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Intuitionistic Fuzzy Goal Geometric Programming Problem (IFG^2P^2) based on Geometric Mean Method

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Abstract

In this paper, a new approach of intuitionistic fuzzy goal programming is proposed. A nonlinear intuitionistic fuzzy goal programming is solved here using geometric programming technique. For this we have used geometric mean method. We have applied the proposed method on industrial waste water treatment design problem. Also there is a comparison of results on industrial waste water treatment design problem with other methods to show the benefit of this method.

Keywords*:* Goal programming, Geometric programming, Non-linear optimization.

1. Introduction

Intuitionistic fuzzy set (IFS) introduced by Atanassov [1], is a growing field of research in different directions. Now a day's goal programming in fuzzy environment is common whereas intuitionistic fuzzy goal programming is rare. In this paper we have worked on intuitionistic fuzzy goal programming problem where equations are non-linear. Geometric programming gives better result than nonlinear programming (K-K-T conditions), which is already described in Ghosh, Roy [2, 3]. Therefore geometric programming is used here to solve nonlinear goal programming problem. Nan et. al. [4], Ghosh, Roy [5] discussed arithmetic mean in intuitionistic fuzzy environment. In this paper we have used geometric mean in intuitionistic fuzzy goal programming. A numerical example and an application on industrial waste water treatment design problem is taken here as an illustration. Previously Shih, Krishnan [6], Evenson [7], Ecker, McNamara [8], Beightler, Philips [9] have illustrated this design using dynamic programming, geometric programming. Later Cao [10] has discussed the same using fuzzy geometric programming. In this paper, we have compared the results of industrial wastewater treatment design problem with the results in another method.

2. Fuzzy Goal Geometric Programming Problem (FG^2P^2)

Find X = $(x_1, x_2 \dots x_q)^T$ (1.1)

so as to

$\widetilde{Minimize}\ f_{j0}$(X), with target value C_{j0}, acceptance tolerance a_{j0}.

subject to f_r(X)≤ C_r, r= 1, 2 … m, X=$(x_1, x_2 \dots x_q)^T$>0.

Membership functions can be written as follows

$$\mu_{f_{j0}}(X) = \begin{cases} 1, & f_{j0}(X) \le C_{j0} \\ 1 - \frac{f_{j0}(X) - C_{j0}}{a_{j0}}, & C_{j0} \le f_{j0}(X) \le C_{j0} + a_{j0} \\ 0, & f_{j0}(X) \ge C_{j0} + a_{j0} \end{cases}$$

Hence the crisp programming from fuzzy goal programming is

Maximize $\mu_{f_{j0}}(X)$, (1.2)

subject to $0 \le \mu_{f_{j0}}(X) \le 1$

f_r(X)≤ C_r, r= 1, 2 … m, X=$(x_1, x_2 \dots x_q)^T$>0.

Model (1.2) can be written as

Maximize α (1.3)

subject to $\mu_{f_{j0}}(X) \ge \alpha$

$f_r(X) \le C_r, r = 1, 2 \dots m$

$0 \le \alpha \le 1$, X=$(x_1, x_2 \dots x_q)^T$>0.

This is equivalent to the following geometric programming problem

Minimize α^{-1} (1.4)

subject to $\frac{f_{j0}(X)}{a_{j0}(1-\alpha) + C_{j0}} \le 1$

$\frac{f_r(X)}{C_r} \le 1, r = 1, 2 \dots$ m,

$0 \le \alpha \le 1$, X=$(x_1, x_2 \dots x_q)^T$>0.

3. Intuitionistic Fuzzy Goal Geometric Programming Problem (I FG^2P^2)

Find X = $(x_1, x_2 \dots\dots x_q)^T$ (2.1)

so as to

$\widetilde{Minimize}^i\ f_{j0}$(X), with target value C_{j0}, acceptance tolerance a_{j0} rejection tolerance b_{j0}.

subject to f_r(X)≤ C_r, r= 1, 2 … m

X=$(x_1, x_2 \dots x_q)^T$>0.

Membership and non-membership functions are

$$\mu_{f_{j0}}(X) = \begin{cases} 1, & f_{j0}(X) \le C_{j0} \\ 1 - \frac{f_{j0}(X) - C_{j0}}{a_{j0}}, & C_{j0} \le f_{j0}(X) \le C_{j0} + a_{j0} \\ 0, & f_{j0}(X) \ge C_{j0} + a_{j0} \end{cases}$$

$$\vartheta_{f_{j0}}(X) = \begin{cases} 0, & f_{j0}(X) \le C_{j0} \\ \frac{f_{j0}(X) - C_{j0}}{b_{j0}}, & C_{j0} \le f_{j0}(X) \le C_{j0} + b_{j0} \\ 1, & f_{j0}(X) \ge C_{j0} + b_{j0} \end{cases}$$

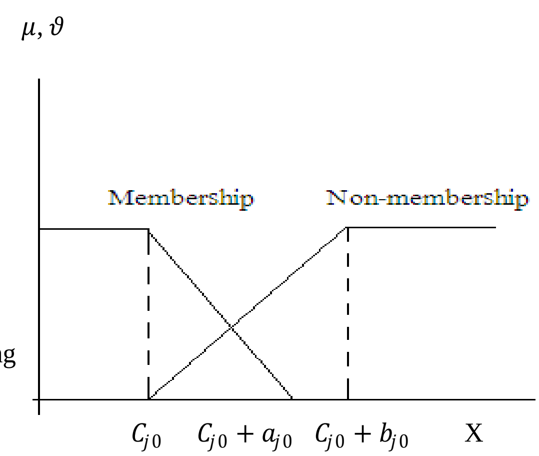

Fig: 1 Membership and non-membership function

Intuitionistic fuzzy goal programming can be transformed into crisp programming model using membership and non-membership function as

Maximize $\mu_{f_{j0}}(X)$, Minimize $\vartheta_{f_{j0}}(X)$ (2.2) subject to $0 \leq \mu_{f_{j0}}(X) + \vartheta_{f_{j0}}(X) \leq 1$

$f_r(\mathrm{X}) \leq C_r$, r= 1, 2 … m

$0 \leq \mu_{f_{j0}}(\mathrm{X}), \vartheta_{f_{j0}}(\mathrm{X}) \leq 1,$

$X=(\mathrm{x}_1, \mathrm{x}_2 \ldots \mathrm{x}_q)^{\mathrm{T}}>0.$

This is equivalent to

Maximize α, Minimize β (2.3)

subject to $\mu_{f_{j0}}(X) \geq \alpha, \vartheta_{f_{j0}}(X) \leq \beta$

$f_r(X) \leq C_r, r = 1, 2 \ldots m$

$0 \leq \alpha + \beta \leq 1, 0 \leq \alpha, \beta \leq 1,$

$X=(\mathrm{x}_1, \mathrm{x}_2 \ldots \mathrm{x}_q)^{\mathrm{T}}>0.$

It is easily seen that Max α is equivalent to Min (1- α) as $0 \leq \alpha \leq 1$. Taking geometric mean, the above model can be written as

Minimize $\beta(1-\alpha)$ (2.4)

subject to $f_{j0}(\mathrm{X}) \leq a_{j0} \times b_{j0}\, \beta(1-\alpha) + C_{j0}$

$f_r(\mathrm{X}) \leq C_r$, r= 1, 2 … m

$0 \leq \alpha + \beta \leq 1, 0 \leq \alpha, \beta \leq 1,$

$X=(\mathrm{x}_1, \mathrm{x}_2 \ldots \mathrm{x}_q)^{\mathrm{T}}>0.$

Let us take $\beta(1-\alpha)$= v>0, then the above model becomes

Minimize v (2.5)

subject to $\frac{f_{j0}(\mathrm{X})}{a_{j0} \times b_{j0} v + C_{j0}} \leq 1$

$\frac{f_r(\mathrm{X})}{C_r} \leq 1$, r= 1, 2 … m

$0 \leq \alpha + \beta \leq 1, 0 \leq \alpha, \beta \leq 1,$

v ∈(0, 1), $X=(\mathrm{x}_1, \mathrm{x}_2 \ldots \mathrm{x}_q)^{\mathrm{T}}>0.$

The above model (2.5) is solved by geometric programming technique with v as parameter.

5. Industrial Wastewater Treatment Design

To optimize the treatment of industrial wastewater, a process flow from a paper and pulp industry has been considered. The treatment units indicate the removal of suspended solid and biological oxygen demand (BOD) from the waste water. In this paper, the design of treatment facilities is based on effluent containing BOD and essentially free from suspended solids. Wastewater treatment is consisted of primary clarification, secondary biological treatment (trickling filter followed by activated sludge or aerated lagoon), sludge disposal and tertiary treatment (coagulation, sedimentation, filtration for effluent of activated sludge and aerated lagoon;

Table-1 Wastewater Treatment Design

Design	Primary	Secondary	Tertiary
1	Primary Clarifier	Trickling Filter & Activated Sludge	Carbon Adsorption
2	Primary Clarifier	Trickling Filter & Aerated Lagoon	Coagulation, Sedimentation, Filtration
3	Primary Clarifier	Activated Sludge	Carbon Adsorption
4	Primary Clarifier	Aerated Lagoon	Coagulation, Sedimentation, Filtration
5	Primary Clarifier	Trickling Filter & Activated Sludge	Coagulation, Sedimentation, Filtration
6	Primary Clarifier	Activated Sludge	Coagulation, Sedimentation, Filtration
7	Primary Clarifier	Activated Sludge	None
8	Primary Clarifier	Trickling Filter & Activated Sludge	None
9	Primary Clarifier	Aerated Lagoon	None
10	Primary Clarifier	Trickling Filter	None

carbon adsorption for effluent of aerated lagoon). There are many combination of wastewater treatment process given in Table-1 (Beightler, Philips (1976)) to remove five day BOD (BOD_5).

In our study, we have taken the first design. There are consecutively four processes (Primary Clarifier, Trickling Filter, Activated Sludge, and Carbon Adsorption).

Primary Clarifier → Trickling Filter ↓

Carbon Adsorption ← Activated Sludge

Let x_i be the percentage of remaining BOD_5 after each step. Then after four processes the remaining percentage of BOD_5 will be $x_1x_2x_3x_4$. Our aim is to minimize the remaining percentage of BOD_5 with minimum annual cost as much as possible. The annual cost of BOD_5 removal by various treatments is shown in Table-2.

Table-2: List of annual costs in different treatments

Design	Treatment	Annual Cost
1	Primary Clarifier	$19.4x_1^{-1.47}$
2	Trickling Filter	$16.8x_2^{-1.66}$
3	Activated Sludge	$91.5x_3^{-0.3}$
4	Carbon Adsorption	$120x_4^{-0.33}$

6. Fuzzy Goal Geometric Programming Problem (FG^2P^2)

Decision maker wants to remove about 98.5% BOD_5 and gives some relaxation 0.1 on this goal. Also he sets another goal as annual cost should be about 300 (thousand \$) and gives flexibility 200 (thousand \$) on this goal.

Then the fuzzy goal programming problem is

$\widetilde{Minimize}\ f_1(x_1,x_2,x_3,x_4) = 19.4x_1^{-1.47} + 16.8x_2^{-1.66} + 91.5\ x_3^{-0.3} + 120\ x_4^{-0.33}$ with target 300, acceptance tolerance 200,

$\widetilde{Minimize}\ f_2(x_1,x_2,x_3,x_4) = x_1x_2x_3x_4$ with target 0.015, acceptance tolerance 0.1,

subject to $x_1,x_2,x_3,x_4 > 0$.

Membership and non-membership functions are given below

$$\mu_{f_1}(x_1,x_2,x_3,x_4) = \begin{cases} 1, & f_1(x_1,x_2,x_3,x_4) \le 300 \\ 1 - \dfrac{f_1(x_1,x_2,x_3,x_4) - 300}{200}, & 300 \le f_1(x_1,x_2,x_3,x_4) \le 500 \\ 0, & f_1(x_1,x_2,x_3,x_4) \ge 500 \end{cases}$$

$$\mu_{f_2}(x_1,x_2,x_3,x_4) = \begin{cases} 1, & f_2(x_1,x_2,x_3,x_4) \le 0.015 \\ 1 - \dfrac{f_2(x_1,x_2,x_3,x_4) - 0.015}{0.1}, & 0.015 \le f_2(x_1,x_2,x_3,x_4) \le 0.115 \\ 0, & f_2(x_1,x_2,x_3,x_4) \ge 0.115 \end{cases}$$

Following (1.2), (1.3) and (1.4), above model can be written into crisp programming problem as

$$\text{Minimize } \alpha^{-1} \qquad (3)$$

$$\text{subject to } 19.4x_1^{-1.47} + 16.8x_2^{-1.66} + 91.5x_3^{-0.3} + 120x_4^{-0.33} \le 200\,(1\text{-}\alpha)+300$$

$$x_1x_2x_3x_4 \le 0.1(1-\alpha) + 0.015,$$

$$x_1,x_2,x_3,x_4 > 0,\ \alpha \in (0,1).$$

Table 3: Optimal values of decision variables and objective functions of model (3)

Dual Variables	Primal Variables	Optimal objective functions	Membership and non-membership functions
δ_{01}^*=1, δ_{11}^*=0.088967278 δ_{12}^*=0.078784276 δ_{13}^*=0.435939662 δ_{14}^*=0.396308784 δ_{21}^*=0.130781899	x_1^*=0.7059559 x_2^*=0.7248393 x_3^*=0.1598653 x_4^*=0.5733523	$f_1^*(x_1,x_2,x_3,x_4)$ = 363.8048 $f_2^*(x_1,x_2,x_3,x_4)$ =0.0469024	$\mu_{f_1}(x_1,x_2,x_3,x_4)$ = 0.680976 $\mu_{f_2}(x_1,x_2,x_3,x_4)$ = 0.680976

Solving it using Cao's geometric programming method taking α as a parameter, where degree of difficulty is 5-(4+1) = 0, we have the results given in Table-3. The table shows that, here 100-0.0469024×100= 95.30976% BOD_5 removes with the cost of 363.8048 (thousand $).

7. Intuitionistic Fuzzy Goal Geometric Programming Problem (I FG^2P^2)

Let decision maker wants to remove about 98.5% BOD_5 and the tolerances of acceptance and rejection on this goal are 0.1 and 0.2 respectively. Also he wants to remove the said amount of BOD_5 within 300 (thousand) $ tolerances of acceptance and rejection on this goal are 200 and 300 respectively. Hence the intuitionistic fuzzy goal programming problem is

$\widetilde{Minimize}^i$ $f_1(x_1,x_2,x_3,x_4)$ = $19.4x_1^{-1.47}$+ $16.8x_2^{-1.66}$+ 91.5 $x_3^{-0.3}$ + 120 $x_4^{-0.33}$ with target 300, acceptance tolerance 200 and rejection tolerance 300

$\widetilde{Minimize}^i$ $f_2(x_1,x_2,x_3,x_4)$ = $x_1x_2x_3x_4$ with target 0.015, acceptance tolerance 0.1 and rejection tolerance 0.2

subject to $x_1,x_2,x_3,x_4 > 0$.

Membership and non-membership functions are given below

$\mu_{f_1}(x_1,x_2,x_3,x_4)$=

$$\begin{cases} 1, & f_1(x_1,x_2,x_3,x_4) \le 300 \\ 1-\dfrac{f_1(x_1,x_2,x_3,x_4)-300}{200}, & 300 \le f_1(x_1,x_2,x_3,x_4) \le 500 \\ 0, & f_1(x_1,x_2,x_3,x_4) \ge 500 \end{cases}$$

$\vartheta_{f_1}(x_1,x_2,x_3,x_4)$=

$$\begin{cases} 0, & f_1(x_1,x_2,x_3,x_4) \le 300 \\ \dfrac{f_1(x_1,x_2,x_3,x_4)-300}{300}, & 300 \le f_1(x_1,x_2,x_3,x_4) \le 600 \\ 1, & f_1(x_1,x_2,x_3,x_4) \ge 600 \end{cases}$$

$\mu_{f_2}(x_1,x_2,x_3,x_4)$=

$$\begin{cases} 1, & f_2(x_1,x_2,x_3,x_4) \le 0.015 \\ 1-\frac{f_2(x_1,x_2,x_3,x_4)-0.015}{0.1}, & 0.015 \le f_2(x_1,x_2,x_3,x_4) \le 0.115 \\ 0, & f_2(x_1,x_2,x_3,x_4) \ge 0.115 \end{cases}$$

$\vartheta_{f_2}(x_1,x_2,x_3,x_4)$=

$$\begin{cases} 0, & f_2(x_1,x_2,x_3,x_4) \le 0.015 \\ \dfrac{f_2(x_1,x_2,x_3,x_4)-0.015}{0.2}, & 0.015 \le f_1(x_1,x_2,x_3,x_4) \le 0.215 \\ 1, & f_1(x_1,x_2,x_3,x_4) \ge 0.215 \end{cases}$$

Following (2.1), (2.2), (2.3) and (2.4), the crisp programming problem is

Minimize v ` (4)

subject to

$$\frac{19.4x_1^{-1.47}+16.8x_2^{-1.66}+91.5x_3^{-0.3}+120x_4^{-0.33}}{200\times 300v+300} \le 1$$

$$\frac{x_1x_2x_3x_4}{0.1\times 0.2v+0.015} \le 1,$$

$$x_1,x_2,x_3,x_4 > 0,\ v \in (0, 1)$$

Solving it using Cao's geometric programming method having degree of difficulty 5-(4+1) = 0, we have the results given in table 4.

Table 4: Optimal values of decision variables and objective functions of model (4)

Dual Variables	Primal Variables	Optimal objective functions	Membership and non-membership functions
δ_{01}^*=1, δ_{11}^*=0.088967278 δ_{12}^*=0.078784276 δ_{13}^*=0.435939662 δ_{14}^*=0.396308784 δ_{21}^*=0.130781899	x_1^*=0.6380199 x_2^*=0.6627170 x_3^*=0.09737155 x_4^*=0.3653206	$f_1^*(x_1,x_2,x_3,x_4)$ =422.1483 $f_2^*(x_1,x_2,x_3,x_4)$ =0.01504072	$\mu_{f_1}(x_1,x_2,x_3,x_4)$ =0.3892583 $\vartheta_{f_1}(x_1,x_2,x_3,x_4)$ =0.4071612 $\mu_{f_2}(x_1,x_2,x_3,x_4)$ =0.9995928 $\vartheta_{f_2}(x_1,x_2,x_3,x_4)$ =0.00020358

The table shows that membership and non-membership functions satisfy all the restrictions as in model (2.2). The percentage of BOD_5 removed from the wastewater is (100 – 0.01504072$\times$ 100) = 98.495928% which attains the set quota by the national standard and the annual total cost is 422.1483 (thousands $).

8. Comparison

Here is a comparison of results between other method and our proposed method.

Table-5: Comparison of results using different methods

Method	Total Annual Cost (Thousand $)	Remaining BOD_5 in waste water	Removed BOD_5
FG^2P^2	363.8048	0.0469024	95.30976%
IFG^2P^2 Arithmetic mean (Ghosh, Roy (2013b))	359.2533	0.04919147	95.080853%
IFG^2P^2 Geometric mean	422.1483	0.01504072	98.495928%

9. Conclusion

We have applied fuzzy and intuitionistic fuzzy goal geometric programming on industrial waste water treatment design. We have compared the results of various methods on industrial waste water treatment design. We have seen that in fuzzy goal geometric programming and intuitionistic fuzzy goal geometric programming with arithmetic mean, percentage of BOD_5 removal is almost same. But 4551.5 $ is saved in intuitionistic fuzzy goal geometric programming with arithmetic mean. In the proposed method, intuitionistic fuzzy goal geometric programming with geometric mean, 98.495928% BOD_5 is removed. Hence for better purification intuitionistic fuzzy goal geometric programming with geometric mean is more appropriate.

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