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Integral Involving H-function of Several Complex Varialbes and Exponential Function

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Naresh Bhati
Department of Mathematics and Statistics
Jai Narain Vyas University,
Jodhpur, India

Rajeev Kumar Gupta
Department of Mathematics and Statistics
Jai Narain Vyas University,
Jodhpur, India, Country

***Abstract*— In this paper we have integrated a H-function of several complex variables with combination of hypergeometric function and an exponential function as product. To perform the integrations, we have used the definite integral listed in "Table of Integrals, Series, and Products" by Gradshteyn and Ryzhik. The integrations obtain in this paper will be useful to solve problems of mathematical, statistical and physical sciences where different kind of functions occurs as product. The result obtained from integration was put in compact form.**

***Keywords*— Multivariable H-function, Exponential function, Hypergeometric function.**

1. INTRODUCTION

The German mathematician Carl Friedrich Gauss (1777-1855) introduced a series called gauss hypergeometric series and represent by

$$ {}_2F_1(\alpha,\beta;\gamma;x)=\sum_{u=0}^{\infty}\frac{(\alpha)_u(\beta)_u}{(\gamma)_u}\frac{x^u}{u!} \qquad (1)$$

where is the Pochhammer symbol defined by

$$(\alpha)_u=\begin{cases}\alpha(\alpha+1)\dots(\alpha+u-1) & u\in N\\ 1 & u=1\end{cases} \qquad (2)$$

The hypergeometric function is defined for |z| < 1, provided that γ is neither zero nor a negative integer. The gaussian hypergeometric function has fundamental importance in theory of special functions. The most of functions used in physical sciences, applied mathematics, engineering etc., are expressible as its special cases.

The H-function of several complex variables, defined H. M. Srivastava and R. Panda in series of their research papers [5]. The H-function defined and represented in terms of a multiple Mellin-Bernes type contour integral as

$$H[z_1,z_2,\dots,z_r]=H^{M,N;m_1,n_1;\dots;m_r,n_r}_{P,Q;p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\left|\begin{matrix}\left(a_j;A_j^{(1)},A_j^{(2)},\dots A_j^{(r)}\right)_{1,P};\left(c_j^{(1)},C_j^{(1)}\right)_{1,p_1};\\ \left(b_j,B_j^{(1)},B_j^{(2)},\dots,B_j^{(r)}\right)_{1,Q};\left(d_j^{(1)},D_j^{(1)}\right)_{1,q_1};\end{matrix}\right.\right. \qquad (3)$$

$$\left.\begin{matrix}\dots;\left(c_j^{(r)},C_j^{(r)}\right)_{1,p_r}\\ \dots;\left(d_j^{(r)},D_j^{(r)}\right)_{1,q_r}\end{matrix}\right]=\frac{1}{(2\pi i)^r}\int_{\Upsilon_1}\dots\int_{\Upsilon_r}\psi(s_1,\dots,s_r)\left\{\prod_{k=0}^{r}\phi_k(s_k)z_k^{s_k}ds_k\right\}$$

denote the H-function of r complex variables $z_1,z_2,\dots,z_r$. Where $i=(-1)^{1/2}$ and

$$\psi(s_1,s_2,\dots,s_r)=\frac{\prod_{j=i}^{M}\Gamma\left(b_j-\sum_{k=1}^{r}B_j^{(k)}s_k\right)\prod_{j=1}^{N}\Gamma\left(1-a_j+\sum_{k=1}^{r}A_j^{(k)}s_k\right)}{\prod_{j=M+1}^{Q}\Gamma\left(1-b_j+\sum_{k=1}^{r}B_j^{(k)}s_k\right)\prod_{j=N+1}^{P}\Gamma\left(a_j-\sum_{k=1}^{r}A_j^{(k)}s_k\right)} \qquad (4)$$

$$\phi_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma\left(d_j^{(k)}-D_j^{(k)}s_k\right)\prod_{j=1}^{n_k}\Gamma\left(1-c_j^{(k)}+C_j^{(k)}s_k\right)}{\prod_{j=m_k+1}^{q_k}\Gamma\left(1-d_j^{(k)}+D_j^{(k)}s_k\right)\prod_{j=n_k+1}^{p_k}\Gamma\left(c_j^{(k)}-C_j^{(k)}s_k\right)} \qquad (5)$$

where $(k=1,2,\dots,r)$

$\begin{cases}a_j,[j=1,2,\dots,P];c_j^{(k)},[j=1,2,\dots,p_i;\forall k\in\{1,2,\dots,r\}]\\ b_j,[j=1,2,\dots,Q];d_j^{(k)},[j=1,2,\dots,q_i;\forall i\in\{1,2,\dots,r\}]\end{cases}$ are complex numbers, and their corresponding related coefficients $\begin{cases}A_j^{(k)},[j=1,2,\dots,P];C_j^{(k)}[j=1,2,\dots,p_k;\forall k\in\{1,2,\dots,r\}]\\ B_j^{(k)},[j=1,2,\dots,Q];D_j^{(k)}[j=1,2,\dots,q_k;\forall k\in\{1,2,\dots,r\}]\end{cases}$ are positive real numbers.

And Υ_k, represent the contours start at the point $\vartheta_k-i\infty$ and goes to the point $\vartheta_k+i\infty$ with $\vartheta_k\in R$, $(k=1,2,\dots,r)$. The integral in (3) is converges absolutely, under the conditions Srivastava et al. [5] if

$$|\arg z_k|<\frac{\pi}{2}\tau_k. \qquad (k=1,2,\dots,r) \qquad (6)$$

Where $\Delta_k\equiv\sum_{j=1}^{P}A_j^{(k)}+\sum_{j=1}^{p_k}C_j^{(k)}-\sum_{j=1}^{Q}B_j^{(k)}-\sum_{j=1}^{q_k}D_j^{(k)}\le 0$

$$\tau_k=\sum_{j=1}^{N}A_j^{(k)}-\sum_{j=N+1}^{P}A_j^{(k)}+\sum_{j=1}^{M}B_j^{(k)}-\sum_{j=M+1}^{Q}B_j^{(k)}+\sum_{j=1}^{n_k}c_j^{(k)}-\sum_{j=n_k+1}^{p_k}C_j^{(k)}+\sum_{j=1}^{m_k}D_j^{(k)}-\sum_{j=m_k+1}^{q_k}D_j^{(k)}>0 \qquad (7)$$

$,\forall(k=1,2,\dots,r)$

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where M,N,P,Q,m_k,n_k,p_k,q_k are positive integer and restricted by the $0\le N\le P$, $Q\ge M\ge 0$, and $q_k\ge m\ge 0$, $p_k\ge n_k\ge 0$, $\forall k\in\{1,2,...,r\}$ and inequalities (6) suitably constrained values of the complex variables $z_1,z_2,...,z_r$. The points $z_k=0,k=1,2,..,r$ and many exceptional parameter values, being tacitly excluded. From Srivastava and Panda [5], we have

$H[z_1,z_2,...,z_r]=o\left(|z_1|^{e_1}....|z_r|^{e_r}\right)\left(\lim_{1\le j\le m_r}\|z_j\|\to 0\right)$, where

$$e_k=\lim_{1\le j\le m_r}\mathrm{Re}\left(\frac{d_j^{(k)}}{D_j^{(k)}}\right)(k=1,2,...,r) \qquad (8)$$

Lemma:

From the table of integration, series and products I.S. Gradshteyn, M.I. Ryzhik [3, (2007): Eq. 3.194 (7), p.316] We need the following integration formula

$$\int_0^\infty x^m(a+bx)^{-n}dx=\frac{a^{m-n+1}}{b^{m+1}}\frac{\Gamma(m+1)\Gamma(n-m-1)}{\Gamma(n)} \qquad (9)$$

$[n>m+1>0,a>0;b>0]$

2. MAIN RESULTS:

In this section we have obtain some integrals involving the product of the basic hypergeometric function with H-function of several complex variable and exponential function.

Theorem 1:

If we take $\rho>\sigma+1>0$; $a>0;b>0$; $\xi\ge 0,\delta\ge 0$; $\lambda\ge 0;\eta\ge 0;$ *and* $\mu_i\ge o,\upsilon_i\ge 0$, $(\forall i=1,2,...,r)$ *then the following integration hold*

$$\int_0^\infty y^\sigma(a+by)^{-\rho}e^{\delta y^\xi}{}_2F_1\left[\alpha,\beta;\gamma;cy^\lambda(a+by)^\eta\right]\times H^{M,N;m_1,n_1;..;m_r,n_r}_{P,Q;p_1,q_1;...,p_r,q_r}\left[\begin{matrix}c_1z_1y^{\mu_1}(a+by)^{\upsilon_1}\\ \vdots\\ c_rz_ry^{\mu_r}(a+by)^{\upsilon_r}\end{matrix}\right]dx=\frac{a^{\sigma-\rho+1}}{b^{\sigma+1}}e^{\delta\left(\frac{a}{b}\right)^\xi}{}_2F_1\left(\alpha,\beta;\gamma;\frac{ca^{\lambda+\eta}}{b^\lambda}\right)\times$$

$$\sum_{k=o}^\infty\sum_{u=0}^\infty H^{M+1,N+1;m_1,n_1;..;m_r,n_r}_{P+2,Q+1;p_1,q_1;...,p_r,q_r}\left[\begin{matrix}a^{\mu_1+\upsilon_1}b^{-\mu_1}c_1z_1\\ \\ a^{\mu_r+\upsilon_r}b^{-\mu_r}c_rz_r\end{matrix}\left|\begin{matrix}(-\sigma-u\lambda-k\xi;\mu_1,..,\mu_r);\\ \left(\begin{matrix}\rho-\eta u-\sigma-u\lambda-k\xi-1;\\ \upsilon_1+\mu_1,..,\upsilon_r+\mu_r\end{matrix}\right);\end{matrix}\right.\right. \qquad (10)$$

$$\left.\begin{matrix}\left(a_j;A_j^{(1)},A_j^{(2)},...A_j^{(r)}\right)_{2,P+1};(\rho-u\eta;\upsilon_1,..,\upsilon_r);\left(c_j^{(1)},C_j^{(1)}\right)_{1,p_1};...;\left(c_j^{(r)},C_j^{(r)}\right)_{1,p_r}\\ \left(b_j,B_j^{(1)},B_j^{(2)},...,B_j^{(r)}\right)_{1,Q+1};\left(d_j^{(1)},D_j^{(1)}\right)_{1,q_1};...;\left(d_j^{(r)},D_j^{(r)}\right)_{1,q_r}\end{matrix}\right]$$

The above integral will be convergence for condition (6), (7) and (8).

PROOF:

$$\int_0^\infty y^\sigma(a+by)^{-\rho}e^{\delta y^\xi}{}_2F_1\left[\alpha,\beta;\gamma;cy^\lambda(a+by)^\eta\right]\times H\left[c_1z_1y^{\mu_1}(a+by)^{\upsilon_1},...,c_rz_ry^{\mu_r}(a+by)^{\upsilon_r}\right]dy$$

Now we replace $e^{\delta y^\xi}$ by $\sum_{k=0}^\infty\frac{(\delta y^\xi)^k}{k!}$ and express the hypergeometric function (1) and H-function of several complex variables (3), then we get

$$=\int_0^\infty y^\sigma(a+by)^{-\rho}\sum_{k=0}^\infty\frac{\delta^k y^{k\xi}}{k!}\sum_{u=0}^\infty\frac{(\alpha)_u(\beta)_u c^u y^{u\lambda}(a+by)^{u\eta}}{(\gamma)_u u!(2\pi i)^r}\int_{\Upsilon_1}...\int_{\Upsilon_r}\psi(s_1,...,s_r)\times$$
$$\phi_1(s_1)...\phi_r(s_r)c_1^{S_1}z_1^{S_1}y^{\mu_1S_1}(a+by)^{\upsilon_1S_1}...c_r^{S_r}z_r^{S_1}y^{\mu_1S_1}(a+by)^{\upsilon_1S_1}dS_1..dS_rdy$$

$$=\int_0^\infty y^\sigma(a+by)^{-\rho}\sum_{k=0}^\infty\sum_{u=0}^\infty\frac{\delta^k c^u(\alpha)_u(\beta)_u y^{u\lambda+k\xi}(a+by)^{u\eta}}{(\gamma)_u k!u!(2\pi i)^r}\int_{\Upsilon_1}...\int_{\Upsilon_r}\psi(s_1,...,s_r)\times$$
$$\phi_1(s_1)...\phi_r(s_r)c_1^{S_1}z_1^{S_1}y^{\mu_1S_1}(a+by)^{\upsilon_1S_1}...c_r^{S_r}z_r^{S_r}y^{\mu_rS_r}(a+by)^{\upsilon_rS_r}dS_1...dS_rdy$$

By interchanging the order of integration and summation, we get

$$=\sum_{k=0}^\infty\sum_{u=0}^\infty\frac{c^u\delta^k(\alpha)_u(\beta)_u}{(\gamma)_u k!u!(2\pi i)^r}\int_{\Upsilon_1}...\int_{\Upsilon_r}\psi(s_1,...,s_r)\phi_1(s_1)...\phi_r(s_r)\times$$
$$\left\{\int_0^\infty y^{\sigma+u\lambda+k\xi+\sum_{i=1}^r\mu_iS_i}(a+by)^{\eta u-\rho+\sum_{i=1}^r\upsilon_iS_i}dy\right\}c_1^{S_1}z_1^{S_1}...c_r^{S_r}z_1^{S_1}dS_1..dS_r$$

$$=\sum_{k=0}^\infty\sum_{u=0}^\infty\frac{c^u\delta^k(\alpha)_u(\beta)_u}{(\gamma)_u k!u!(2\pi i)^r}\int_{\Upsilon_1}...\int_{\Upsilon_r}\frac{a^{\sigma+u\lambda+k\xi+\sum_{i=1}^r\mu_iS_i+\eta u-\rho+\sum_{i=1}^r\upsilon_iS_i+1}}{b^{\sigma+u\lambda+k\xi+\sum_{i=1}^r\mu_iS_i+1}}$$
$$\frac{\Gamma\left(\sigma+u\lambda+k\xi+\sum_{i=1}^r\mu_iS_i+1\right)\Gamma\left\{\rho-\eta u-\sum_{i=1}^r\upsilon_iS_i-\sigma-u\lambda-k\xi-\sum_{i=1}^r\mu_iS_i-1\right\}}{\Gamma\left(\rho-\eta u-\sum_{i=1}^r\upsilon_iS_i\right)}$$
$$\psi(s_1,...,s_r)\phi_1(s_1)...\phi_r(s_r)c_1^{S_1}z_1^{S_1}...c_r^{S_r}z_1^{S_1}dS_1..dS_r$$

$$=\frac{a^{\sigma-\rho+1}}{b^{\sigma+1}}\sum_{k=0}^\infty\sum_{u=0}^\infty\frac{c^u\delta^k(\alpha)_u(\beta)_u a^{u\lambda+\eta u+k\xi}}{(\gamma)_u b^{k\xi+u\lambda}k!u!(2\pi i)^r}\times\int_{\Upsilon_1}...\int_{\Upsilon_r}\frac{a^{\sum_{i=1}^r\mu_iS_i+\sum_{i=1}^r\upsilon_iS_i}}{b^{\sum_{i=1}^r\mu_iS_i}}\times$$
$$\frac{\Gamma\left(\sigma+u\lambda+k\xi+\sum_{i=1}^r\mu_iS_i+1\right)\Gamma\left(\rho-\eta u-\sigma-u\lambda-k\xi-\sum_{i=1}^r\mu_iS_i-\sum_{i=1}^r\upsilon_iS_i-1\right)}{\Gamma\left(\rho-\eta u-\sum_{i=1}^r\upsilon_iS_i\right)}$$
$$\psi(s_1,...,s_r)\phi_1(s_1)...\phi_r(s_r)c_1^{S_1}z_1^{S_1}...c_r^{S_r}z_r^{S_r}dS_1..dS_r$$

by virtue, interpreting of equation (1) and (3), we obtain the required result.

$$=\frac{a^{\sigma-\rho+1}}{b^{\sigma+1}}e^{\delta\left(\frac{a}{b}\right)^\xi}{}_2F_1\left(\alpha,\beta;\gamma;\frac{ca^{\lambda+\eta}}{b^\lambda}\right)\sum_{k=o}^\infty\sum_{u=0}^\infty H^{M+1,N+1;m_1,n_1;..;m_r,n_r}_{P+2,Q+1;p_1,q_1;...,p_r,q_r}$$
$$\left[\begin{matrix}a^{\mu_1+\upsilon_1}b^{-\mu_1}c_1z_1\\ \\ a^{\mu_r+\upsilon_r}b^{-\mu_r}c_rz_r\end{matrix}\left|\begin{matrix}(-\sigma-u\lambda-k\xi;\mu_1,..,\mu_r);\left(a_j;A_j^{(1)},A_j^{(2)},...A_j^{(r)}\right)_{2,P+1};\\ \\ (\rho-\eta u-\sigma-u\lambda-k\xi-1;\upsilon_1+\mu_1,..,\upsilon_r+\mu_r);\end{matrix}\right.\right.$$
$$\left.\begin{matrix}(\rho-u\eta;\upsilon_1,..,\upsilon_r);\left(c_j^{(1)},C_j^{(1)}\right)_{1,p_1};...;\left(c_j^{(r)},C_j^{(r)}\right)_{1,p_r}\\ \left(b_j,B_j^{(1)},B_j^{(2)},...,B_j^{(r)}\right)_{1,Q+1};\left(d_j^{(1)},D_j^{(1)}\right)_{1,q_1};...;\left(d_j^{(r)},D_j^{(r)}\right)_{1,q_r}\end{matrix}\right]$$

Hence theorem 1 is proved.

Theorem 2:

If we take $\rho>\sigma+1>0$; $a>0;b>0$; $\xi\ge 0,\delta\ge 0$; $\lambda\ge 0;\eta\ge 0$ *and* $\mu_i\ge o,\upsilon_i\ge 0$; $(\forall i=1,2,...,r)$ *then the following integration hold*

$$\int_0^\infty y^\sigma(a+by)^{-\rho}e^{\delta y^\xi}{}_2F_1\left[\alpha,\beta;\gamma;cy^\lambda(a+by)^\eta\right]\times H^{M,N;m_1,n_1;..;m_r,n_r}_{P,Q;p_1,q_1;...,p_r,q_r}\left[\begin{matrix}c_1z_1y^{-\mu_1}(a+by)^{-\upsilon_1}\\ \vdots\\ c_rz_ry^{-\mu_r}(a+by)^{-\upsilon_r}\end{matrix}\right]dx=\frac{a^{\sigma-\rho+1}}{b^{\sigma+1}}e^{\delta\left(\frac{a}{b}\right)^\xi}{}_2F_1\left(\alpha,\beta;\gamma;\frac{ca^{\lambda+\eta}}{b^\lambda}\right)$$

$$\sum_{k=o}^\infty\sum_{u=0}^\infty H^{M+1,N+1;m_1,n_1;..;m_r,n_r}_{P+1,Q+2;p_1,q_1;...,p_r,q_r}\left[\begin{matrix}a^{-\mu_1-\upsilon_1}b^{\mu_1}c_1z_1\\ \\ a^{-\mu_r-\upsilon_r}b^{\mu_r}c_rz_r\end{matrix}\left|\begin{matrix}\left(\begin{matrix}2+\sigma+k\xi+u\lambda+u\eta-\rho;\\ \mu_1+\upsilon_1,...,\mu_r+\upsilon_r\end{matrix}\right);\\ (1+\sigma+k\xi+u\lambda;\mu_1,..,\mu_r);\end{matrix}\right.\right. \qquad (11)$$

$$\left.\begin{matrix}\left(a_j;A_j^{(1)},A_j^{(2)},...A_j^{(r)}\right)_{2,P+1};\left(c_j^{(1)},C_j^{(1)}\right)_{1,p_1};...;\left(c_j^{(r)},C_j^{(r)}\right)_{1,p_r}\\ \left(b_j,B_j^{(1)},B_j^{(2)},...,B_j^{(r)}\right)_{2,Q+1};(1-\rho+u\eta;\upsilon_1,..,\upsilon_r);\left(d_j^{(1)},D_j^{(1)}\right)_{1,q_1};..;\left(d_j^{(r)},D_j^{(r)}\right)_{1,q_r}\end{matrix}\right]$$

The above integral will be convergence for condition (6), (7) and (8).

Theorem 3:

If we take $\rho > \sigma + 1 > 0;\quad a > 0; b > 0;\quad \xi \ge 0, \delta \ge 0;\quad \lambda \ge 0; \eta \ge 0$ *and* $\mu_i \ge o, \upsilon_i \ge 0;\ (\forall i = 1,2,...,r)$ *then the following integration hold*

$$\int_0^\infty y^\sigma (a+by)^{-\rho} e^{\delta y^\xi} {}_2F_1\left[\alpha,\beta;\gamma;cy^\lambda (a+by)^\eta\right] \times H^{M,N;m_1,n_1;...;m_r,n_r}_{P,Q;p_1,q_1;...;p_r,q_r}\left[\begin{matrix} c_1z_1y^{-\mu_1}(a+by)^{\upsilon_1} \\ \vdots \\ c_rz_ry^{-\mu_r}(a+by)^{\upsilon_r}\end{matrix}\right]dx = \frac{a^{\sigma-\rho+1}}{b^{\sigma+1}} e^{\delta\left(\frac{a}{b}\right)^\xi} {}_2F_1\left(\alpha,\beta;\gamma;\frac{ca^{\lambda+\eta}}{b^\lambda}\right)$$

$$\sum_{k=0}^\infty\sum_{u=0}^\infty H^{M+1,N+1;m_1,n_1;...;m_r,n_r}_{P+2,Q+1;p_1,q_1;...;p_r,q_r}\left[\begin{matrix} a^{\upsilon_1-\mu_1}b^{\mu_1}c_1z_1 \\ \vdots \\ a^{\upsilon_r-\mu_r}b^{\mu_r}c_rz_r\end{matrix}\middle| \begin{matrix} \left(\begin{matrix}2+\sigma+k\xi+u\lambda+u\eta-\rho; \\ \mu_1-\upsilon_1,...,\mu_r-\upsilon_r\end{matrix}\right); (a_j;A_j^{(1)},A_j^{(2)},...A_j^{(r)})_{2,P+1}; (\rho-u\eta;\upsilon_1,...,\upsilon_r); (c_j^{(1)},C_j^{(1)})_{1,p_1};...;(c_j^{(r)},C_j^{(r)})_{1,p_r} \\ (1+\sigma+k\xi+u\lambda;\mu_1,...,\mu_r); (b_j,B_j^{(1)},B_j^{(2)},...,B_j^{(r)})_{2,Q+1}; (d_j^{(1)},D_j^{(1)})_{1,q_1};...;(d_j^{(r)},D_j^{(r)})_{1,q_r}\end{matrix}\right] \tag{12}$$

The above integral will be convergence for condition (6), (7) and (8).

Theorem 4:

If we take $\rho > \sigma + 1 > 0;\quad a > 0; b > 0;\quad \xi \ge 0, \delta \ge 0;\quad \lambda \ge 0;\quad \eta \ge 0;$ *and* $\mu_i \ge o, \upsilon_i \ge 0; (\forall i = 1,2,...,r)$ *then the following integration hold*

$$\int_0^\infty y^\sigma (a+by)^{-\rho} e^{\delta y^\xi} {}_2F_1\left[\alpha,\beta;\gamma;cy^\lambda (a+by)^\eta\right] \times H^{M,N;m_1,n_1;...;m_r,n_r}_{P,Q;p_1,q_1;...;p_r,q_r}\left[\begin{matrix} c_1z_1y^{\mu_1}(a+by)^{-\upsilon_1} \\ \vdots \\ c_rz_ry^{\mu_r}(a+by)^{-\upsilon_r}\end{matrix}\right]dx = \frac{a^{\sigma-\rho+1}}{b^{\sigma+1}} e^{\delta\left(\frac{a}{b}\right)^\xi} {}_2F_1\left(\alpha,\beta;\gamma;\frac{ca^{\lambda+\eta}}{b^\lambda}\right)$$

$$\sum_{k=0}^\infty\sum_{u=0}^\infty H^{M,N+2;m_1,n_1;...;m_r,n_r}_{P+2,Q+1;p_1,q_1;...;p_r,q_r}\left[\begin{matrix} a^{\mu_1-\upsilon_1}b^{-\mu_1}c_1z_1 \\ \vdots \\ a^{\mu_r-\upsilon_r}b^{-\mu_r}c_rz_r\end{matrix}\middle| \begin{matrix} \left(\begin{matrix}2+\sigma+k\xi+u\lambda+u\eta-\rho; \\ \upsilon_1-\mu_1,...,\upsilon_r-\mu_r\end{matrix}\right); \left(\begin{matrix}-\sigma-\lambda u-k\xi; \\ \mu_1,...,\mu_r\end{matrix}\right); (a_j;A_j^{(1)},A_j^{(2)},...A_j^{(r)})_{2,P+1}; (c_j^{(1)},C_j^{(1)})_{1,p_1};...;(c_j^{(r)},C_j^{(r)})_{1,p_r} \\ (b_j,B_j^{(1)},B_j^{(2)},...,B_j^{(r)})_{1,Q}; (1+\sigma+k\xi+u\lambda;\mu_1,...,\mu_r); (d_j^{(1)},D_j^{(1)})_{1,q_1};...;(d_j^{(r)},D_j^{(r)})_{1,q_r}\end{matrix}\right] \tag{13}$$

The above integral will be convergence for condition (6), (7) and (8). The integrals (13) to (15) can be proved on lines similar to those of integral (12)

3. PARTICULAR CASES:

If we take $r = 2, \eta = 0$ and $b = 1$ in eq. (10), (11), (12) and (13) then Eq. (10) reduce to (14) as follows

$$\int_0^\infty y^\sigma (a+y)^{-\rho} e^{\delta y^\xi} {}_2F_1\left[\alpha,\beta;\gamma;cy^\lambda\right] H^{M,N;m_1,n_1;m_2,n_2}_{P,Q;p_1,q_1;p_2,q_2}\left[\begin{matrix} c_1z_1y^{\mu_1}(a+y)^{\upsilon_1} \\ c_2z_2y^{\mu_2}(a+y)^{\upsilon_2}\end{matrix}\right]dx$$

$$= a^{\sigma-\rho+1}e^{\delta a^\xi} {}_2F_1\left(\alpha,\beta;\gamma;ca^\lambda\right)\sum_{k=0}^\infty\sum_{u=0}^\infty H^{M+1,N+1;m_1,n_1;m_2,n_2}_{P+2,Q+1;p_1,q_1;p_2,q_2}\left[\begin{matrix} a^{\mu_1+\upsilon_1}c_1z_1 \\ a^{\mu_2+\upsilon_2}c_rz_r\end{matrix}\middle| \begin{matrix} \left(\begin{matrix}-\sigma-u\lambda-k\xi; \\ \mu_1,\mu_2\end{matrix}\right); (a_j;A_j^{(1)},A_j^{(2)})_{2,P+1}; (\rho;\upsilon_1,\upsilon_2); (c_j^{(1)},C_j^{(1)})_{1,p_1}; (c_j^{(2)},C_j^{(2)})_{1,p_2} \\ \left(\begin{matrix}\rho-\sigma-u\lambda-k\xi-1; \\ \upsilon_1+\mu_1,\upsilon_2+\mu_2\end{matrix}\right); (b_j,B_j^{(1)},B_j^{(2)})_{1,Q+1}; (d_j^{(1)},D_j^{(1)})_{1,q_1}; (d_j^{(2)},D_j^{(2)})_{1,q_r}\end{matrix}\right] \tag{14}$$

Eq. (11) reduce to (15) as follows

$$\int_0^\infty y^\sigma (a+y)^{-\rho} e^{\delta y^\xi} {}_2F_1\left[\alpha,\beta;\gamma;cy^\lambda\right] H^{M,N;m_1,n_1;m_2,n_2}_{P,Q;p_1,q_1;p_2,q_2}\left[\begin{matrix} c_1z_1y^{-\mu_1}(a+y)^{-\upsilon_1} \\ c_2z_2y^{-\mu_2}(a+y)^{-\upsilon_2}\end{matrix}\right]dx$$

$$= a^{\sigma-\rho+1}e^{\delta a^\xi} {}_2F_1\left(\alpha,\beta;\gamma;ca^\lambda\right)\sum_{k=0}^\infty\sum_{u=0}^\infty H^{M+1,N+1;m_1,n_1;m_2,n_2}_{P+2,Q+1;p_1,q_1;p_2,q_2}\left[\begin{matrix} a^{-\mu_1-\upsilon_1}c_1z_1 \\ a^{\mu_2+\upsilon_2}c_2z_2\end{matrix}\middle| \begin{matrix} \left(\begin{matrix}2+\sigma+k\xi+u\lambda-\rho; \\ \mu_1+\upsilon_1,\mu_2+\upsilon_2\end{matrix}\right); (a_j;A_j^{(1)},A_j^{(2)})_{2,P+1}; (c_j^{(1)},C_j^{(1)})_{1,p_1}; (c_j^{(2)},C_j^{(2)})_{1,p_2} \\ \left(\begin{matrix}1+\sigma+k\xi+u\lambda; \\ \mu_1,\mu_2\end{matrix}\right); (b_j,B_j^{(1)},B_j^{(2)})_{1,Q+1}; \left(\begin{matrix}1-\rho; \\ \upsilon_1,..,\upsilon_r\end{matrix}\right); (d_j^{(1)},D_j^{(1)})_{1,q_1}; (d_j^{(2)},D_j^{(2)})_{1,q_r}\end{matrix}\right] \tag{15}$$

Eq. (12) reduce to (16) as follows

$$\int_0^\infty y^\sigma (a+y)^{-\rho} e^{\delta y^\xi} {}_2F_1\left[\alpha,\beta;\gamma;cy^\lambda\right] H^{M,N;m_1,n_1;m_2,n_2}_{P,Q;p_1,q_1;p_2,q_2}\left[\begin{matrix} c_1z_1y^{-\mu_1}(a+y)^{\upsilon_1} \\ c_2z_2y^{-\mu_2}(a+y)^{\upsilon_2}\end{matrix}\right]dx$$

$$= a^{\sigma-\rho+1}e^{\delta a^\xi} {}_2F_1\left(\alpha,\beta;\gamma;ca^\lambda\right)\sum_{k=0}^\infty\sum_{u=0}^\infty H^{M+1,N+1;m_1,n_1;m_2,n_2}_{P+2,Q+1;p_1,q_1;p_2,q_2}$$

$$\left[\begin{matrix} a^{\upsilon_1-\mu_1}c_1z_1 \\ a^{\upsilon_2-\mu_2}c_2z_2\end{matrix}\middle| \begin{matrix} (2+\sigma+k\xi+u\lambda-\rho;\mu_1-\upsilon_1,\mu_2-\upsilon_2); (a_j;A_j^{(1)},A_j^{(2)})_{2,P+1}; (\rho;\upsilon_1,...,\upsilon_r); (c_j^{(1)},C_j^{(1)})_{1,p_1}; (c_j^{(2)},C_j^{(2)})_{1,p_2} \\ (1+\sigma+k\xi+u\lambda;\mu_1,\mu_2); (b_j,B_j^{(1)},B_j^{(2)})_{2,Q+1}; (d_j^{(1)},D_j^{(1)})_{1,q_1}; (d_j^{(2)},D_j^{(2)})_{1,q_2}\end{matrix}\right] \tag{16}$$

Eq. (13) reduce to (17) as follows

$$\int_0^\infty y^\sigma (a+y)^{-\rho} e^{\delta y^\xi} {}_2F_1\left[\alpha,\beta;\gamma;cy^\lambda\right] H^{M,N;m_1,n_1;m_2,n_2}_{P,Q;p_1,q_1;p_2,q_2}\left[\begin{matrix} c_1z_1y^{\mu_1}(a+y)^{-\upsilon_1} \\ c_2z_2y^{\mu_2}(a+y)^{-\upsilon_2}\end{matrix}\right]dx$$

$$= a^{\sigma-\rho+1}e^{\delta a^\xi} {}_2F_1\left(\alpha,\beta;\gamma;ca^\lambda\right)\sum_{k=0}^\infty\sum_{u=0}^\infty H^{M,N+2;m_1,n_1;m_2,n_2}_{P+2,Q+1;p_1,q_1;p_2,q_2}$$

$$\left[\begin{matrix} a^{\mu_1-\upsilon_1}c_1z_1 \\ a^{\mu_2-\upsilon_2}c_2z_2\end{matrix}\middle| \begin{matrix} (2+\sigma+k\xi+u\lambda-\rho;\upsilon_1-\mu_1,\upsilon_2-\mu_2); (-\sigma-\lambda u-k\xi;\mu_1,\mu_2); (a_j;A_j^{(1)},A_j^{(2)})_{2,P+1}; (c_j^{(1)},C_j^{(1)})_{1,p_1}; (c_j^{(2)},C_j^{(2)})_{1,p_2} \\ (b_j,B_j^{(1)},B_j^{(2)})_{1,Q}; (1+\sigma+k\xi+u\lambda;\mu_1,\mu_2); (d_j^{(1)},D_j^{(1)})_{1,q_1}; (d_j^{(2)},D_j^{(2)})_{1,q_2}\end{matrix}\right] \tag{17}$$

4. CONCLUSION:

The H-function of several complex variables, is quite basic in nature. Therefore, on specializing the parameters of this function, we may obtain various other special functions such as Meijer's G-function, Fox's H-function, Wright's generalized hypergeometric function, Wright's generalized Bessel function, Whittaker function, generalized hypergeometric function, Mac-Robert's E-function, modified Bessel function, Bessel function of first kind, binomial function, exponential function, etc. as its special cases, and therefore, various unified integrals can be obtained as special cases of our results.

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