# Improvement In Power System State Estimation By Use Of Phasor Measurement Unit 

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#### Abstract

This paper mainly focuses on a placement algorithm of phasor measurement unit which enhance the accuracy of the state estimation. The algorithm developed will ascertain a list of buses with low quench of estimator accuracy. The optimal placement of phasor measurement unit is determined with respect to their characteristic of parameters measured by using phasor measurement unit. The test results of the above are tested on IEEE 6-bus system, IEEE 9bus system and IEEE 14-bus system.


Keywords-- phasor measurement unit, state estimation, weighted least square.

## 1. Introduction

The concept of state estimation is originally proposed by Schweppe in 1970s. Over the last 3 decades of power system, the state estimation will help in resolving the parameter of power system. The State estimation is conventionally explained by the weighted least square algorithm (WLS) [1] with conventional measurements of system voltage magnitude and system current.

The Recently developed technique of phasor measurement unit will able to measure the system voltage and system current in the power system. It produces a great impact on the state estimation parameter measurement. In reference [8], a detailed review on development of the phasor measurement unit made by Phadke. The phasor measurement unit at substation will allows us for direct measurement of the state of the network. It makes the whole state estimator as a linear estimator. However, it may be reasonable in replacement of all the existing measurements system with phasor measurements in the power system due to accuracy and quickly analyzing the behavior of system parameter.

The main objective is to optimally situate a phasor measurement unit to obtain maximum benefit in regard to state estimation. Several methods [2] are proposed in relation of PMU placements as annealing method, graph theoretic
procedures, depth first, tabu search method and genetic algorithms. They are proposed in determination of a minimum set of phasor measurement unit to make the network more observable. There is also several meter assignment methods proposed in literature in determination of an optimal set of measurements. The criterion of determining the minimum number of the measurement matrix proposed in an optimal meter placement [11] method.

The authors [14] concentrate on the accuracy, redundancy and cost requirements while determine the placement of measurement devices. In an incremental meter placement [9], the State Estimation solutions will improve their accuracy and robustness. The criterion of variances of state estimation errors is proposed in incremental meter placement.
In this paper, a placement algorithm for phasor measurement unit is proposed. Its objective is to improve the accuracy and robustness of the solution of state estimation when an inadequate number of PMUs can be added to the power system. The algorithm will determine the phasor measurement unit placement by using the main three aspects as: system topological structure, accuracy and redundancy. The algorithm has been tested on the IEEE 6-bus test system [7], IEEE 9bus test system [3] and IEEE 14-bus test system [13].
Phasor Measurements Unit


Fig. 1: Phasor Measurements Unit
For the practical purposes the Fig. 2 will shows a 4-bus system, which has single PMU at bus 1 . It
has one voltage phasor measurement and three current phasor measurements [12], namely $\mathrm{V}_{1} \angle \theta_{1}, \mathrm{I}_{1} \angle \delta_{1}, \mathrm{I}_{2} \angle \delta_{2}$ and $\mathrm{I}_{3} \angle \delta_{3}$.


Fig 2: Single PMU Measurement Model
The investigators of Virginia Polytechnic Institute [8] will develop the phasor measurement unit in around 1970s. They state that the PMU has capability of providing a synchronized real time measurement of current phasors and voltage phasors. The Synchronization within the buses is achieved at the same-time sampling of current and voltage waveforms using by timing signals from the Global Positioning System [10]. A PMU placed on a bus makes that the bus and all its fellow buses are observable. This compose the incremental placement of PMU are different from standard measurements placement [4].

A brief review of state estimation in power system is presented in section 2. The proposed algorithm in relation to placement of phasor measurement unit is presented in Section 3. The test results are carried out on the IEEE 6-bus, IEEE 9 -bus and IEEE 14-bus test systems are given in Section 4. Finally some concluding notes are presented in Section 5.

## 2. Power System State Estimation

The WLS state estimation minimizes the weighted sum of squares of the residuals. Consider the set of measurements given by the vector $z$ :

$$
z=\left[\begin{array}{l}
z_{1}  \tag{1}\\
z_{2} \\
z_{3} \\
\cdot \\
\cdot \\
z_{m}
\end{array}\right]=\left[\begin{array}{l}
h_{1}\left(x_{1}, x_{2}, x_{3}, \ldots \ldots x_{n}\right) \\
h_{2}\left(x_{1}, x_{2}, x_{3}, \ldots \ldots x_{n}\right) \\
h_{3}\left(x_{1}, x_{2}, x_{3}, \ldots \ldots x_{n}\right) \\
\cdot \\
\cdot \\
h_{m}\left(x_{1}, x_{2}, x_{3}, \ldots \ldots . x_{n}\right)
\end{array}\right]+\left[\begin{array}{l}
e_{1} \\
e_{2} \\
e_{3} \\
\cdot \\
\cdot \\
e_{m}
\end{array}\right]=h(x)+e
$$

Where $z$ is the measurement vector ( $m$-vector), $x$ is the true state vector ( $n$-vector, $n<m$ )
$h^{T}=\left[h_{1}(x), h_{2}(x), h_{3}(x), \ldots . h_{m}(x)\right]$
$h_{i}(x)$ is the nonlinear function relating measurement $i$ to the state vector $x$
$x^{T}=\left[\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3} \ldots \ldots \ldots . . \mathrm{x}_{\mathrm{n}}\right]$ is system state vector
$e^{T}=\left[\mathrm{e}_{1}, \mathrm{e}_{2}, \mathrm{e}_{3} \ldots \ldots . \mathrm{e}_{\mathrm{m}}\right]$ is the vector of measurement errors.

The Weighted Least Square estimator will minimize the following objective function:
$J(x)=\sum_{i=1}^{m} \frac{\left(z_{i}-h_{i}(x)\right)^{2}}{R_{i i}}=[z-h(x)]^{T} R^{-1}[z-h(x)]$

The non-linear function $g(x)$ can be expanded into its Taylor series around the state vector $x^{k}$ neglecting the higher order terms.

$$
\begin{equation*}
g(x)=g\left(x^{k}\right)+G\left(x^{k}\right)\left(x-x^{k}\right)+\ldots \ldots .=0 \tag{4}
\end{equation*}
$$

An iterative solution scheme known as the Gauss-Newton method is used to solve above equation:

$$
\begin{equation*}
x^{k+1}=x^{k}-\left[G\left(x^{k}\right)\right]^{-1} \cdot g\left(x^{k}\right) \tag{5}
\end{equation*}
$$

where, $k$ is the iteration index and $x^{k}$ is the solution vector at iteration $k . G(x)$ is called the gain matrix, and expressed by:

$$
\begin{equation*}
G(x)=\frac{\partial g\left(x^{k}\right)}{\partial x}=H^{T}\left(x^{k}\right) R^{-1} H\left(x^{k}\right) \tag{6}
\end{equation*}
$$

$g\left(x^{k}\right)=-H^{T}\left(x^{k}\right) R^{-1}\left[z-h\left(x^{k}\right)\right]$

Generally, the gain matrix is quite sparse and decomposed into its triangular factors. At each iteration $k$, the following sparse linear sets of equations are solved using forward/backward substitutions, where $\Delta x^{k+1}=x^{k+1}-x^{k}$ :

$$
\begin{align*}
& {\left[G\left(x^{k}\right)\right] \Delta x^{k+1}=} \\
& H^{T}\left(x^{k}\right) R^{-1}\left[z-h\left(x^{k}\right)\right]=H^{T}\left(x^{k}\right) \cdot R^{-1} \Delta z^{k} \tag{8}
\end{align*}
$$

These iterations are going on until the maximum variable difference satisfies the condition, ' Max $\left|\Delta x^{k}\right|<\varepsilon$ '. A detailed flow-chart of this algorithm is shown in next section.

## 3. The Proposed PMU Placement Algorithm

The objective of the proposed algorithm in relation to placement of PMU is to reduce the variances of the Weighted Least Square State Estimation error. Also help in increases the local redundancy at the same time. While a PMU placed on a specific bus makes that bus and its entire neighbor buses are observable, buses are formed a group by using that single PMU placement [5]. The weakest bus group is determined by the local redundancy ranking and Weighted Least Square State Estimation error variance ranking. The local redundancy rankings are determined by the method described in [1]. The ranking of accuracy are determined by the diagonal elements of the Weighted Least Square State Estimation covariance matrix.

The WLS with PMU algorithm is illustrate as follows:

1. Start iterations, set the iteration index $\mathrm{k}=0$.
2. Initialize the state vector $x^{k}$, typically as a flat start.
3. Calculate the gain matrix, $\boldsymbol{G}\left(x^{k}\right)$.
4.Calculate the right hand side
$t^{k}=H^{T}\left(x^{k}\right) R^{-1}\left[z-h\left(x^{k}\right)\right]$
4. Decompose $G\left(x^{k}\right)$ and solve for $\Delta x^{k}$
5. Test for convergence, Max $\left|\Delta x^{k}\right|<\varepsilon$ ?
6. If no, update $x^{k+1}=x^{k}+\Delta x^{k}, \mathrm{k}=\mathrm{k}+1$ and go to step 3. Else, stop.
The above algorithm essentially involves the following computations in each iteration, k ;
7. Calculation of the RHS parameter
$t^{k}=H^{T}\left(x^{k}\right) R^{-1}\left[z-h\left(x^{k}\right)\right]$
(a) Calculating the measurement function, $\mathrm{h}\left(\mathrm{x}^{\mathrm{k}}\right)$.
(b) Building the measurement Jacobian, H ( $\mathrm{x}^{\mathrm{k}}$ ).
8. Calculation of $\mathrm{G}\left(\mathrm{x}^{\mathrm{k}}\right)$ and solve for $\Delta x^{k}$.
(a) Building the gain matrix, $\mathrm{G}\left(\mathrm{x}^{\mathrm{k}}\right)$.
(b) Decomposing G ( $\mathrm{x}^{\mathrm{k}}$ ) into its Cholesky factors.
(c) Performing the forward/back substitutions to solve for $\Delta x^{k+1}$.

## 4. Simulation Results

In this section, the proposed algorithm in relation to placement of PMU in the previous section has been tested on the IEEE 6-bus test system [7], IEEE 9-bus test system [3] and IEEE 14-bus test system [13]. In respect of the system accuracy and reliability, PMU can deliver more precise measurement data [6]. Several cases to be tested with PMUs added to the conventional measurement set. The simulations and analysis of different cases are as shown in Table 1 are done with several IEEE bus systems in the next section.

TABLE 1
Different cases PMU addition in IEEE System

| Cases | Measurements |
| :---: | :---: |
| 1 | Conventional with No PMUs |
| P | Only PMUs |

For investigate the system accuracy with or without PMU on system variables, some cases are tested with the help of MATLAB simulink software. The testing parameters are available on conventional method with or without PMU.

TABLE 2
PMU LOCATIONS FOR IEEE SYSTEM

| Type of System | PMU locations at Bus |
| :---: | :---: |
| IEEE 6 System | Bus 2 |
| IEEE 9 System | Bus 2 |
| IEEE 14 System | Bus 2 |

The circuit diagram will be shown as in Fig. 3 for IEEE 6.


Fig. 3: IEEE 6 Bus System
Similarly the placement of PMU to be done on IEEE 9-bus test system and IEEE 14-bus test system.

In this segment, IEEE bus systems as IEEE 6 bus system, IEEE 9 bus system and IEEE 14-bus
test system are tested with their respective cases to find out the consequences of the PMUs to the precision of the estimated variables.

The settings for error standard deviations for measurements are shown in Table 3. A PMU has much smaller error deviations than conventional measurements as 0.0001 .

TABLE 3
Measurement S.D. for the Test

| Real <br> Power <br> (Max ) | Real <br> Power <br> (Min ) | Reactive <br> Power <br> (Max ) | Reactive <br> Power <br> (Min ) |
| :---: | :---: | :---: | :---: |
| 0.001 | 0.0001 | 0.0001 | 0.0001 |

The parameters measured are Real Power and Reactive Power (flow \& injected) measurements. The variation of parameters with or without PMU easily reflected in the fig. $4-15$ as below:




Fig. 6 : GRAPH BETWEEN P (S D) vs BUS NO.


Fig. 7 : GRAPH BETWEENP (S D) vs BUS NO.


Fig. 8 : GRAPH BETWENP (S D) vs BUS NO.


Fig. 9 : GRAPH BETWEEN P (S D) vs BUS NO.



Fig. 11 : GRAPH BETWEEN Q (S D) vs BUS NO.



Fig. 13 : GRAPH BETWE $Q$ (S D) vs BUS NO.


Fig. 14 : GRAPH BETWEENQ(SD) vs BUS NO.


## 5. Conclusions

An algorithm in relation to PMU placement for power system state estimation has been presented. The algorithm establishes the optimal placement of PMU to exterminate the critical measurement. While at the same time it develops the accuracy of Weighted Least Square State Estimation and its measurement with their effective redundancy level. The algorithm determines the optimal placement of the PMU based on criterion which takes into account critical measurement and PMU characteristics. The test results on IEEE 6-bus system, IEEE 9-bus system and IEEE 14-bus test system shows that the addition of PMU measurement will improves the performance of the power system state estimation significantly at real time configurations.

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