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# Inovative Metohods For Modeling Of Petroleum Mechanical Sistems Using Almost Periodic Functions

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## Abstract

*A method of boring research dynamics is the use of almost periodic functions resorting classical sense and almost periodic probability to study the dynamics of petroleum facilities, taking into account a large number as random factors.*

## 1. Introduction

The dynamics of machinery and plant oil-fields equipement operations for extraction requires the substantiation of equivalent mathematical models, generally comprising a number of discrete masses (concentrated), joined by elastic links or elements with distributed parameters.

Given the complexity there is established a number of simplifying assumptions, considering that the masses are concentrated rigid bodies, elastic connecting elements have mass, and the influences of nature are not considered random.

Crucial to solving the corresponding dynamic problem, its systems work is equivalent to building mathematical models and simplifying assumptions election.



## 2. Problem formulation

Be the equation of motion, written in handling drum shaft in the general case:

$$J_r \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} = M_m - M_r \qquad (1)$$

where:
Jr is reduced mass moment of inertia;
ω - angular velocity roads pump;
Mm - engine torque, Mm = Mm(ω);
Mr - reduced when handling drum resistant tree,
Mr = Mr(v) = Mr(kω)

Relation (1) is the classic expression of equivalent mathematical model of a working system, in this case: the operating system. Taking into account expressions (4.1.6.6 and 4.1.6.56, [1]), expression engine when taking into account random phenomena:

$$M_m \equiv M_m(\omega, a_k, t) = M_m(\omega) - \sum_{k=1}^{m}\sum_{i=1}^{n} p_{ik}(a_k)\cdot q_{ik}(t) \qquad (2)$$

and of the resistance, taking into account the relations (4.1.6.7 and 4.1.6.55, [1]), is:

$$M_r \equiv M_r(v, b_h, t) = M_r(\omega) + \sum_{h=1}^{l}\sum_{j=1}^{q} r_{jh}(b_h)\cdot s_{jh}(t) \qquad (3)$$

where:
Mr(ω) are reduced moments;

Rjh(bh) resistant forces rjh (bh) - random reduced to the actuator of the operating system

$$J_r \cdot \frac{d\omega}{dt} = M_m(\omega) - \sum_{k=1}^{m}\sum_{i=1}^{\infty} p_{ik}(a_k)\cdot q_{ik}(t) - M_r(\omega) - \sum_{h=1}^{l}\sum_{j=1}^{\infty} r_{jh}(b_h)\cdot s_{jh}(t) \qquad (4)$$

equivalent mathematical model representing the operating system generally unconventional for the dynamic study, where:
Mm(ω), Mr(ω) is deterministic components, and:

$$\sum_k \sum_i M_{ik}; \qquad \sum_h \sum_j F_{jh} \qquad (4')$$

random components model

*Mik* and *Fjh* as and (4), obtained by generations and reduce random moments drum handling resistant tree. Solving the mathematical model (4) is not accessible at this point.

From studies, if functions Fjh Mik and certain conditions, solving the model becomes available, can be obtained the general form of the model solution.

This phase starts to form premises simulation steps using digital programs of specific applications in mechanical drives, the electromechanical field in general and oil in particular.
*Definitions:*

$$P_{\varepsilon}(t) = \sum_{i=1}^{n} c_{ik}\varphi_{ik}(t) = \sum_{i=1}^{n} c_{ik}\cdot e^{(\sqrt{-1}\cdot\lambda_{ik}\cdot t)} \qquad (5)$$

$$P_{\varepsilon'}(t) = \sum_{i=1}^{q} c'_{jh}\,\varphi'_{jh}(t) = \sum_{i=1}^{q} c'_{jh}\cdot e^{(\sqrt{-1}\cdot\lambda'_{jh}\cdot t)} \qquad (6)$$

*Mik* and *Fjh* say are random functions almost periodic in the classical sense ***FAPC***.

These families of functions, the customizations on the actual situation will be function that will create the mathematical model study.

The relationship (4) becomes:

$$J_r \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} = M_m(\omega) - \sum_{k=1}^{m}\sum_{i=1}^{n} c_{ik}\cdot\varphi_{ik}(t) - M_r(\omega) - \sum_{h=1}^{l}\sum_{j=1}^{q} c'_{jh}\cdot\ \varphi'_{jh}(t)$$

(7)

If conditions (5) and (6) are not met by approximating the trigonometric polynomials (7) is recommended to use random trigonometric polynomials, of the form:

$$P_{\varepsilon}(a_k,t)=\sum_{i=1}^{n}c_{ik}(a_k)\cdot\ \varphi_{ik}(t)=\sum_{i=1}^{n}c_{ik}(a_k)\cdot e^{(\sqrt{-1}\cdot\lambda_{ik}\cdot t)} \tag{8}$$

$$P_{\varepsilon'}(b_h,t)=\sum_{j=1}^{q}c'_{jh}(b_h)\cdot\ \varphi'_{jh}(t)=\sum_{j=1}^{q}c'_{jh}(b_h)\cdot e^{(\sqrt{-1}\cdot\lambda'_{jh}\cdot t)} \tag{9}$$

If polynomials (8) and (9) satisfy the conditions (5) and (6), we say that functions are functions Fjh Mik and almost periodic random probability ***FAPP***.

$$J_r\cdot\frac{d\omega}{dt}=M_m(\omega)-\sum_{k=1}^{m}\sum_{i=1}^{n}c_{ik}(a_k)\cdot\ \varphi_{ik}(t)-M_r(\omega)-\sum_{h=1}^{l}\sum_{j=1}^{q}c'_{jh}(b_h)\cdot\ \varphi'_{jh}(t) \tag{10}$$

Consider a linear variation in the difference deterministic components:

$$M_m(\omega)-M_r(\omega)=\mu\cdot\omega \tag{11}$$

justified by the analysis of possible cases where the DC electric drive.

If, Mm(ω)=A – B·ω, and Mr(ω)= A'- B'·ω and substituting (11) in (9) or (10), we obtain:

$$J_r\cdot\frac{d\omega}{dt}=\mu\cdot\omega-\sum_{k=1}^{m}\sum_{i=1}^{n}c_{ik}\cdot\varphi_{ik}(t)-\sum_{h=1}^{l}\sum_{j=1}^{q}c'_{jh}\cdot\ \varphi'_{jh}(t) \tag{12}$$

Mathematical model solution (12), is: case of almost periodicity in the classical sense of random phenomena, namely:

$$J_r\cdot\frac{d\omega}{dt}=\mu\cdot\omega-\sum_{k=1}^{m}\sum_{i=1}^{n}c_{ik}(a_k)\cdot\varphi_{ik}(t)-\sum_{h=1}^{l}\sum_{j=1}^{q}c'_{jh}(b_h)\cdot\varphi'_{jh}(t) \tag{13}$$

case of almost periodicity in the probability of phenomena.

Relations (4) and (13) the general form of generalized mathematical model for system dynamics study of flexibility to both forms of almost periodicity of random phenomena.
Mathematical model solution (12), is:

$$\omega(t)=-\frac{1}{J_r}\cdot\sum_{k=1}^{m}\sum_{i=1}^{n}\left[\int_t^{\infty}e^{\frac{\mu(t-u)}{J_r}}\cdot c_{ik}\cdot\varphi_{ik}(u)du\right]-\frac{1}{J_r}\cdot\sum_{h=1}^{l}\sum_{j=1}^{q}\left[\int_t^{\infty}e^{\frac{\mu(t-u)}{J_r}}\cdot c'_{jh}\cdot\varphi'_{jh}(u)du\right], \tag{14}$$

For $\mu > 0$ (14) and $\mu < 0$ in (15)

$$\omega(t)=\frac{1}{J_r}\cdot\sum_{k=1}^{m}\sum_{i=1}^{n}\left[\int_{-\infty}^{t}e^{\frac{\mu(t-u)}{J_r}}\cdot c_{ik}\cdot\varphi_{ik}(u)du\right]+\frac{1}{J_r}\cdot\sum_{h=1}^{l}\sum_{j=1}^{q}\left[\int_{-\infty}^{t}e^{\frac{\mu(t-u)}{J_r}}\cdot c'_{jh}\cdot\ \varphi'_{jh}(u)du\right], \tag{15}$$

Model solution (15) is similar to relations (12), (13)

$$\omega(a_k,b_h,t)=$$
$$=-\frac{1}{J_r}\cdot\sum_{k=1}^{m}\sum_{i=1}^{n}\left[\int_t^{\infty}e^{\frac{\mu(t-u)}{J_r}}\cdot c_{ik}(a_k)\cdot\varphi_{ik}(u)du\right]-\frac{1}{J_r}\cdot\sum_{h=1}^{l}\sum_{j=1}^{q}\left[\int_t^{\infty}e^{\frac{\mu(t-u)}{J_r}}\cdot c'_{jh}(b_h)\cdot\varphi'_{jh}(u)du\right], \tag{16}$$

for $\mu < 0$, results:

$$\omega(a_k,b_h,t)=$$
$$=\frac{1}{J_r}\cdot\sum_{k=1}^{m}\sum_{i=1}^{n}\left[\int_{-\infty}^{t}e^{\frac{\mu(t-u)}{J_r}}\cdot c_{ik}(a_k)\cdot\varphi_{ik}(u)du\right]+\frac{1}{J_r}\cdot\sum_{h=1}^{l}\sum_{j=1}^{q}\left[\int_{-\infty}^{t}e^{\frac{\mu(t-u)}{J_r}}\cdot c'_{jh}(b_h)\cdot\varphi'_{jh}(u)du\right] \tag{17}$$

## 3. Analysis of possible cases

If $\mu > 0$, k = 1 and h = 1, which means that the influence is considered a single random variable from engine to the actuator.

a1) Where almost periodicity in the classical sense. We have the result form:

$$\omega(t)=\frac{e^{-\mu\cdot t}}{J_r}\cdot\sum_{i=1}^{n}\left[c_{i1}\cdot\frac{1}{\left(\frac{\mu}{J_r}\right)^2+(\omega_0)^2}\cdot\left(-\frac{\mu}{J_r}\cdot\sin(\omega_0 t)-i\omega_0\cdot\cos(\omega_0 t)\right)\right]$$
$$+\frac{e^{-\mu\cdot t}}{J_r}\cdot\sum_{j=1}^{q}\left\{c'_{j1}\cdot\left\{\frac{\sin(\omega_0 t)}{\left(\frac{\mu}{J_r}\right)^2+(\omega_0)^2}\cdot\left[j\omega_0\cdot\sin(\varphi_0)-\frac{\mu}{J_r}\cdot\cos(\varphi_0)\right]-\right.\right.$$
$$\left.\left.-\frac{\cos(\omega_0 t)}{\left(\frac{\mu}{J_r}\right)^2+(\omega_0)^2}\cdot\left[j\omega_0\cdot\cos(\varphi_0)-\frac{\mu}{J_r}\cdot\sin(\varphi_0)\right]\right\}\right\}+\omega_0;$$

(18,a)

a2) If the probability of almost periodicity

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$$\omega(t)=\frac{e^{-\mu\cdot t}}{J_r}\cdot\sum_{i=1}^{n}\left[c_{i1}(t_1)\cdot\frac{1}{\left(\frac{\mu}{J_r}\right)^2+(\omega_0)^2}\cdot\left(-\frac{\mu}{J_r}\cdot\sin(\omega_0 t)-i\omega_0\cdot\cos(\omega_0 t)\right)\right]$$

$$+\frac{e^{-\mu\cdot t_1}}{J_r}\cdot\sum_{j=1}^{q}\left\{c'_{j1}(t_1)\cdot\left\{\frac{\sin(\omega_0 t)}{\left(\frac{\mu}{J_r}\right)^2+(\omega_0)^2}\cdot\left[j\omega_0\cdot\sin(\varphi_0)-\frac{\mu}{J_r}\cdot\cos(\varphi_0)\right]-\right.\right.$$

$$\left.\left.-\frac{\cos(\omega_0 t)}{\left(\frac{\mu}{J_r}\right)^2+(\omega_0)^2}\cdot\left[j\omega_0\cdot\cos(\varphi_0)-\frac{\mu}{J_r}\cdot\sin(\varphi_0)\right]\right\}\right\}+\omega_0; \quad (18,b)$$

b) Where A and B according to and considerations point to remain valid. The general solution of the model (14) becomes

$$\omega(t)=\frac{2A}{B}-\frac{\Omega(t)}{B}-\left(\frac{A}{B}-\omega_0\right)\cdot e^{-\frac{B}{J_r}\cdot t} \quad (19)$$

The function expression (19) involved constants A, B, whose expressions are known.

For explicit expression Ω(t) is presented as result-ing from ra-tionalmentul above. Significance of measurements involved in relationship (19) is known.

Where:

$$\Omega(t)=-\frac{B}{J_r}\cdot\sum_{i=1}^{n}\left[c_{i1}\cdot\frac{e^{-\mu\cdot t}}{\left(-\frac{B}{J_r}\right)^2+(\omega_0)^2}\cdot\left(\frac{B}{J_r}\cdot\sin(\omega_0 t)-i\omega_0\cdot\cos(\omega_0 t)\right)\right]-$$

$$-\frac{B\cdot e^{-\mu\cdot t}}{J_r}\cdot\sum_{j=1}^{q}\left\{c'_{j1}\cdot\left\{\frac{\sin(\omega_0 t)}{\left(-\frac{B}{J_r}\right)^2+(\omega_0)^2}\cdot\left[j\omega_0\cdot\sin(\varphi_0)+\frac{B}{J_r}\cdot\cos(\varphi_0)\right]-\right.\right.$$

$$\left.\left.-\frac{\cos(\omega_0 t)}{\left(-\frac{B}{J_r}\right)^2+(\omega_0)^2}\cdot\left[j\omega_0\cdot\cos(\varphi_0)+\frac{B}{J_r}\cdot\sin(\varphi_0)\right]\right\}\right\}+\omega_0. \quad (19')$$

Figures at the end paper are simulated using computer applications.

c) Cases studied previously, are cases of work-specific systems has consistently yield. Using relations, the functionality of the system actuator considering

$$M_m(\omega)=A-B\cdot\omega \quad (20)$$

is given by expression (11) becomes:

$$M_m(\omega)=M_r(\omega)=(A-B\cdot\omega)\left(1-\frac{k_F}{k\cdot x}\cdot\eta(t)\eta\right) \quad (21)$$

valid for systems with variable yield, where kf is the conversion factor resistance

$$M_r=k_F M \quad (22)$$

It proposes the following notations:

$$A(t)=A\cdot\left(1-\frac{k_F}{k\cdot x}\cdot\eta(t)\eta\right)\quad B(t)=B\cdot\left(1-\frac{k_F}{k\cdot x}\cdot\eta(t)\eta\right) \quad (23)$$

The general solution of the model (19), in this case, becomes:

$$\omega(t)=\frac{2A(t)}{B(t)}-\frac{\Omega_x(t)}{B(t)}-\left(\frac{A(t)}{B(t)}-\omega_0\right)\cdot e^{-\frac{B(t)}{J_r}t} \quad (24)$$

where :

$$\Omega_x(t)=-\frac{B(t)}{J_r}\sum_{i=1}^{n}\left\{c_{i1}\cdot\int_t^{\infty}e^{-\frac{B(t)}{J_r}(t-u)}\cdot\sin(\omega_0 u)du\right\}-$$

$$-\frac{B(t)}{J_r}\sum_{j=1}^{q}\left\{c'_{j1}\cdot\int_t^{\infty}e^{-\frac{B(t)}{J_r}(t-u)}\cdot\sin(\omega_0 u+\varphi_0)du\right\}; \quad (25)$$

case of almost periodicity in the classical sense.

Case of almost periodicity in probability under the conditions agreed, the general solution is (24) or (25), but coefficients $c_{i1}$ şi $c'_{j1}$ will take values $c_{i1}(t_1)$ and $c'_{j1}(t_1)$. This is a general case with real phenomena which are more specifically studied. This results in the use and usefulness of the proposed approach to modeling mechanical systems of these functions.

## 4. Analysis of the structure for the oil field equipment

Plant fluids from wells with pumping progressive cavity pumps, submersible (Figure 1) have the following main components: Operating system (SM ─ ST ─ SCA) in the rotating pump surface mounted ;

Pipes (P) that are screwed to the stator to be placed in column operation..Pumping rods (PR )which transmit the rotation of the rotor drive system with progressive cavity pump, pipes are placed in the probe, with the rotor screw to the bottom of the ram pump road.

Progressive cavity pump (PCP), submersible known as other names that screw pump, eccentric screw pump, rotary pump thaw, or thaw pump rotating twister. The following figures are numerical applications of these results.

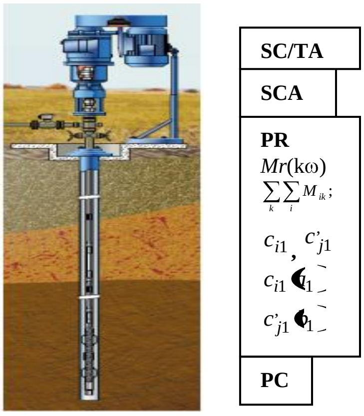


**Figure 1** Installation of pumps with progressive cavity pump

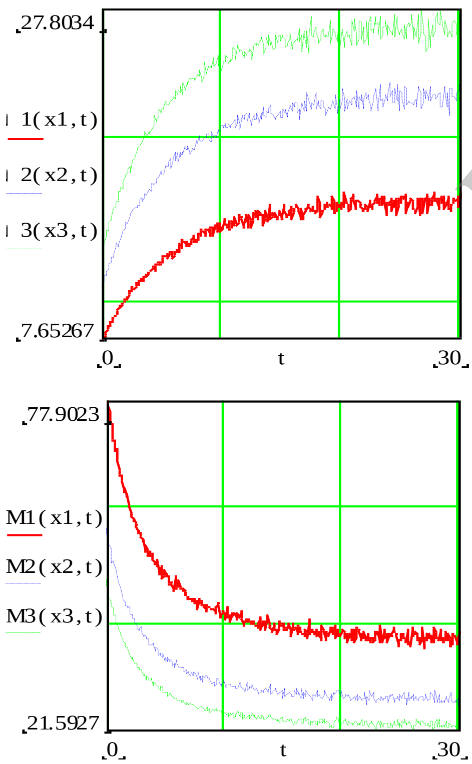


**Figure 2** Variation of angular velocity and momentum with variable yield case of almost periodicity in the classical sense

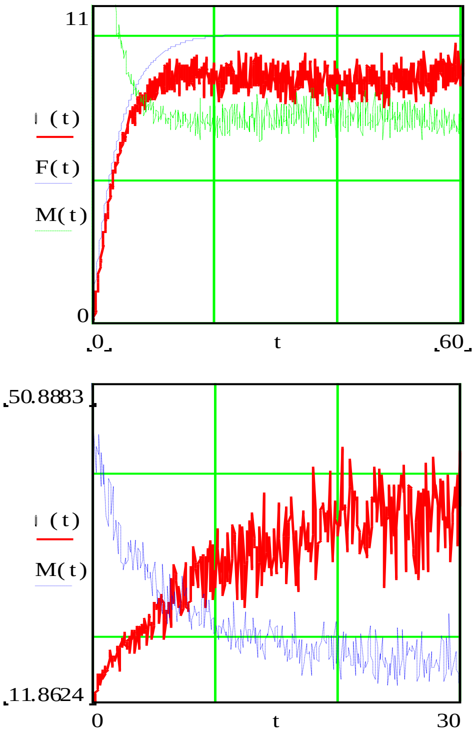


**Figure 3** Variation of angular speed and engine torque case of almost periodicity in the probability



## 4. Conclusions

In this article it presented the mathematical model of the whole drive system composed of equipment from the top of the gasket seal Pomar and pump assembly.

This was due to the fact that in establishing the functional caracteristics of the pumping sistem, the deterministic trearment of the mathematical model has shown that a series of influiences linked to the action of some random factors are not found in the analysis and/or the synthesis of the system and of the functional characteristic.

An element of novelty innovation in the author's view is the introduction of the almost periodic functions in the classical sense and of the probability almost periodic functions allow the identification of the influence of random factors.

Using these "test" functions transfers the problematic of the study from the domain of the mathematic statistics to that of mathematic analisysis, which is more accessible to the requirements.

The results are materialized by developing a computerized calculation program whose results are presented in graphical simulation system operation after at various depth for pumps.

A great importance in the dynamic study of a working system in technological pumping operation is given to the way in which the system structure is set.

A first concern in this regard was to set the structure of the working system in order to design the dynamic simulation as a continuous system.

By comparison with mathematical models established by already existing energy method proposed by researchers, some original contributions have been made by including the mathematical equations of the model.

They are represented by the effect given by the gel resistance of the drilling mud and of the "plunger", by additional hydrodynamic pressure occurrence during bit run back operation. It has also been considered that energy losses of the drill string result from viscous and dry amortization.

Produced mechanical waves can become dangerous for both the road string (area of threaded joints) and the surface guidance structure on which the waves have effect.

## 4. Acknowledgements

I wish to express our homage to Octav Onicescut, a Romanian mathematician, whose work made it posible to the work of a to achieve this research paper.

We also thank our colleagues manufacturing engineers without whose input we could not have verifyed in practice the results of research and software developed



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