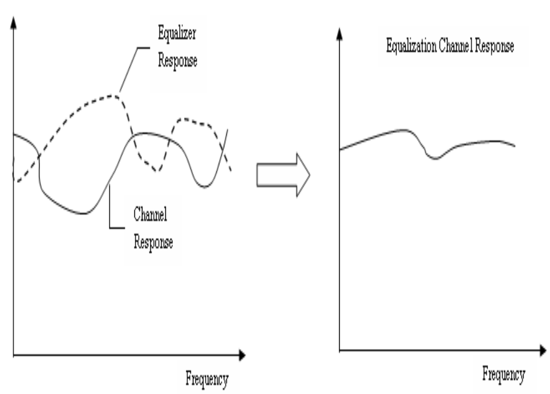

Figure.1 Performance of Equalizer

1.4 Performance Measures in Adaptive Systems

Six performance measures will be discussed in the following sections such as convergence rate, minimum mean square error, computational complexity, stability, robustness, and filter length.Measures are studied from literature [5, 7] that are generally considered in channel equalization problem.

1.4.1 Convergence Rate

The convergence rate determines the rate at which the filter converges to its resultant state. Usually a faster convergence rate is a desired characteristic of an adaptive system. Convergence rate is not, however, independent of all of the other performance characteristics. There will be a trade off, in other performance criteria, for an improved convergence rate and there will be a decreased convergence performance for an increase in other performance. For example, if the convergence rate is increased, the stability characteristics will decrease, making the system more likely to diverge instead of converge to the proper solution. Likewise, a decrease in convergence rate can cause the system to become more stable. This shows that the convergence rate can only be considered in relation to the other performance metrics, not by itself with no regards to the rest of the system. The convergence rate is defined as the number of iterations required for the algorithm to converge to its steady state mean square error**.**Convergence is the most important factor to observe in the adaptive channel equalization using different algorithms. If the filter coefficients used in the adaptive filter algorithm did not converge, the code could get problem. In this simulation, we used the standard signals as white noise (as the input signals), the low pass filter (model the impulse response) to check the operation of the algorithm. If the problem still exists, then we verify the convergence factor μ. By varying this factor, we can control and adjust the convergence of the adaptive filter algorithm. It is easy to see that if at any index , w(n0)=w0 then we have , the algorithm converges meaning completely, e(n)=0 and w(n)=w0 for all n>=n0. Conventionally, the input to the LMS algorithm as well as the desired response are random processes and under such case, convergence is reached in mean, i.e., for appropriate choice of the step size, we have E[w(n)]= w0as n tends to infinite. This, in turn, requires the so-called “independence” assumption requiring statistical independence between x(n) and w(n).

1.4.2 Minimum Mean Square Error

The minimum mean square error (MSE) is a metric indicating how well a system can adapt to a given solution. A small minimum MSE is an indication that the adaptive system has accurately modeled, predicted, adapted and/or converged to a solution for the system. A very large MSE usually indicates that the adaptive filter cannot accurately model the given system or the initial state of the adaptive filter is an inadequate starting point to cause the adaptive filter to converge. There are a number of factors which will help to determine the minimum MSE including, but not limited to; quantization noise, order of adaptive system, measurement noise, and error of the gradient due to finite step size.

1.4.3 Computational Complexity

Computational complexity is particularly important in real time adaptive filter applications. When a real time system is being implemented, there are hardware limitations that may affect the performance of the system. A highly complex algorithm will require much greater hardware resources than a simplistic algorithm.

1.4.4 Stability

Stability is probably the most important performance measure for the adaptive system. By the nature of the adaptive system, there are very few completely asymptotically stable systems that can be realized. In most cases the systems that are implemented are marginally stable, with the stability determined by the initial conditions, transfer function of the system and the step size of the input.

1.4.5 Robustness

The robustness of a system is directly related to the stability of a system. Robustness is a measure of how well the system can resist both input and quantization noise. The algorithm is said to be robust if it is capable of analyzing the signal parameters even the assumptions or formulation used for the analysis are not supported.

1.4.6 Filter Length

The filter length of the adaptive system is inherently tied to many of the other performance measures. The length of the filter specifies how accurately a given system can be modeled by the adaptive filter. In addition, the filter length affects the convergence rate, by increasing or decreasing computation time, it can affect the stability of the system, at certain step sizes, and it affects the minimum MSE. If the filter length of the system is increased, the number of computations will increase, decreasing the maximum convergence rate. Conversely, if the filter length is decreased, the number of computations will decrease, increasing the maximum convergence rate. For stability, due to an increase in length of the filter for a given system, you may add additional poles or zeroes that may be smaller than those that already exist. In this case the maximum step size, or maximum convergence rate, will have to be decrease to maintain stability. Finally, if the system is under specified, meaning there is not enough pole and/or zeroes to model the system, the mean square error will converge to a nonzero constant. If the system is over specified, meaning it has too many poles and/or zeroes for the system model, it will have the potential to converge to zero, but increased calculations will affect the maximum convergence rate possible from the literature [7].

2. MMSE Equalization

From the literature [8, 9] it is observed that updating is very much needed to the algorithm which helps to drive the equalizer The Sampled signal after MMSE Equalizer can be expressed in matrix form as

$$\hat{s} = w^{H} y(i) \qquad (2)$$

where

$$y(i) = H^{T}(i)S(i)+n(i) \qquad (3)$$

where M is the length of the MMSE equalizer

$$W = [w_1, w_2, w_3, \dots\dots\dots w_M]^T \qquad (4)$$

is the equalizer coefficients vector; Then the error signal e(i) is given by

$$e(i) = d(i) - \hat{s}(i) \qquad (5)$$

where d (i) is the desired response.

For MMSE equalizer,

$$d(i) = S(i + D), \qquad (6)$$

where D is a time delay parameter which is L +1 usually. The MMSE criterion is used to derive the optimal equalizer coefficients vector w:

$$w = minimize\ E\{|e|^2\} \qquad (7)$$

We make the assumption that signal s (i) and noise n(i) are independent identity distribution stochastic. This assumption is considered from literature [10].Variable and uncorrelated each other, and then the equalizer coefficients vector w can be expressed as

$$w = (H^H H + \frac{1}{SNR} I)^{-1} H^H \delta_D \qquad (8)$$

where

$$\delta_D = [0 \dots \dots \dots . I_D; 0 \dots \dots \dots 0]^T_{1\times(L+M-1)} \quad (9)$$

$SNR = \frac{\sigma_s^2}{\sigma_n^2}$ denotes the signal noise ratio.

I is M×M identity matrix.

To reduce the complexity caused by matrix inversion of ideal MMSE equalizer, we propose an adaptive MMSE equalizer algorithm. According to Eqn.(7) and Eqn.(8), the mean square error (MSE) J can be expressed as

$$J(w) = E[e(i)e(i)^*] = \sigma_s^2 - W^H p - P^H W + W^H RW \quad (10)$$

where autocorrelation matrix and the cross correlation vector are given as

$$R = E[y(i)y^H(i)] \quad (11)$$
$$P = E[y(i)\,d^*(i)] \quad (12)$$

σ_s^2 denotes the signal power, $(.)^*$ represents conjugate operation. Because the wireless channel is time varying, the equalizer coefficients vector w must be updated real time. Conventional adaptive algorithm requires reference signal d(i) , while in the downlink of code multiplexed pilot CDMA systems, d(i) is difficult to distil. To resolve this problem, the steepest decent method is used. From Eqn. (11), the gradient vector is

$$\frac{\partial J(w)}{w} = -2p + 2Rw \quad (13)$$

Then the equalizer updating equation is

$$w(i+1) = w(i) + 2\mu[p - Rw(i)] \quad (14)$$

where parameter μ is a positive real-valued constant which controls the size of the incremental correction applied to the equalizer coefficients vector. For the autocorrelation matrix:

$$R = E[y(i)y^H(i)] \quad (15)$$
$$R = E[S(i)S^H(i)]\,\{H^H(i)H(i)\}^T + E[n(i)n^H(i)] \quad (16)$$
$$R = \sigma_s^2\{H^H(i)H(i)\}^T + \sigma_n^2 I \quad (17)$$

and the cross-correlation vector

$$P = E[y(i)d^*(i)] = E[(H^T(i)S(i) + n(s))S^*(i\text{-}D)] \quad (18)$$
$$P = \sigma_s^2 H^T(i)\delta_D \quad (19)$$

From Eqn (8, 14, and 17) we can obtain the time recursive equation of MMSE equalizer by

$$w(i+1) = w(i) + 2\mu\sigma_s^2[H^T(i)\delta_D - \{H^H(i)H(i)\}^T + \frac{1}{SNR} I w(i)] \quad (20)$$

As can be seen from Eqn.(7), the updating process avoids the matrix inversion operation. On the other hand, the updating process abstains the requirement to store the autocorrelation matrix R(i) and only the equalizer coefficients vector of last time is needed. From Eqn (14) we know, the channel convolution matrix H(i) is required to update the equalizer coefficients vector from the literature [11] hence Eqn (20) is the updating equation for MMSE equalizer.

2.1 MMSE equalizer with variants of LMS:

MMSE equalizer is placed and all four algorithms are used to update the filter coefficients. As in the Figure.1, the system follows the procedure to get the noise reduced signal. The transmitted symbols are passed through the channel where Gaussian noise is added to it. Noise corrupted signal is given to equalizer whose output is compared to the training sequence to obtain the error. The loop run till the error becomes zero in ideal case, till the minimum error was obtained. With the help of MATLAB code, it is executed and simulated considered from literature [12].

The results were obtained in such a way that NLMS Sign gives better Equalized outputs when compared to other algorithms. Better BER plot is obtained, also the improvement in the performance is obtained in BER vs SNR plot. The Transmitted symbols, received symbols, equalized symbols are observed in the Simulation results. Convergence is a measure for the performance measurement. The convergence plot is also obtained with the help of mat lab software. Data and the training symbols are considered. AWGN noise which is complex is considered and it is simulated to obtain the noisy signal. Channel is considered and it is normalized. QPSK symbol sequence is taken along with SNR in dB. So noise power is adjusted with SNR in dB. Here the updating equation is

w=w+ mu*conj(e(i))*X(:,i+10) (21)

Iterating to convergence usually means continuing the iterations until some error tolerance is reached. Here iterations are made till the values reach the stable values; this shows in convergence plot, this process of spinning of values is observed in literature [13].

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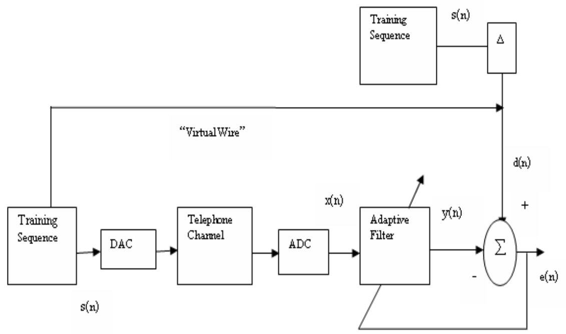

Figure .2 Adaptive Channel Equalization Model

2.2 Simulation Results

Running the simulation produces in four scatter plots that display the transmitted symbols, and the received symbols, equalized symbols and convergence plots respectively. The transmitted symbols plot is a scatter plot of all the transmitted symbols where T=3000 is the total number of data, M=2000 is total number of training symbols. It includes the received symbols plot which shows the pattern of received symbols, equalized symbols plot which shows the pattern of the equalized symbols. Using LMS algorithm, convergence plot which shows that error is zero till the training symbols n=1000.After wards the error started increasing; this shows that LMS algorithm with MMSE equalizer needs to switch to another algorithm. In fact, the equalizer is busy trying to adapt its weights appropriately. The following third scatter plot shows the equalized signal very early in the simulation in which a constellation is surrounded by Gaussian clouds.

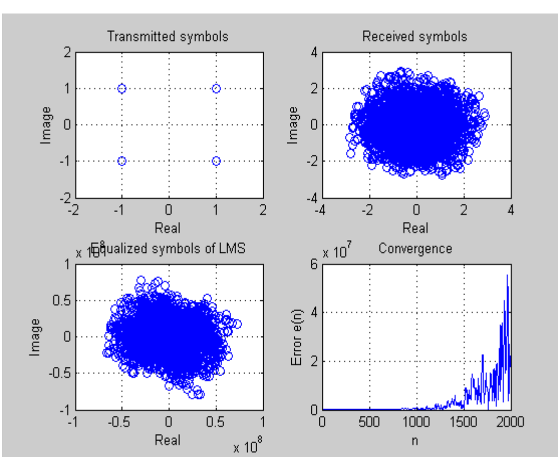

Figure.3 Scatter plots for LMS

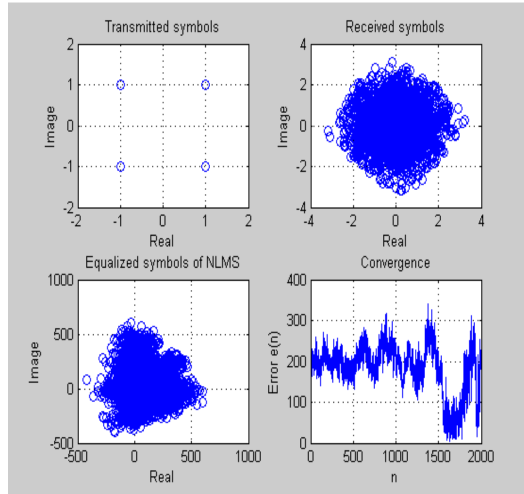

Figure.4 Scatter plots for NLMS

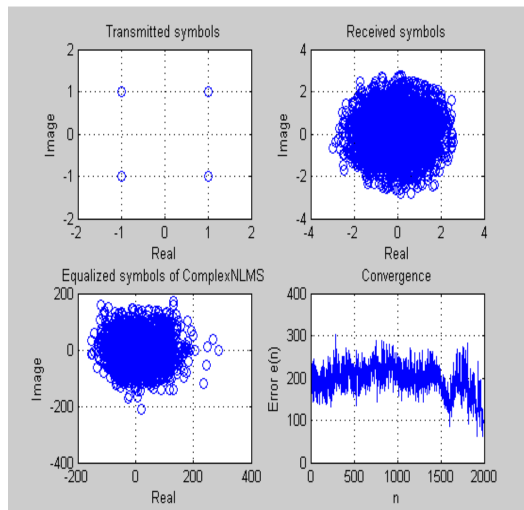

Figure.5 Scatter plots for Complex LMS

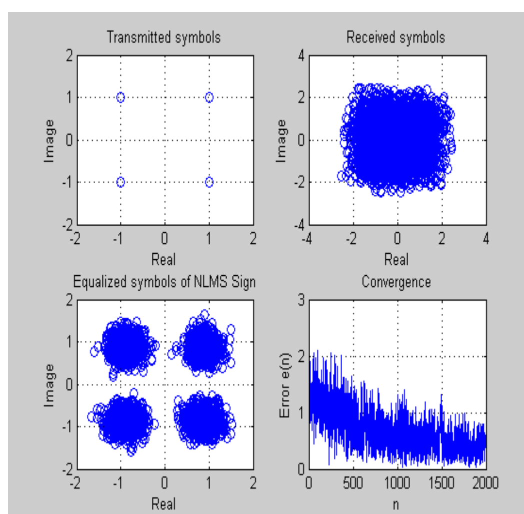

Figure.6 Scatter plots for NLMS Sign

The above Figures 3, 4, 5 shows the equalized signal in its unsteady state for three algorithms LMS, NLMS, Complex LMS and in Figure 6 equalized signal is in steady state for NLMS Sign algorithm. In Figure. 6, the convergence plot is available whose convergence rate is high. All the groups of constellations are in between-0.5×10^8 to 0.5×10^8 for LMS. In NLMS algorithm, the group of constellations is in between-500 to +500 on the real and imaginary axis where as in Complex NLMS plot, all the group of constellations are in between -200 to +200 on the real and imaginary axis. Initially error is 2, slowly it decays to 1. When the training symbols exceeds n=500 then error becomes 0 till n=2000.This shows that NLMS Sign algorithm is better than the prior algorithms. So, bit error rate verses signal to noise ratio plot is taken only for NLMS Sign algorithm in Figure 7.

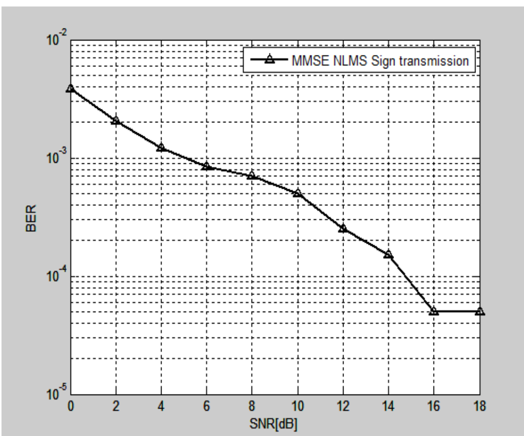

Figure.7 BER vs., SNR plot for NLMS Sign

3. Conclusion

The basic goal of this project is to investigate the application of an algorithm based on adaptive filtering with averaging in channel equalization. Here the main concern is to achieve a high convergence rate in order to meet the requirements for short training time and good tracking properties. In this light the obtained results show that the MMSE equalizer is very promising. Its main advantages could be summarized as follows as high adaptation rate, comparable to that of the RLS equalizer, low computational complexity and possible robustness in fixed-point implementations with good convergence rate. The efficient of equalization and convergences is too good. The time complexity is very less and more efficient for advance communication systems.

Finally, this study is analyzed the effect of the MMSE equalizer performance of the LMS algorithm, NLMS algorithm, Complex NLMS algorithm and NLMS Sign algorithm. Each algorithm was demonstrated the convergence plot and BER vs. SNR plot also. Among NLMS Sign has a significant role in the performance of MMSE equalizer and is an important design criterion. This obtained BER plot is better than plot in literature [14].

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