Joint Optimization of Interpolated FIR Filter

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Abstract

Finite Impulse Response (FIR) filters are often used in phase-sensitive applications because they can always be designed to have linear phase. They are also inherently stable because all of the poles lie at the origin. Unlike infinite impulse response (IIR) filters, FIR filters cannot be designed using the well established electrical circuit filter design methods because the concept of a finite impulse response does not exist in electronics. This necessitates accurate FIR design techniques. Common implementations are created by truncating Fourier series coefficients to a certain length or using a window function to minimize the abrupt ends caused by truncation.In this paper joint optimization of interpolated FIR filter has been discussed and it is found that joint optimization is a better technique that can reduce filter order and computational complexity to a large extent.

Keywords:*FIRFilter,IFIR,Interpolation,optimization*

I. INTRODUCTION

FIR filters are having linear phase and are stable as all poles lie at the origin so they are used in phase sensitive applications.Implementation of FIR filters is different from that of IIR filters as they cannot be designed using electric circuit filter design methods.FIR filters are implemented by truncating Fourier series coefficients to a certain length using window functions which minimize abrupt ends.Certain window function will be having more favourable characteristics compared to the other.FIR filters can also be designed using computer aided design of equi-ripple linear phase filters.It is very easy to create FIR filters using these methods in MATLAB.However FIR filters are having higher order than IIR filter with equivalent magnitude spectrum.Interpolated Finite impulse response method can be used to reduce extra computational complexity that accompanies higherfilter order.

a. Interpolation method

Interpolation is required when it is necessary to change from one sampling rate to another.In speech processing system speech parameters are computed at low sampling rate for low bit rate storage or transmission whereas for constructing synthetic speech signal from low bit rate representation speech parameters are required at higher sampling rates[**1**]which needs increased sampling rate by using digital interpolation process.Sampling rate reduction is required in converting delta modulation representation of a waveform to pulse code modulation representation.[**2**] Efficient digital realization of a frequency multiplexed single sideband system has been obtained[**3**] by performing complicated filtering functions at low sampling rate and simpler functions at high sampling rate for grouping several channels into frequency multiplexed format**.**The process of digital signal interpolation is fundamental to signal processing. It is used in many contexts, most typically for conversion between sampling rates. This paper explores efficient designs of digital interpolation systems for integer up-samplefactors.Interpolation of a signal by an integer up-sample factor can be accomplished by processing the signal, x[n], with the cascade of an expander and low-pass filter, as shown in Figure 1-1. If the input signal x[n] has sampling frequency f, this results in the up-sampled and interpolated output signal y[n] at the increased sampling frequency L_f.

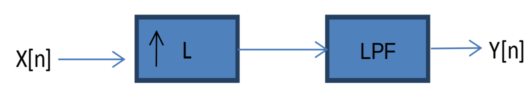


Fig 1 Interpolation System

This paper studies the tradeoffs in the design of such interpolation systems for integerupsample factors. The metric used for comparison between systemdesigns is computational cost, measured in multiplies per output sample

b. FILTER ORDER ESTIMATION

Various methods can be used to determine minimum length a filter needs to meet required specifications but they donot always give correct filter order.The smallest integer value that lies above the estimation can be checked for accuracy. The parameters given include normalized pass-band edge angular frequencyω_p,normalized stop band edge angular frequencyω_s,peak pass band rippleδ_p,and peak stopband ripple δ_s.Various window designs uses parameters such as sidelobelevel to control ripples in pass-band and stop-band.An example of adjustable window is Dolph-Chebyshevwindow.This window method results in smallest transition bands and equi-ripple behavior due to specified side-lobelevel[**4**].

Linear phase FIR filters can be designed using window method[**5**].In this an infinite length ideal filter is truncated by finite length window function.

Kaiser's Formula [**6**], is given by

$$N=\frac{-20\log_{10}(\sqrt{\delta_p \delta s})-13}{14.6(\omega_s-\omega_p)/2\pi} \qquad (1.1)$$

The Kaiser window [**6**] is defined as

$$\omega_n=\frac{I_0\left(\beta\sqrt{1-(n/M)^2}\right)}{I_0}, M\le n\le M \qquad (1.2)$$

Where β is the adjustable parameter that controls δ and I_0 is the modified zeroth order Bessel function.To determine β first the minimum attenuation in the stop-band is found with

$\alpha=-20\log_{10}\delta_s$

and then following estimation function is used [**7**].

$$\beta=\begin{cases}0.1102(\alpha_s-8.7), & \alpha_s>50,\\ 0.584(\alpha_s-21)^{0.4}+0.078(\alpha_s-21), & 21\le\alpha_s\le 50,\\ 0, & \alpha_s<21.\end{cases} \qquad (1.3)$$

c. EQUIRIPPLE FILTER

Parks McClellan method is a computer aided iterative approach to reach specification within a certain error e(w) to exhibit equi-ripplebehavior.This method is popular due to its flexibilities[**8**].It is based on a result called alternation theorem which gives set of conditions so that filter design is optimal.According to which polynomial of order M cannot have more than M zeros.There are four types of linear phase real coefficient FIR filters. Type 1 filters have the form

$$H(z)=\sum_{n=0}^{N-1}h(n)z^{-n}=h(0)+h(1)z^{-1}+........+h(N-1)z^{-(N-1)} \qquad (2.1)$$

where N is even and

$$h(n)=h(N-n) \qquad (2.2)$$

so that

$$H(e^{j\omega})=e^{-j\omega M}\sum_{n=0}^{M}b_n\cos(\omega n) \qquad (2.3)$$

where M=N/2. The factor $e^{-j\omega M}$ representing the linear phase part will be ignored in all discussions. Since Type 2, 3, and 4 filters can be expressed in terms of Type 1 filters, the theory and design of linear phase filters is centered around the design of the coefficients {b_n} in the sum $\sum_{n=0}^{M}b_n\cos(\omega n)$

This summation is used to approximate a real desired response D(w) in 0 < w < 7 with a specified weighting function W(w) > 0 on the error. The approximation error is

$$E(\omega)=D(\omega)-\sum_{n=0}^{M}b_n\cos(\omega n) \qquad (2.4)$$

Whereas the weighted error of approximation is defined by

$$E^{\wedge}{}_b(\omega)=W(\omega)[D(\omega)-\sum_{n=0}^{M}b_n\cos(\omega n)] \qquad (2.5)$$

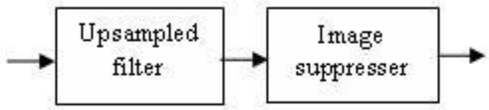
II. METHODOLOGY

In case of FIR filters filter order is inversely proportional to transition bandwidth.In joint optimization design problem is broken into two stages,Anupsampling filter to reduce number of multipliers and image suppressor filter which removes the images created by the upsampling filter.

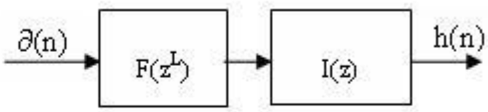
In case of IFIR filter we design a filter for multiple L of the transition.Then impulse response is upsampled by a factor equal to multiple of the transition width,L.Upsampling introduces zeros resulting in larger delay meeting original specifications without introducing extra multipliers.

$F(z^L)$ is an upsampled version of the shaping filter known as up-sampledfilter.Due to up-sampling spectral replicas appear within the nyquistinterval.These replicas are removed by image suppressor or interpolator as it reconstructs the sparse impulse response given by $F(z^L)$.It suppresses the undesired pass-band image due to up-sampling to obtain the desired overall response.Though extra filter introduces additional multipliers there is overall computational saving.The implementation is shown in Figure1 and expressed in equation (4).

$$H(z)=F(z^L)G(z) \qquad (3.1)$$



2(a)



2(b)

Figure 2: The IFIR implementation. (a) An upsampled filter is cascaded with an image suppressor filter to attain an overall design with a reduced computational cost.(b)Implementation of IFIR filter

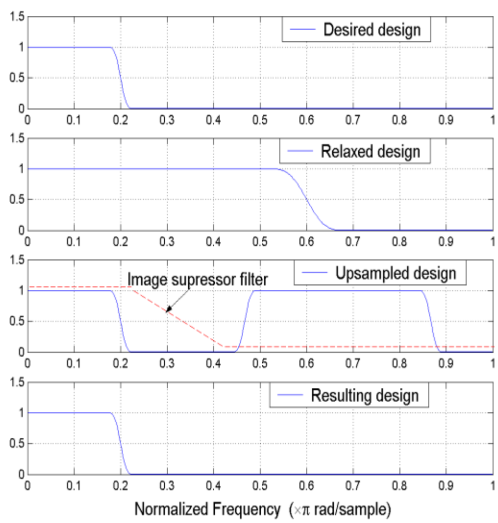


Figure 3: Illustration of the IFIR design paradigm. Two filters are used to attain stringent transition width specifications with reduced total multiplier count when compared to a single filter design.

The "relaxed" design is approximately one third the length of the desired design, if the latter were to be designed directly. The up-sampled design has the same transition width as the desired design.However spectral replicas are introduced due to up-sampling.Image suppressor filter removes these replicas thereby reducing the number of multipliers needed to meet the given specifications.In order to design an IFIR filter up-sampling factor must be found from[1].The largest value of L is given by

$$L_{\max} = \left\lceil \frac{\pi}{\omega_s} \right\rceil \qquad (3.1)$$

$$L_{opt}=[2\pi/w_p+w_s+\sqrt{2\pi}(w_s-w_p)] \quad (3.2)$$

Parameters giving the design cost of IFIR filter before and after joint optimization as a result of simulation done in MATLAB are

Design cost of Interpolated FIR (IFIR)
Number of Multipliers : 208
Number of Adders : 206
Number of States : 802
Multiplications per Input Sample : 208
Additions per Input Sample : 206

Design cost of IFIR filter using joint optimization
Number of Multipliers : 152
Number of Adders : 150
Number of States : 730
Multiplications per Input Sample : 152
Additions per Input Sample : 150

An optimal stretching factor results in a much simpler design of an imaging filter.With increase in L shaping filter decreases in complexity and imaging filter increases in complexity[**9**].When $L_{max}$is chosen,the imaging filter will beat a maximum and the shaping filter at a minimum. Equation3.2 is used to resolve these disparities.In case of IFIR filters pass-band ripples of the two filters are combined in a disorderly fashion.They can add up in such a way that sum of the sum of the two peak pass-band ripples exceeds the original set of specifications.By jointly optimizing the design of two filters a much cleaner pass-band can be attained.joint optimization results in a filter that can meet the same specifications resulting in further reduction in number of multipliers.Joint optimization results in savings in terms of multipliers in case of image suppressor filter.

Result & Conclusion

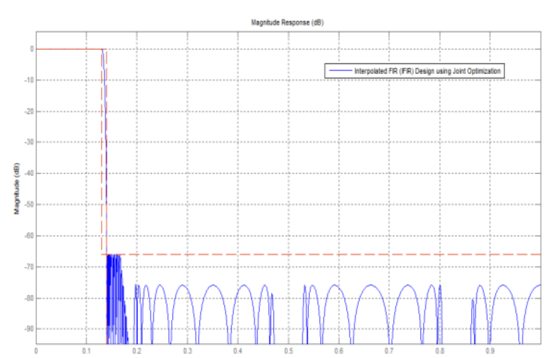


Fig 4 Magnitude response using Joint Optimization Technique

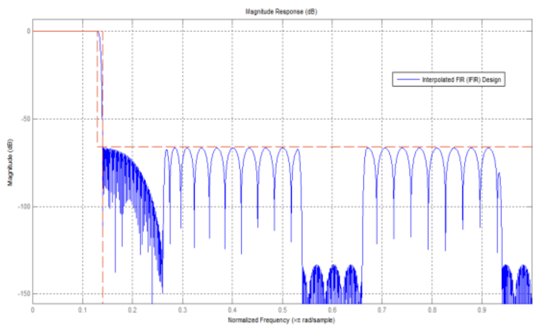


Fig. 5 Magnitude response using Interpolation Technique

Comparison between Interpolated FIR and Joint Optimization

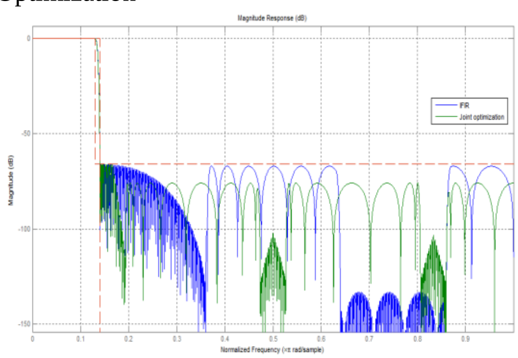


Fig 6 Comparison of Magnitude response of Interpolated FIR and Joint Optimization.

Joint optimization of IFIR filter is a better technique in terms of number of multipliers used as compared to other techniques such as IFIR,ParksMcClellan filter designing method etc.Results show that in case of joint optimization number of multipliers used is 152 whereas in case of IFIR design it is 208 thereby reducing the computational complexity to a large extend for the same filter order by appropriately choosing the up-sampling factor for joint optimization.

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