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Inverse Thermoelastic Problem of Heat Conduction with Internal Heat Generation in Circular Plate

Vijay B Patil
Assistance Professor
in R.A.I.T.,Nerul,Navi Mumbai(M.S.)India.

B. R. Ahirrao
Head and Associate Professor in
Department of Mathematics, Z.B. Patil college, Dhule

Abstract

This paper consist of the inverse thermoelastic problem of heat conduction with internal heat generation for the determination of unknown temperature, displacement and stress function by using finite Hankel transform and Finite fourier cosine integral transform. The results are obtained in the form of infinite series.

Keywords: Inverse thermoelastic problem, Temperature distribution, thin circular plate.

1. Introduction

Nowacki, W. investigated the state of stress in a thick circular plate due to a temperature field. Noda etc determined the thermal stesses in circular plate.Recently N.W.Khobragade and Hamna parveen investigated the thermal stresses of thick circular plate due to heat generation. N.L.Khobragade and K.C.Deshmukh determined thermal deformation in a thin circualr plate due to partially distributed heat supply.Gaikwad and Ghadle determined an inverse quasi-static thermoelastic problem in a circular plate using the Hankel transform technique. The result are obtained in series form in terms of Bessel's function.

2. Statement of the problem

Consider a thin circular plate of thickness h occupying the space

D: $0 \le r \le a, 0 \le z \le h$. The differential equation governing the displacement function $U(r,z)$ as

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+\nu)a_t T \qquad (1)$$

With $U = 0$ at $r = 0$ and $r = a$

ν and a_t are the Poissio's ratio and the linear coefficient of the thermal expansion of the material of the disc respectively and $T(r,z)$ is the temperature of the disc satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2)$$

Where k and α thermal conductivity and thermal diffusivity of the material of the disc subject to the initial conditions

$$T(r,z,0) = T_0 \qquad (3)$$

The boundary conditions and interior condition are

$$\left[\frac{\partial T(r,z,t)}{\partial r}\right]_{r=0} = F_1\ (z,t) \qquad (4)$$

$$\left[\frac{\partial T(r,z,t)}{\partial r}\right]_{r=a} = F_2\ (z,t) \qquad (5)$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=0} = F_3\ (r,t) \qquad (6)$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=\xi} = f(r,\xi,t)\text{(Known)} \qquad (7)$$

$$[T(r,z,t)]_{z=h} = G(r,h,t) \quad \text{(Unknown)} \qquad (8)$$

The stress functions σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = -2\mu\frac{1}{r}\frac{\partial U}{\partial r} \qquad (9)$$

$$\sigma_{\theta\theta} = -2\mu\frac{\partial^2 U}{\partial r^2} \qquad (10)$$

Where μ is the Lame's constant, while each of stress function σ_{rz}, σ_{zz}, $\sigma_{\theta z}$ are zero within the plate in the plane state of stress.

The equations (1) to (10) constitute the mathematical formulation of the problem under consideration.

3. Solution of the problem

If f(x) satisfies Dirchlet's condition in the interval (0,a) then its finite Hankel transform in that range is defined to be

$$\bar{f}_{\square}(\lambda_n) = \int_0^a x f(x) J_{\square}(\lambda_n x)\,dx \qquad (11)$$

Where λ_n is the root of the transcendental equation ,

$$J_{\square}(\lambda_n a) = 0 \qquad (12)$$

Then at any point of (0,a) at which the function f(x) is continuous,

$$f(x) = \frac{2}{a^2}\sum_{n=1}^{\infty} \bar{f}_{\square}(\lambda_n)\frac{J_{\square}(\lambda_n x)}{[J'_{\square}(\lambda_n a)]2} \qquad (13)$$

Where the sum is taken over all the positive roots of the equation (12),

An operational property is given by,

$$H_{\square}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] = \frac{\lambda_n}{2}\left[-H_{\square-1}\left\{\frac{\partial f}{\partial x}\right\} + H_{\square+1}\left\{\frac{\partial f}{\partial x}\right\}\right] \qquad (14)$$

If f(z) satisfies Dirichelet's condition in the interval (0,h) and if for that range its finite Fourier Cosine transform is defined to be

$$\bar{f}_c(m) = \int_0^h f(z)\cos\frac{m\pi z}{h}\,dz \qquad (15)$$

Then at each point (0,h) at which f(z) is continuous, Inverse finite Fourier Cosine transform is given by

$$f(z) = \frac{\bar{f}_c(0)}{h} + \frac{2}{h}\sum_{m=1}^{\infty}\bar{f}_c(m)\cos\frac{m\pi z}{h} \qquad (16)$$

Applying finite Hankel transform again finite cosine transform and then their inverses stated in (11) to (16), to equations (2) to (8) ones obtain

$$T = \frac{4}{a^2\xi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\,\phi dt + T_0 - \int \phi dt]\frac{cosm\pi z}{\xi}\times\frac{J_{?}(r\lambda_n)}{[J'_{?}(a\lambda_n)]^2} \qquad (17)$$

$$G = \frac{4}{a^2\xi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\,\phi dt + T_0 - \int \phi dt]\frac{cosm\pi\, h}{\xi}\times\frac{J_{?}(r\lambda_n)}{[J'_{?}(a\lambda_n)]^2} \qquad (18)$$

Where , $\phi = \alpha[\frac{\lambda_n}{2}[-H_{?-1}\{F_1\} + H_{?+1}\{F_2\}] + (-1)^m\bar{f}^* - F_3 + \frac{\bar{g}^*}{k}]$

and, $\bar{f}^*$ denotes the finite cosine transform of $\bar{f}$ and $\bar{f}$ denotes the finite Hankel transform of $'f'$. $\bar{g}^*$ denotes the finite cosine transform of $\bar{g}$ and $\bar{g}$ denotes the finite Hankel transform of $'g'$.

4. Determination of thermoelastic displacement

Substituting the value of T from (17) in (1) it gets,

$$U(r,z,t) = -(1+\nu)a_t\frac{4}{a^2\xi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\,\phi dt + T_0 - \int \phi dt]\frac{cosm\pi z}{\xi}\times\frac{J_{?}(r\lambda_n)}{[J'_{?}(a\lambda_n)]^2} \qquad (19)$$

5. Determination of stress functions

Using (19) in (9) and (10) the stress functions are obtained as

$$\sigma_{rr} = \frac{4?(1+\nu)a_t}{ra^2\xi^2}[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\,\phi dt + T_0 - \int \phi dt]\frac{cosm\pi z}{\xi}\times\frac{\lambda_n J'_{?}(r\lambda_n)}{[J'_{?}(a\lambda_n)]^2} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\,\phi' dt + T_0 - \int \phi' dt]\frac{cosm\pi z}{\xi}\times\frac{J_{?}(r\lambda_n)}{[J'_{?}(a\lambda_n)]^2}] \qquad (20)$$

$$\sigma_{\theta\theta} = \frac{4?(1+\nu)a_t}{ra^2\xi^2}[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\,\phi dt + T_0 - \int \phi dt]\frac{cosm\pi z}{\xi}\times\frac{(\lambda_n)^2 J''_{?}(r\lambda_n)}{[J'_{?}(a\lambda_n)]^2} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\,\phi' dt + T_0 - \int \phi' dt]\frac{cosm\pi z}{\xi}\times\frac{\lambda_n J'_{?}(r\lambda_n)}{[J'_{?}(a\lambda_n)]^2} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\,\phi' dt + T_0 - \int \phi' dt]\frac{cosm\pi z}{\xi}\times\frac{\lambda_n J'_{?}(r\lambda_n)}{[J'_{?}(a\lambda_n)]^2} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\,\phi'' dt + T_0 - \int \phi'' dt]\frac{cosm\pi z}{\xi}\times\frac{J_{?}(r\lambda_n)}{[J'_{?}(a\lambda_n)]^2}] \qquad (21)$$

Where, $\phi' = \alpha[\frac{\lambda_n}{2}[-H_{?-1}\{F_1\} + H_{?+1}\{F_2\}] + (-1)^m(\bar{f}^*)' - F'_3 + \frac{(\bar{g}^*)'}{k}]$

and, $\phi'' = \alpha[\frac{\lambda_n}{2}[-H_{?-1}\{F_1\} + H_{?+1}\{F_2\}] + (-1)^m(\bar{f}^*)'' - F'_3 + \frac{(\bar{g}^*)''}{k}]$

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7. Special case

$$f(r,z,t) = T_0 e^{z} r^{\square} h(a-r) \quad (22)$$

$$g(r,z,t) = \delta(r-r_0)\delta(z-z_0)\delta(t-t_0) \quad (23)$$

Where δ is Dirac delta function.

Applying finite Hankel transform and then finite cosine transform to (22) and (23) it gets,

$$\bar{f}^* = \frac{a^{\square+1} h^2 J_{\square+1}(a\lambda_n)[(-1)^m e^h - 1]}{\lambda_n (h^2+m^2\pi^2)} \quad (24)$$

$$\bar{g}^* = A \quad (25)$$

Substituting values from (24) and (25) in (17) to (21) it obtains

$$T = \frac{4}{a^2\xi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t} \alpha[\frac{\lambda_n}{2}[-H_{\square-1}\{F_1\} + H_{\square+1}\{F_2\}] + (-1)^m\{\frac{a^{\square+1} h^2 J_{\square+1}(a\lambda_n)[(-1)^m e^h - 1]}{\lambda_n (h^2+m^2\pi^2)}\} - F_3 + \frac{A}{k}]dt + T_0 - \int \alpha\ [\frac{\lambda_n}{2}[-H_{\square-1}\{F_1\} + H_{\square+1}\{F_2\}] + (-1)^m\{\frac{a^{\square+1} h^2 J_{\square+1}(a\lambda_n)[(-1)^m e^h - 1]}{\lambda_n (h^2+m^2\pi^2)}\} - F_3 + \frac{A}{k}]dt\, \frac{cos m\pi z}{\xi} \times \frac{J_{\square}(r\lambda_n)}{[J'_{\square}(a\lambda_n)]^2} \quad (26)$$

$$G = \frac{4}{a^2\xi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t} \alpha[\frac{\lambda_n}{2}[-H_{\square-1}\{F_1\} + H_{\square+1}\{F_2\}] + (-1)^m\{\frac{a^{\square+1} h^2 J_{\square+1}(a\lambda_n)[(-1)^m e^h - 1]}{\lambda_n (h^2+m^2\pi^2)}\} - F_3 + \frac{A}{k}]dt + T_0 - \int \alpha\ [\frac{\lambda_n}{2}[-H_{\square-1}\{F_1\} + H_{\square+1}\{F_2\}] + (-1)^m\{\frac{a^{\square+1} h^2 J_{\square+1}(a\lambda_n)[(-1)^m e^h - 1]}{\lambda_n (h^2+m^2\pi^2)}\} - F_3 + \frac{A}{k}]dt\, \frac{cos m\pi\ h}{\xi} \times \frac{J_{\square}(r\lambda_n)}{[J'_{\square}(a\lambda_n)]^2} \quad (27)$$

Thermoelastic displacement is given by

$$U(r,z,t) = -\alpha(1+\nu)a_t \frac{4}{a^2\xi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t} \alpha[\frac{\lambda_n}{2}[-H_{\square-1}\{F_1\} + H_{\square+1}\{F_2\}] + (-1)^m\{\frac{a^{\square+1} h^2 J_{\square+1}(a\lambda_n)[(-1)^m e^h - 1]}{\lambda_n (h^2+m^2\pi^2)}\} - F_3 + \frac{A}{k}]dt + T_0 - \int \alpha\ [\frac{\lambda_n}{2}[-H_{\square-1}\{F_1\} + H_{\square+1}\{F_2\}] + (-1)^m\{\frac{a^{\square+1} h^2 J_{\square+1}(a\lambda_n)[(-1)^m e^h - 1]}{\lambda_n (h^2+m^2\pi^2)}\} - F_3 + \frac{A}{k}]dt\, \frac{cos m\pi z}{\xi} \times \frac{J_{\square}(r\lambda_n)}{[J'_{\square}(a\lambda_n)]^2} \quad (28)$$

Stress functions are given by

$$\sigma_{rr} = \frac{4\square(1+\nu)a_t}{ra^2\xi^2}[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\phi dt + T_0 - \int \phi dt]\frac{cos m\pi z}{\xi} \times \frac{\lambda_n J'_{\square}(r\lambda_n)}{[J'_{\square}(a\lambda_n)]^2} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\phi' dt + T_0 - \int \phi' dt]\frac{cos m\pi z}{\xi} \times \frac{J_{\square}(r\lambda_n)}{[J'_{\square}(a\lambda_n)]^2}] \quad (29)$$

$$\sigma_{\theta\theta} = \frac{4\square(1+\nu)a_t}{ra^2\xi^2}[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\phi dt + T_0 - \int \phi dt]\frac{cos m\pi z}{\xi} \times \frac{(\lambda_n)^2 J''_{\square}(r\lambda_n)}{[J'_{\square}(a\lambda_n)]^2} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\phi' dt + T_0 - \int \phi' dt]\frac{cos m\pi z}{\xi} \times \frac{\lambda_n J'_{\square}(r\lambda_n)}{[J'_{\square}(a\lambda_n)]^2} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\phi' dt + T_0 - \int \phi' dt]\frac{cos m\pi z}{\xi} \times \frac{\lambda_n J'_{\square}(r\lambda_n)}{[J'_{\square}(a\lambda_n)]^2} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[e^{-\alpha\frac{m^2\pi^2}{\xi^2}t} \int e^{\alpha\frac{m^2\pi^2}{\xi^2}t}\phi'' dt + T_0 - \int \phi'' dt]\frac{cos m\pi z}{\xi} \times \frac{J_{\square}(r\lambda_n)}{[J'_{\square}(a\lambda_n)]^2}] \quad (30)$$

Where, $\phi' = \alpha[\frac{\lambda_n}{2}[-H_{\square-1}\{F_1\} + H_{\square+1}\{F_2\}] + (-1)^m(\bar{f}^*)' - F'_3 + \frac{(\bar{g}^*)'}{k}]$

and, $\phi'' = \alpha[\frac{\lambda_n}{2}[-H_{\square-1}\{F_1\} + H_{\square+1}\{F_2\}] + (-1)^m(\bar{f}^*)'' - F'_3 + \frac{(\bar{g}^*)''}{k}]$

$$\bar{f}^* = \frac{a^{\square+1} h^2 J_{\square+1}(a\lambda_n)[(-1)^m e^h - 1]}{\lambda_n (h^2+m^2\pi^2)}$$

$$\bar{g}^* = A$$

8.Numerical Results:

Take a=3m, h=5m, ξ=1.5m, k=0.014 then,

$$T = \frac{4}{3^2 1.5^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [e^{-\alpha \frac{m^2\pi^2}{1.5^2} t} \int e^{\alpha \frac{m^2\pi^2}{1.5^2} t}$$
$$\alpha[\frac{\lambda_n}{2} [-H_{?-1}\{F_1\} + H_{?+1}\{F_2\}]$$
$$+(-1)^m \{\frac{3^{?+1} 5^2 J_{?+1}(3\lambda_n)[(-1)^m e^h - 1]}{\lambda_n (5^2 + m^2\pi^2)}\} - F_3 +$$
$$\frac{A}{0.014}]dt + T_0 - \int \alpha \ [\frac{\lambda_n}{2} [-H_{?-1}\{F_1\} + H_{?+1}\{F_2\}]$$
$$+(-1)^m \{\frac{3^{?+1} 5^2 J_{?+1}(3\lambda_n)[(-1)^m e^h - 1]}{\lambda_n (5^2 + m^2\pi^2)}\} - F_3 +$$
$$\frac{A}{0.014}]dt \frac{cos m\pi z}{0.014} \times \frac{J_{?}(r\lambda_n)}{[J'_{?}(3\lambda_n)]^2}$$

9.Conclusion:

In this paper temperature, distribution thermoelasic displacement, thermal stresses have been determined for thin circular plate with internal heat generation. Temperature distribution and thermal stresses have been investigated by using Hankel transform and finite fourier cosine transform. The results are obtained in the form of Bessel's function and infinite series.

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