

Investigation of Non-Newtonian Fluids Flow Behavior in a Double Step Expansion Channel: Part 1

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Abstract— a numerical study is performed to predict the behavior of non-newtonian turbulent fluid flow in a double step sudden expansion channel. The non-newtonian fluid flow is investigated for the range of Reynolds number from 4000 to 16000 and power law index from 0.4 to 1.3. The double step sudden expansion channel is investigated for expansion ratio values: 1.5, 2, and 3, aspect ratio values: 1, 2, and 4, step ratio values: 1, 2, 3, 4, and 6. A commercial ANSYS Fluent 15.0 code is adopted to obtain the numerical results while the Gambit 2.4 is for meshing the problem configuration. The obtained results show that we can use the first step as a controller of the second step to change friction coefficient, turbulence intensity and reattachment length.

Keywords— Double Step Channel, Non-Newtonian Fluid Flow, Sudden Expansion, Turbulent.

I. INTRODUCTION

An expanding section of a fluid-carrying channel is called a sudden expansion, in which the kinetic energy of the fluid is converted to static pressure head. This geometry is encountered in multiple equipment [1]. One of the industrial important applications is to use it in combustion chambers of an aircraft and head box of the paper pulp machine. Understanding the flow structure of a sudden expansion needs more research work. Many attempts to modify or improve the flow structure have been performed. Up to date, the double step sudden expansion does not covered by a large number of studies. Therefore, the authors here try to submit a three-dimensional detailed study for double step expansion of non-newtonian turbulent flow.

Many researchers studied the sudden expansion or non-newtonian fluids flow such as: Podolsak et al. [2] determined dynamic shear properties of three distinctive non-Newtonian fluids, aiming to investigate their flow behavior through tube with abrupt contractions and expansions by using modified Quemada model. He found that the Corrections of inertial, and entrance effects from Newtonian results can be approximately applied to non-Newtonian results. Manica et al. [3] tried to simulate incompressible Newtonian and non-Newtonian small

Reynolds number flow through ducts with 1:3 sudden expansion for Reynolds numbers range from 40 to 140 settled by utilizing the finite differences explicit R-K time-stepping scheme. Power law model applied to predict shear-thinning and shear-thickening behavior in expansions and made a comparison with an analytical solution. Abu-Mulaweh [4], reviewed a comprehensive of the flow and heat transfer results of laminar mixed convection flow over vertical, horizontal and inclined backward and contraction steps. Then he gave detailed summary of the effect of several parameters such as step height, Reynolds number and expansion ratio on temperature difference between the buoyancy force, the heated wall and the stream core and on the flow and thermal zone after the step. He reported several correlations to calculate the reattachment lengths of circulation regions. Oliveira [5] studied the stream of non-Newtonian fluids with consistent shear viscosity through 1:3 symmetrical sudden expansion. The considered geometry was planar. The constitutive model, which followed the modified FENE-CR equation, was valid to relative dilute solutions fluids. For the non-Newtonian case, the transition depended on both the concentration and the extensibility parameters of the model, and the trend was for the pitchfork bifurcation to occur at higher Reynolds numbers. The given results comprised size and strength of the recirculation zones, and bifurcation diagrams. Poole et al. [6] [7] [8] made experimental study of back-word facing step for turbulent flow of thixotropic, shear thinning, and shear thickening liquids. They demonstrated the impact on the reattachment length caused by variations in the maximum turbulence intensity at separation for expansion ratio 1.25 to 32 and aspect ratio 13.3. No major differences were found between the turbulent and mean flow characteristics of the Newtonian and non-Newtonian fluid flows. Neofytou [9] investigated numerically the effects on the threshold of transition between symmetry and asymmetry due to the flow through a 1:2 symmetrical sudden expansion by the attributes of generalized Newtonian fluids. The study included both shear thickening and shear-thinning fluids covering a range of the index n of the Power-Law model

from 0.3 to 3.0 with the use of the Casson model. Yılmaz [10] studied numerically turbulent forced heat transfer for double contraction step flow. The bottom wall of the duct was heated by uniform temperature and the temperature of flow of the stream core was colder than the wall. He employed the standard $k-\epsilon$ turbulence model to investigate modeling of turbulence flow to double contraction step. The effect on heat transfer by Reynolds numbers, step heights and step lengths on and fluid flow was investigated. The second step could be used as a control device. Ternik et al. [11] studied numerically laminar incompressible non-Newtonian fluid flow through a symmetrical sudden expansion to calculate Reynolds number critical value. Lowers the onset of the bifurcation at the critical value of the Reynolds number and increased the reattachment length. The results for the Quadratic model was compared to the results obtained with the Power law. Zdanski et al. [12] [13] studied the melting polymer laminar flow in sudden expansion to mapping the viscous heating effect. The main results demonstrated that the flow parameters, such as pressure drop along the ducts and Nusselt number at walls, were affected by viscous heating. Sousa [14] investigated experimentally flow of a Newtonian fluid and a non-Newtonian fluid through square to square sudden contractions to characterize the flow and provided quantitative data in a complex three-dimensional flow for low Reynolds number. Roul in [15] investigated numerically pressure drop in sudden expansions for multi-phase flow of oil-water emulsions. Moreover, he calculated the velocity profiles and pressure drop across sudden expansion, by computing axial pressure profiles in the regions of fully developed duct flow upstream and downstream of the duct expansion, to the transitional cross section. The numerical results are validated with experimental data and are found to be in good agreement.

This field has many effected parameters. Some of them are recording to the fluid type like viscosity, density and power law index. Another depend on the geometry dimension ratios such as expansion ratio, aspect ratio, and step ratio. However, other parameters depend on the inlet flow variable as velocity value and profile and turbulence intensity. All these parameters will effect in friction coefficient and the reattachment length.

II. PROBLEM DESCRIPTION:

In present study, we deal with a rectangular channel with two equal expansion steps (h) separated by a distance S with inlet height D_{in} and D_1 at the first expansion and D_{out} at the second expansion. The deep of the geometry is W and the length from the first expansion to outlet is L as shown in fig. 1. Some of the important studied parameters are shown in table 1.

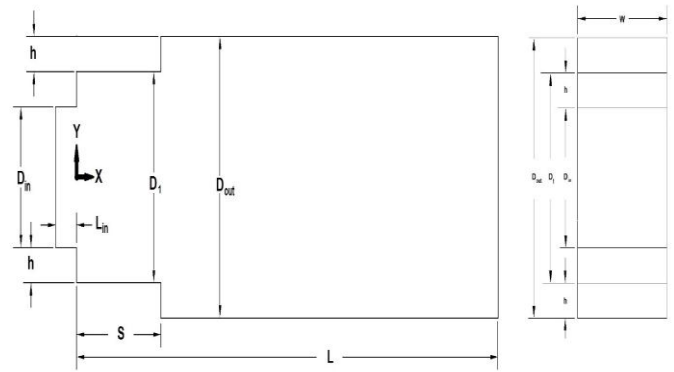


Fig. 1. The geometry and dimension of double steps sudden expansion duct.

TABLE I. THE IMPORTANT STUDIED PARAMETERS

Parameter	Range
Reynolds number (Re)	4000-160000
Power law index (n)	0.4-1.3
Expansion Ratio (ER=D/D)	1.5-3
Step ratio (SR=S/h)	1-6
Aspect ratio (AR=W/D)	1-4

III. MATHEMATICAL MODEL AND NUMERICAL ANALYSIS:

The fluids are assumed incompressible and viscous dissipation is neglected. The governing equations of the flow are depicted as follows:

Continuity equations

$$\frac{\partial \rho u_i}{\partial x_i} = 0 \quad (1)$$

Momentum equations

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial (-\overline{u_i u_j})}{\partial x_j} + \mu \frac{\partial^2 u_i}{\partial x_i^2} - f_i \quad (2)$$

A. The Power-law fluid:

The power law model covers the shear thickening or thinning behavior of materials.

$$\sigma = K \dot{\gamma}^n \quad (3)$$

Where n is the Power-law index or shear index, and K is the consistency factor [16] [17].

The viscosity in power law model is expressed as

$$\mu = \tau_{ij} / \dot{\gamma}_{ij} \quad (4)$$

B. The $k-\epsilon$ model

The turbulent kinetic energy and the dissipation at turbulent kinetic energy are represented by two transport equations as below [18]:

$$\frac{\partial \rho k}{\partial t} + \frac{\partial}{\partial x_j} (\rho k u_j) = -\overline{u_i u_j} \frac{\partial u_i}{\partial x_j} - \rho \epsilon + \frac{\partial}{\partial x_j} \left\{ \left(\frac{v_t}{\sigma_k} + \mu \right) \frac{\partial k}{\partial x_j} \right\} + c_{fk} G_v \quad (5)$$

$$\frac{\partial \rho \epsilon}{\partial t} + \frac{\partial}{\partial x_j} (\rho \epsilon u_j) = -c_{\epsilon 1} \frac{\epsilon}{k} \overline{u_i u_j} \frac{\partial u_i}{\partial x_j} - c_{\epsilon 2} \rho \frac{\epsilon^2}{k}$$

$$+ \frac{\partial}{\partial x_j} \left\{ \left(\frac{v_\varepsilon}{\sigma_\varepsilon} + \mu \right) \frac{\partial \varepsilon}{\partial x_j} \right\} + c_{\varepsilon 2} c_{f \varepsilon} \frac{\varepsilon}{k} G_v \quad (6)$$

C. Boundary Conditions:

Inlet boundary:

- u=uniform value.
- v=w=0.

Walls boundary:

- Non-slip boundary condition are imposed.
- Standard wall functions [16].

Outlet boundary:

- Zero pressure

The governing flow and transport turbulence equations are discretized on non-uniform mesh of different densities. Constricting the mesh was done by a Gambit 2.4 and ANSYS fluent 15.0 adopted for solution these discretized equations.

D. Grid independency:

To ensure the mesh do not effect on studied parameters, a different number of mesh were tested for each geometry case.

Table 2 illustrates the mesh density and percentage error. It can be seen the mesh density is sensitive for Re value and sudden expansion. The mesh 845,127 for Re=4000 & ER=1.5, 1,245,701 for Re=16000 & ER=1.5, 2,145,214 for Re=16000 & ER=2 and 3,124,514 for Re=16000 & ER=3 are adopted in this work.

TABLE II. THE CHANGE OF FRICTION COEFFICIENT ERROR WITH CHANGING MESH

Re	ER	mesh no.	C _f	Error %
4000	1.5	52,415	0.009195	7.3209796
4000	1.5	154,215	0.008901	4.2671491
4000	1.5	845,127	0.008406	1.3725433
16000	1.5	142,575	0.004149	10.911620
16000	1.5	845,764	0.003925	5.8229065
16000	1.5	1,245,701	0.003633	1.7252344
16000	2	842,751	0.001975	12.782638
16000	2	1,427,542	0.001845	6.5936966
16000	2	2,145,214	0.001698	1.4827756
16000	3	1,452,785	0.001755	28.040484
16000	3	2,451,724	0.001146	10.213255
16000	3	3,124,514	0.001234	2.3472527

IV. RESULTS AND DISCUSSION:

The obtained results are represented for Re range from 4000 to 16000, n from 0.4 to 1.3, SR from 1 to 6, ER from 1.5 to 3 and AR from 1 to 4.

A. Verification

The considered model in this work is validated with previous works [6] as shown in fig. 2. It can be noticed that a good agreement between the results are obtained.

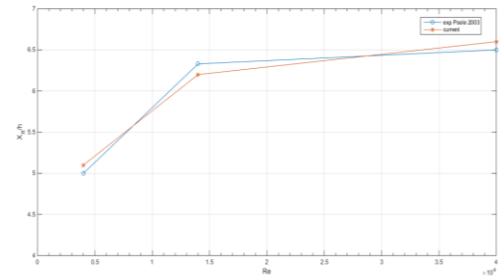


Fig. 2. the reattachment length VS. Re. RMSE=0.0125.

B. Velocity profile and local values

A channel with double step expansion have two circulation regions separately or merge in a single circulation region that depend on the X_R of the first step and the distance between the two steps.

If the X_R of the first step exceed the distance between steps that will mix the two circulation in complex one. For separated circulation regions, fig. 3 shows the velocity profile after expansion at the first step and second step. It can be seen the negative velocity that refer to reverse flow (circulation) between dimensionless axial distance (x_r) values from 0 to 4 and no circulation after that from 4 to 6. In addition, the same for the second step and reverse flow from 6 to 10 and no circulation at large values after it.

Fig. 4 illustrates the variation of X_R along the z-direction. It clears the X_R increasing with Re increasing But the X_R increase near the right and left wall more than at the core of flow because the effect of the wall boundary layer.

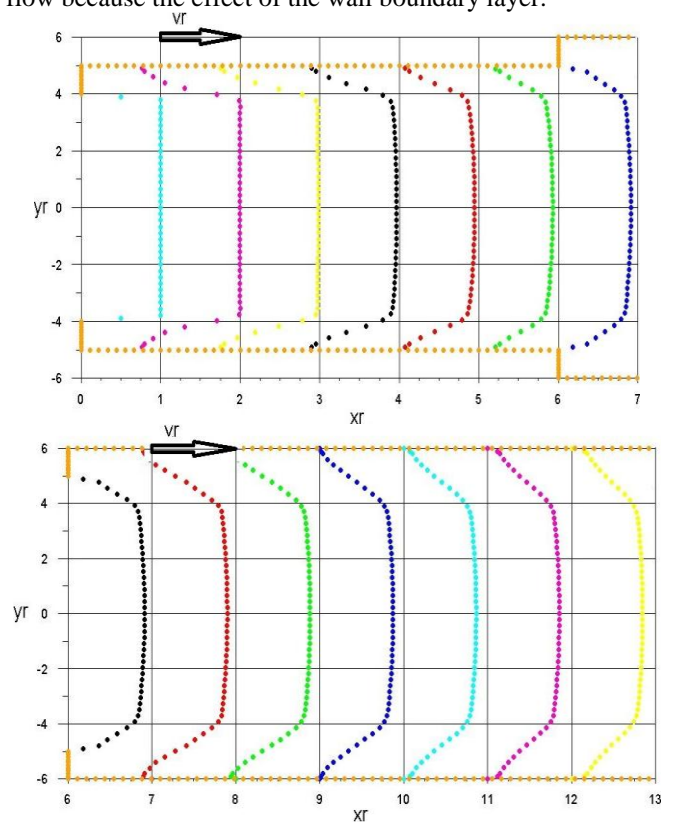


Fig. 3. Velocity profile along the double step channel

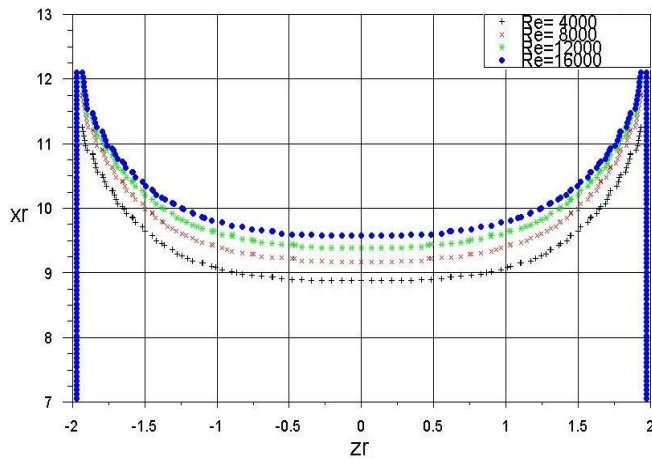


Fig. 4. X_R of the second step vs. zr for $n=1$.

Fig. 5 describes the shear stress along the lower channel wall. It shows the negative peaks of shear beyond to the circulation regions. In addition, it clears that the peaks on the first step is bigger than the second step and the increasing of ER gives higher shear values. Also, the region between the two circulation regions has high shear stress because of that the boundary layers at it is very thin and has high velocity gradient. At ER equal to 3 the shear stress has a peak after the second circulation region because of the bifurcation. At all cases, the shear establishes at constant values due to fully development.

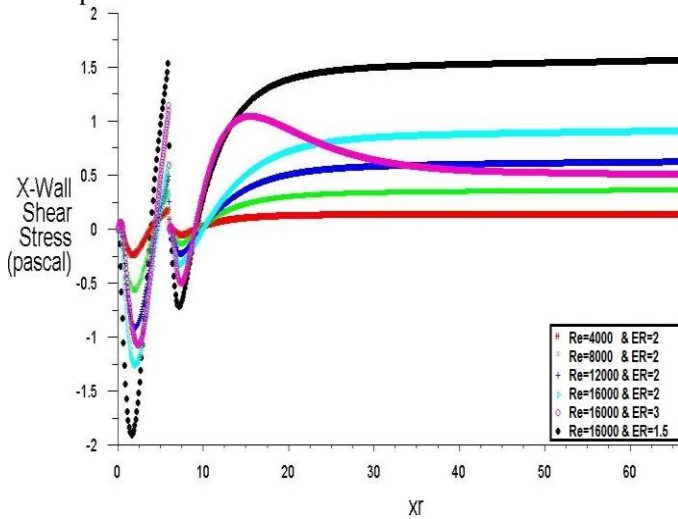


Fig. 5. x -component of shear stress vs. xr at centerline of the lower wall for $n=1$.

C. Contour and stream lines

The index n effects on the X_R , and consequently affects the size of circulation region. The most decreasing influence appears at index value equal to 0.7 as in fig. 6 (b). However, it increases with Re at constant index n as in fig. 7.

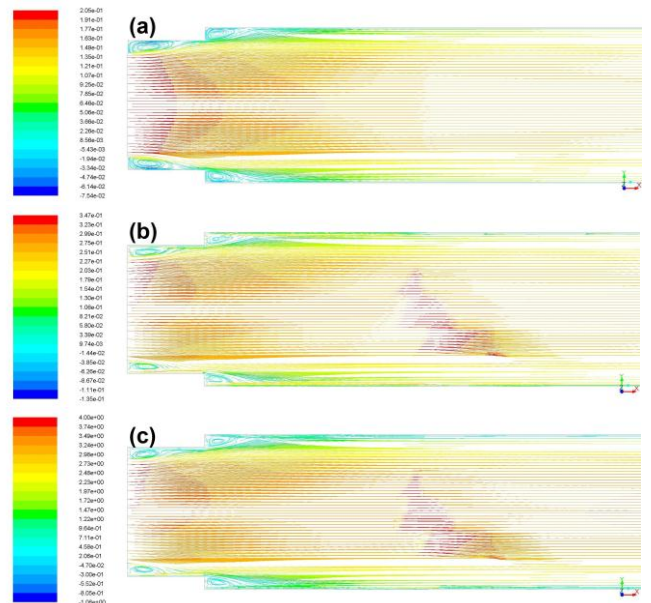


Fig. 6. Streamlines of $Re=16000$ and $ER=1.5$, (a) $n=0.4$ (b) $n=0.7$ (c) $n=1.3$.

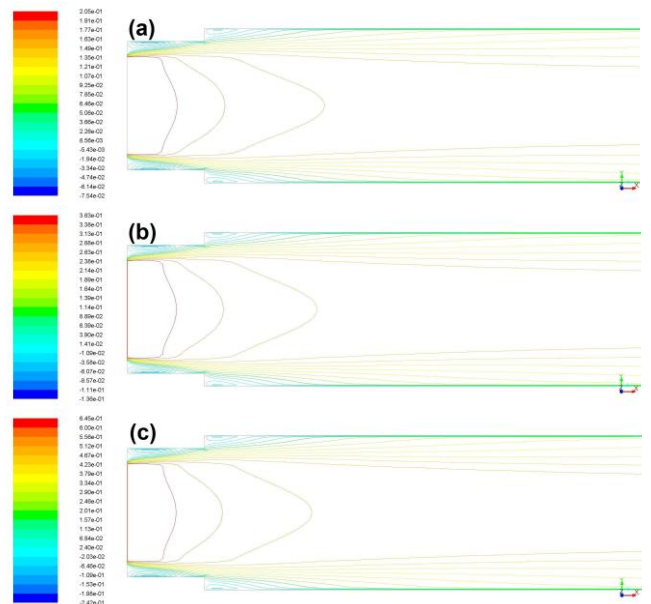


Fig. 7. Velocity contour of $n=1$ (a) $Re=4000$ (a) $Re=8000$ (a) $Re=12000$.

D. Parameters effect on friction coefficient (C_f)

The C_f depends on the wall shear stress and the kinetic energy. Therefore, any change in them will change the C_f value.

Fig. 8 shows that the increasing in AR decreases the C_f index n 0.4, 0.7, 1 but increases for index n equal to 1.3. This increasing or decreasing will be negligible for large AR. At constant Re , the kinetic energy is constant but the AR increase the lower wall area so change the average shear stress on that wall. That changing will be increasing for $n=1.3$ because of the large viscosity and decreasing for other because of the small viscosity but disappear near sidewall because of sidewall effect.

In other hand, the ER increasing at constant AR will decrease the kinetic energy so the C_f will decrease as in fig. 9.

The index n increasing rapidly the C_f at the ranges from 0.4 to 0.7 and from 1.0 to 1.3 and slightly increasing or decreasing between 0.7 and 1.0 depend on Re as in fig. 10. Because of the equilibrium between shear forces and kinetic forces.

Fig. 11 elucidates the behavior of C_f with Re . It can be seen that increasing of Re will decrease the C_f but not as well as Blasius equation because these Re lay in low turbulent region.

The SR controls the mixing between the two circulation regions. So if it is large than the X_R of the first step, the flow be less circulation and less shear therefore the C_f decreases with the SR increasing but each index n needs a different SR to mix. The perfect SR of maximum mixing for each index n gives the worst C_f as in fig. 12.

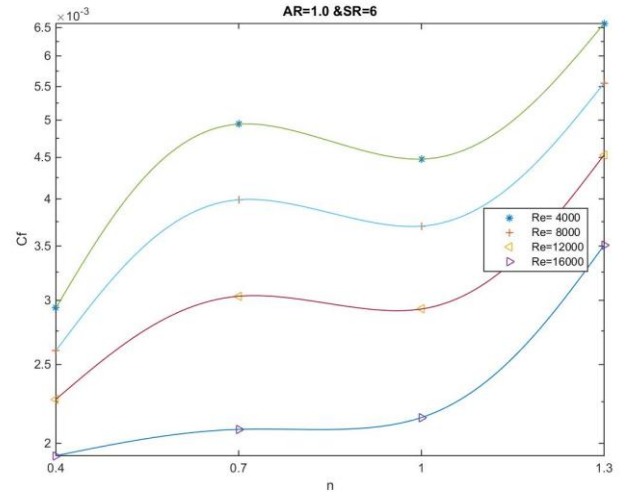


Fig. 10. C_f vs. index n of $SR=6$ and $AR=1$.

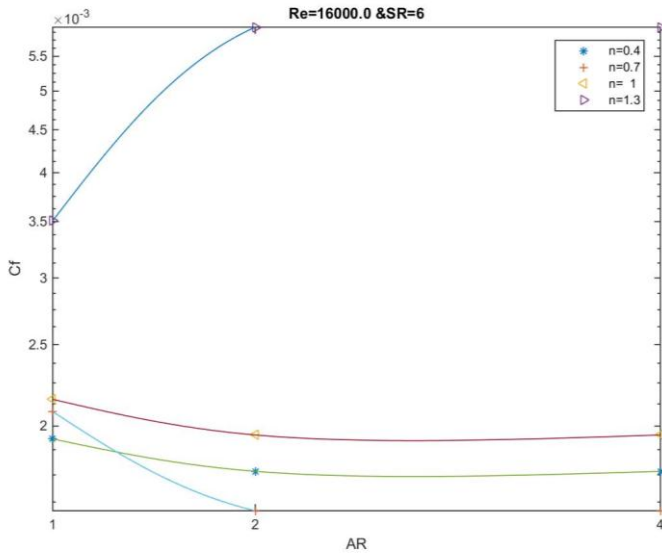


Fig. 8. C_f vs. AR of $Re=16000$ and $SR=6$.

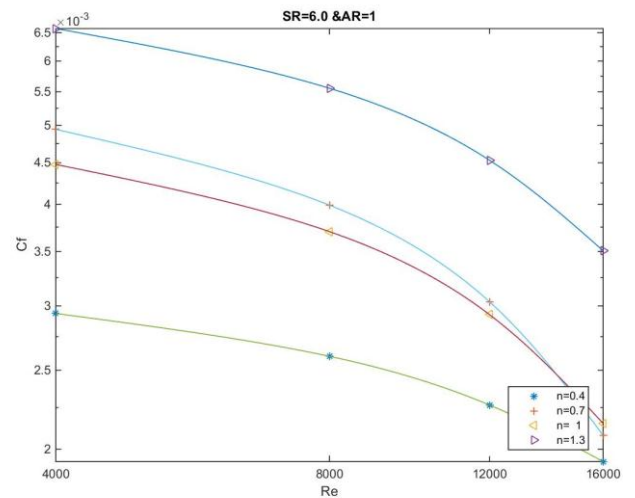


Fig. 11. C_f vs. Re of $SR=6$ $AR=1$.

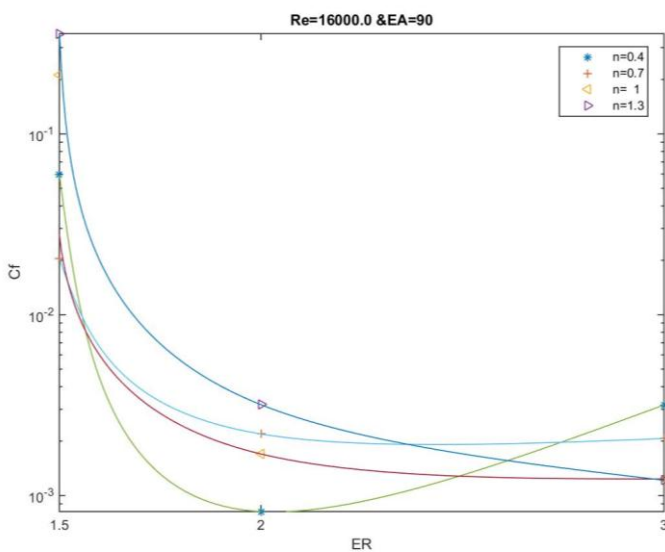


Fig. 9. C_f vs. ER of $Re=16000$.

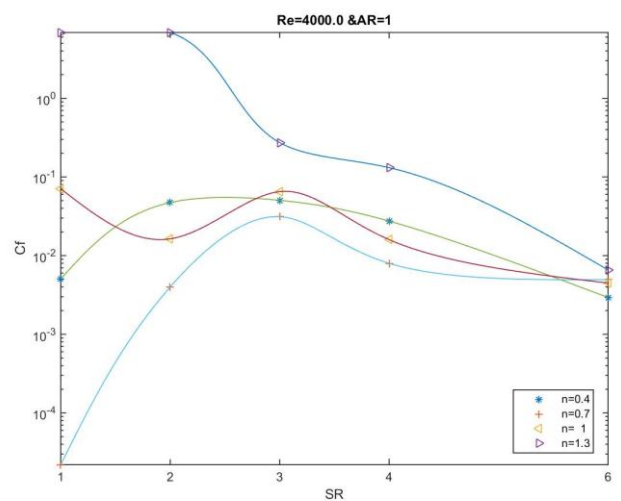


Fig. 12. C_f vs. SR of $Re=4000$ and $AR=1$.

E. Parameters effect on X_R

The increasing of AR will increase the hydraulic diameter so the Re that evaluated using hydraulic diameter will increase too. But the increasing of AR decrease the near sidewall effect. Therefore the increasing of AR will decrease the X_R for all index n as in fig. 13.

Fig. 14 illustrates the behavior of different index n. It clears that each one has different behavior. For index n equals to 0.4 or 1.0, the X_R increases with ER increasing until it reaches to 2 but after that begin to decrease. For 0.7, the X_R will increase with ER and establish at specific value. However, at 1.3 it always increases. This different behaviors above ER equal to 2 beyond to bifurcation occurring.

The index n increasing always increases the X_R at larger than 0.7 but from 0.4 to 0.7 will decrease as in fig. 15 for each Re greater than 8000.

Fig. 16 clearly shows the increasing of X_R with Re for all index n. But the SR decreases the X_R in the mixing region ($SR < 4$) and increases it at larger that. Therefore the minimum X_R happens at SR equal to 4 as in fig. 17.

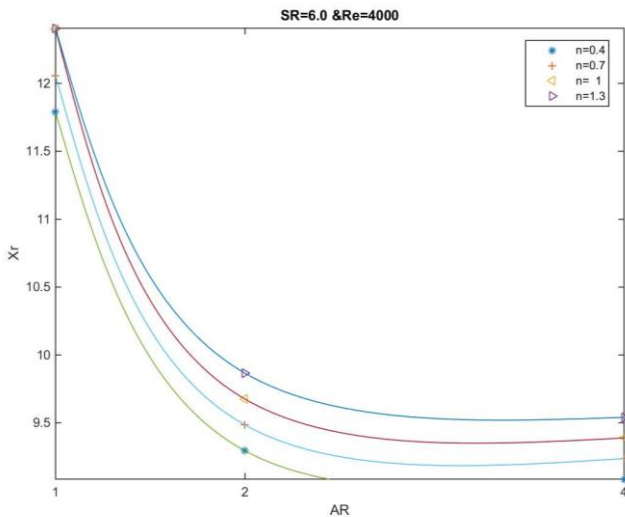


Fig. 13. X_R vs. AR of SR=6 and Re=4000.

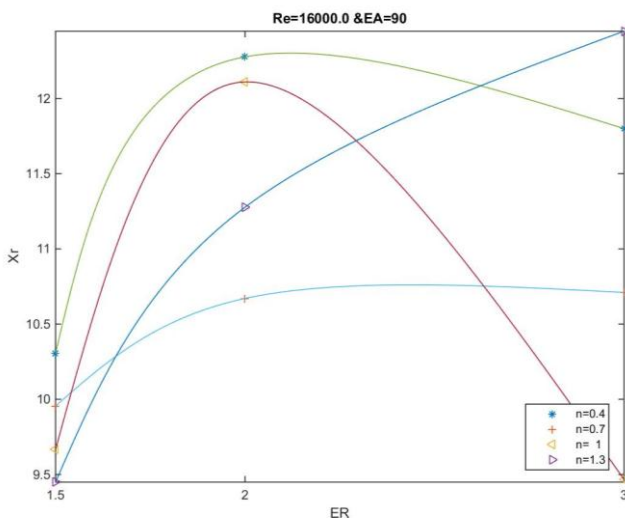


Fig. 14. X_R vs. ER of SR=6 and Re=16000.

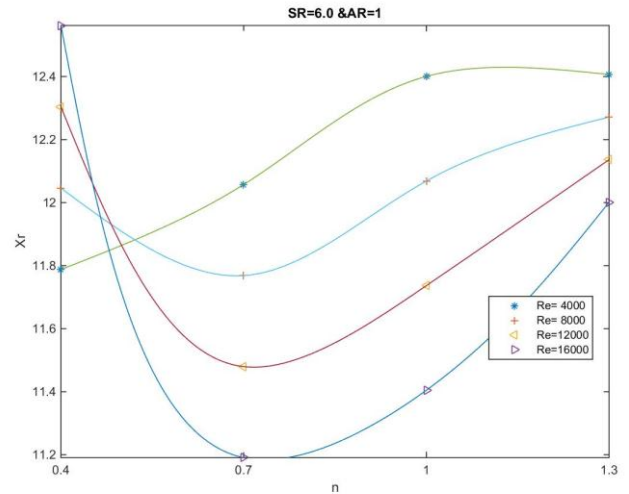


Fig. 15. X_R vs. index n of SR=6 and AR=1.

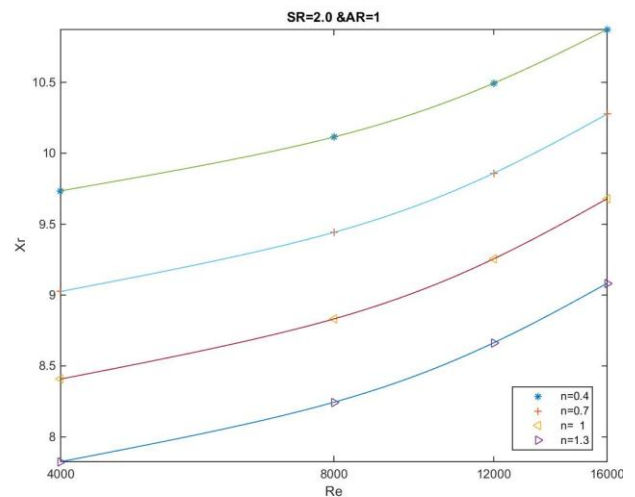


Fig. 16. X_R vs. Re of SR=2 and AR=1.

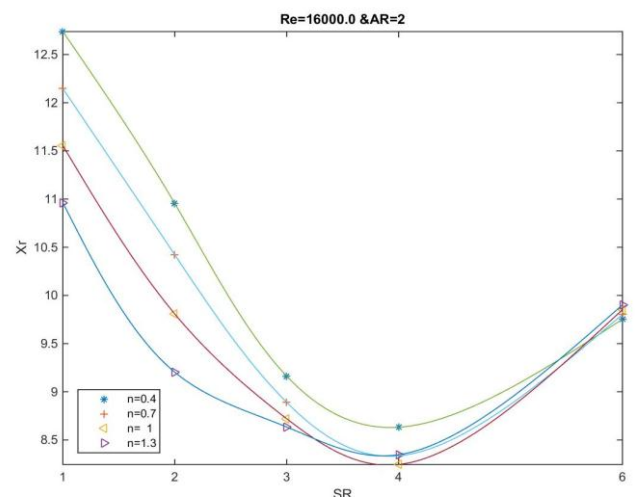


Fig. 17. X_R vs. SR of Re=16000 and AR=2.

F. The steps interaction

Many parameters effect on circulation region but in double step there is the effect of first step on the second step. When the two steps are so far greater than $10 \cdot ER \cdot D_{in}$ there is no effect from the first step to the second step. However, when they be near less than $10 \cdot ER \cdot D_{in}$ the indirect effect will be appear. Because of the velocity profile shape and velocity gradient near wall. So, the region near core flow is effected more than the region near sidewall. Therefore, the reattachment near center is less than near sidewall as in fig. 18 and 19.

However, when the two steps be more close the direct effect will appear and the first circulation region begin to mix with the second region. This mixing will reduce the first circulation region size and increase the second. If the mixing begin at ER less than 2 the symmetric is continuous as in fig. 20 and 21.

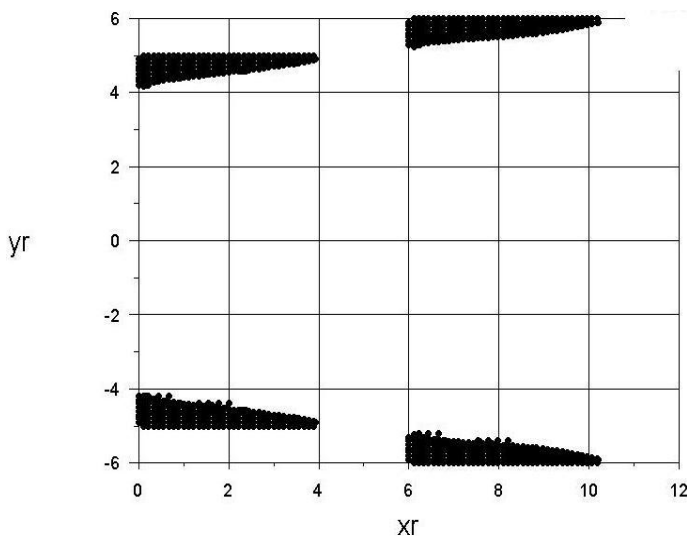


Fig. 18. Reverse flow region of Re=16000 in x-y plain.

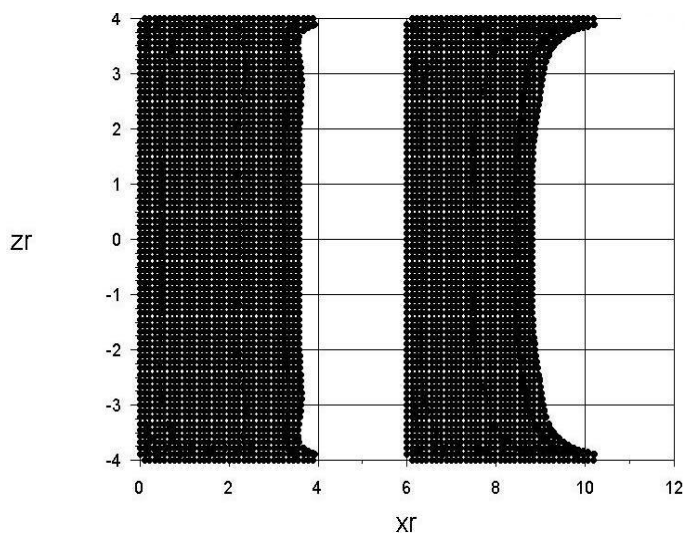


Fig. 19. Reverse flow region of Re=16000 in x-z plain.

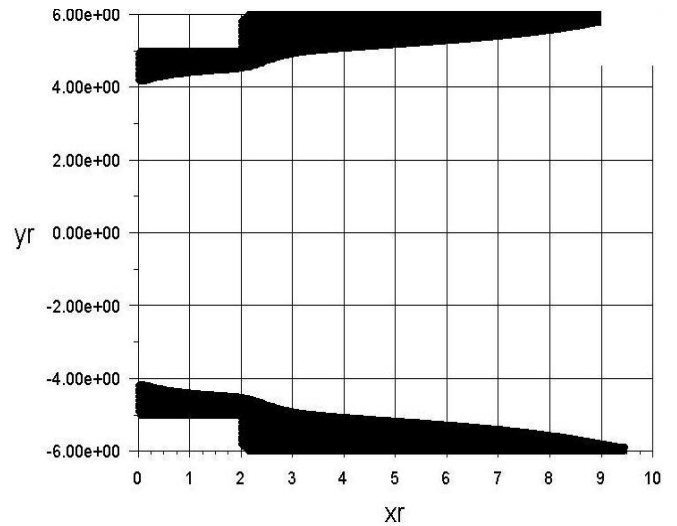


Fig. 20. Reverse flow region of Re=4000 and SR=2 in x-y plain.

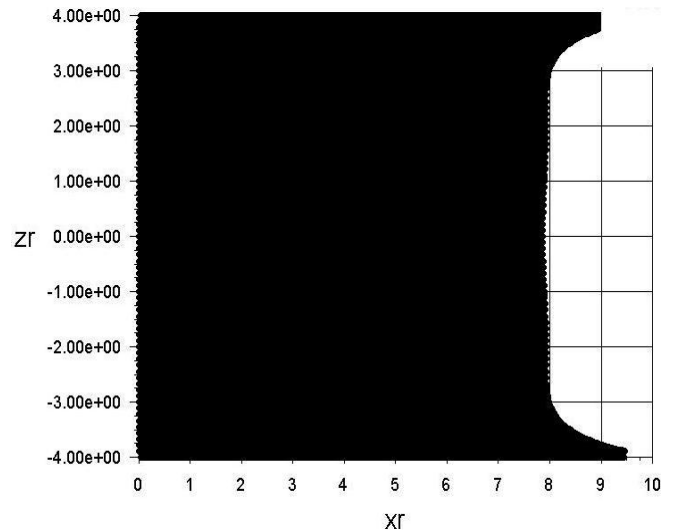


Fig. 21. Reverse flow region of Re=4000 and SR=2 in x-z plain.

G. The bifurcation

At ER larger than 2 the mixing happen and the size of the second circulation region will be very huge so cannot stable therefor the bifurcation begin as in fig. 22, 23 and 24. This phenomenon beyond to the pressure gradient different between the upper and lower circulation regions. The usually local pressure at beginning of reverse flow region is less than the core flow local pressure. The local pressures are equal at the distance of center of circulation region. Then the reverse flow local pressure become to be greater than the local pressure of the core flow. The two side circulation regions have a different pressure but the different become large with bifurcation. Therefor at the beginning of circulation region, the core flow deflects to the less pressure side until reaches the circulation center distance and begins to generate more pressure at the small circulation region and feeding the biggest. Therefore, the big side be biggest and the small side be smallest until pressure equilibrium as clear in fig. 25.

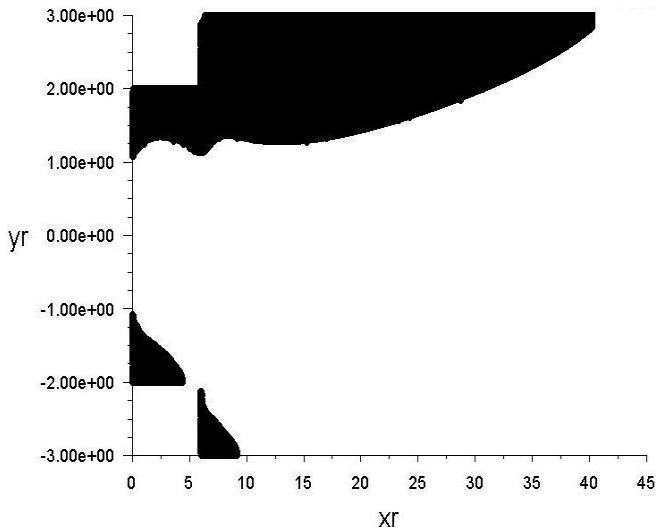


Fig. 22. Reverse flow region of $Re=16000$, $n=0.7$ and $SR=6$ in $x-z$ plain.

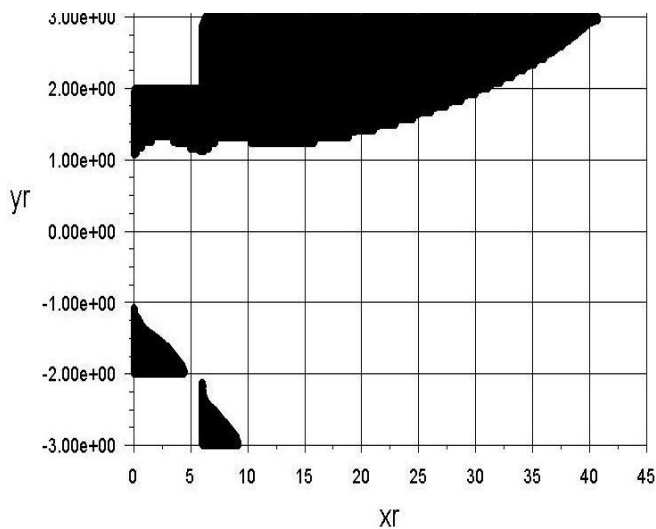


Fig. 23. Reverse flow region of $Re=16000$, $n=1.0$ and $SR=6$ in $x-z$ plain.

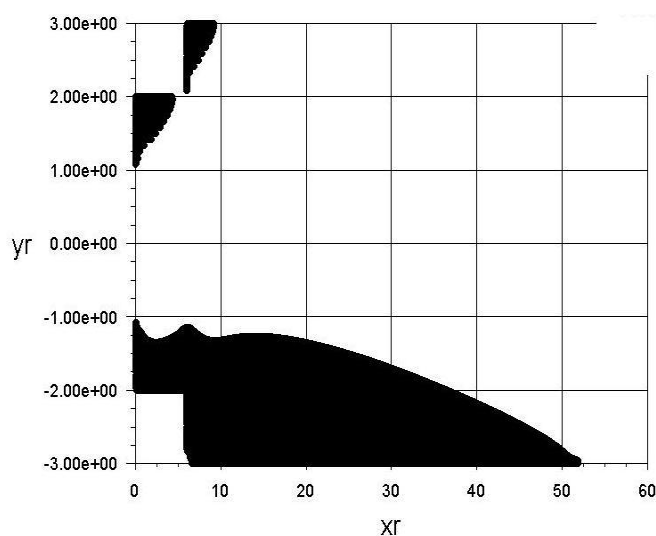


Fig. 24. Reverse flow region of $Re=16000$, $n=1.3$ and $SR=6$ in $x-z$ plain.

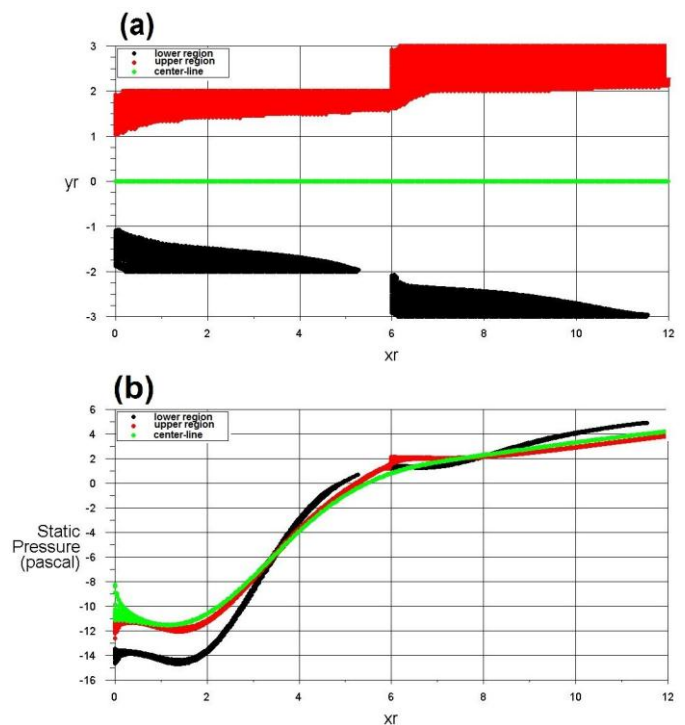


Fig. 25. Reverse flow regions and local pressure for $n=1$ a $Re=6000$.

V. CONCLUSION:

In this paper the following conclusions can be documented

- The SR is a good controller to change flow structure and reduce the bifurcation phenomenon.
- The bifurcation usually occurs at ER less than 2 in the classical sudden expansion step but with double step occurs larger than 2.

REFERENCES

- [1] D. K. Mandal, S. Bandyopadhyay and S. Chakrabarti, "A numerical study on the flow through a plane symmetric sudden expansion with a fence viewed as a diffuser," *MultiCraft International Journal of Engineering, Science and Technology*, vol. 3, no. 8, pp. 210-233, 2011.
- [2] A. K. Podolsak, C. Tiu and T. N. Fang, "Flow of non-Newtonian fluids through tubes with abrupt expansions and contractions (square wave tubes)," *J. Non-Newtonian Fluid Mech.*, no. 71, pp. 25-39, 1997.
- [3] R. Manica and A. L. de Bortoli, "Simulation of Incompressible Non-Newtonian Flows Through Channels with Sudden Expansion Using the Power-Law Model," *TEMA Tend. Mat. Apl. Comput.*, vol. 4, no. 3, pp. 333-340, 2003.
- [4] H. I. Abu-Mulaweh, "A review of research on laminar mixed convection flow over backward- and forward-facing steps," *International Journal of Thermal Sciences*, no. 42, p. 897-909, 2003.
- [5] P. J. Oliveira, "Asymmetric flows of viscoelastic fluids in symmetric planar expansion geometries," *J. Non-Newtonian Fluid Mech.*, no. 114, p. 33-63, 2003.
- [6] R. J. Poole and M. P. Escudier, "Turbulent flow of non-Newtonian liquids over a backward-facing step," *J. Non-Newtonian Fluid Mech.*, no. 109, p. 177-191, 2003.
- [7] R. J. Poole and M. P. Escudier, "Turbulent flow of viscoelastic liquids through an axisymmetric sudden expansion," *J. Non-Newtonian Fluid Mech.*, no. 117, p. 25-46, 2004.
- [8] R. J. Poole, F. T. Pinho, M. A. Alvesc and P. J. Oliveira, "The effect of expansion ratio for creeping expansion flows of UCM fluids," *J. Non-Newtonian Fluid Mech.*, no. doi:10.1016/j.jnnfm.2009.06.004, 2009.
- [9] P. Neofytou, "Transition to asymmetry of generalised Newtonian fluid flows through a symmetric sudden expansion," *J. Non-Newtonian Fluid Mech.*, no. 133, p. 132-140, 2006.
- [10] I. Yilmaz and H. F. Oztop, "Turbulence forced convection heat transfer over double forward facing step flow," *International Communications in Heat and Mass Transfer*, no. 33, p. 508-517, 2006.

- [11] P. Ternik, M. Jure and Z. Zuni, "Non-Newtonian fluid flow through a planar symmetric expansion: Shear-thickening fluids," *J. Non-Newtonian Fluid Mech.*, no. 135, p. 136–148, 2006.
- [12] P. S. B. Zdanski, M. V. Jr. and A. P. Dias, "POLYMER MELT FLOW IN SUDDEN EXPANSIONS: THE EFFECTS OF VISCOUS HEATING," *Engenharia Térmica (Thermal Engineering)*, vol. 7, no. 1, pp. 65-70, 2008.
- [13] P. S. B. Zdanski and M. V. Jr., "Three-dimensional polymer melt flow in sudden expansions: Non-isothermal flow topology," *International Journal of Heat and Mass Transfer*, no. 52, p. 3585–3594, 2009.
- [14] P. C. Sousa, P. M. Coelho, M. S. N. Oliveira and M. A. Alves, "Three-dimensional flow of Newtonian and Boger fluids in square-square contractions," *J. Non-Newtonian Fluid Mech.*, no. 160, p. 122–139, 2009.
- [15] M. K. Roul and L. K. Sahoo, "CFD modeling of pressure drop caused by two-phase flow of oil/water emulsions through sudden expansions," *international Journal of Engineering Research and Applications (IJERA)*, vol. 2, no. 6, pp. 1047-1054, 2012.
- [16] R. P. Chhabra and J. F. Richardson, *non-Newtonian flow and applied rheology*, Oxford: Butterworth-Heinemann, 2008.
- [17] E.-A. Brujan, *Cavitation in Non-Newtonian Fluids*, Berlin: Springer-Verlag, 2011.
- [18] M.-L. Zhang, C. W. Li and Y.-M. Shen, "A 3D non-linear k–e turbulent model for prediction of flow and mass transport in channel with vegetation," *Applied Mathematical Modelling*, no. 34, p. 1021–1031, 2010.