Kalman Filter Based Channel Estimation

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***Abstract*— The paper addresses the problem of channel tracking for time varying channels. The accurate knowledge of the channel aids equalization and symbol detection. The Kalman filter is proposed to address the problem of channel tracking in fading environment. It is due to the fact that Kalman filter is optimal estimator. The random data bits are BPSK-modulated and transmitted over the channel. The channels under consideration are AWGN, Ricean and Rayleigh. The Kalman filter operates over the data bits to estimate the channel. At the receiver end demodulation and detection is performed. The above model is implemented in MATLAB. The simulation results prove that Kalman filter is an excellent tracker and predictor. The mean square error converges quickly. The results are compared on the basis of SNR and MMSE for the three mentioned channels. The Kalman filter finds its application in channel tracking in OFDM systems.**

Keywords*— *Kalman Filter; Channel Estimation; Semi-Blind; MIM; MSE.

I. INTRODUCTION

The work explores the problem of channel tracking in Multiple-Input-Multiple-Output (MIMO) systems through the use of Kalman filter. The Kalman filter is an optimal estimator i.e. it minimizes the mean square error. The filter works in decision feedback or training mode and utilizes the data received to update the channel matrix. The updated matrix can be used to predict the channel or perform optimal symbol detection or to construct an equalizer. The channel assumed here is frequency-selective. Multiple antennas are used to transmit the modulated data bits. The bits are distorted during transmission. The channel introduces amplitude and phase shifts due to its frequency-selective and time-varying nature. This also leads to Intersymbol Interference (ISI). It is resolved through channel equalization. In order for the receiver to acquire the original bits, it needs to take into account these unknown changes.

Here, Kalman filter based channel predictor is proposed.The goal of the filter is to minimize the mean square error of the equalized signals before they reach the receiver. Distinctive feature of Kalman Filter is that its mathematical formulation is described in terms of state-space concepts and also its solution is computed recursively applying without modification to stationary as well as non-stationary environments.

The work involves application of Kalman filter to track the channel matrix in AWGN, Rayleigh and Ricean environment. The results are compared on the basis of Signal to Noise Ratio (SNR) andMean Square Error (MSE).

A. OUTLINE

The section 1 introduces the wireless channel and the challenges in the wireless environment including the Rayleigh and Ricean environment. A brief introduction to the MIMO channels and channel estimation is made. The section 2 focusses on the basics of Kalman filter. The section 3 defines the system model and reports the simulation results. The conclusion and future scope are described later.

B. LITERATURE REVIEW

An introduction to Kalman filter is provided in [7] and [8]. The basics of fading and equalization can be studied from [12]. [1] proposed a scalar Kalman filter based approach for improved signal to noise ratio estimation in the presence of AWGN and fading environment. The mean square error (MSE) and bias are used as performance measures. Simulation result show that at low SNR, Kalman filter can improve the performance of the SNR estimators . Channel equalizer based on adaptive Kalman filter has been presented in [2]. The performance indices are MSE and SNR. The results are compared for different communication systems like QAM (4-QAM to up to 64-QAM) in the AWGN environment. The problem of channel tracking and equalization for MIMO time varying frequency-selective channels is addressed in [3]. A low-order autoregressive model approximates the MIMO channel variation and facilitates tracking via a Kalman filter. In [4] the pilot symbols and Extended Kalman Filter (EKF) algorithm are applied and compared with results of LS and LMS algorithms. The EKF outperforms the traditional methods. The EKF is used to track and predict a sine wave in Rayleigh fading channel in [5]. The LMS and RMS errors have been compared for AWGN, Ricean and Rayleigh fading environment in [6]. The error is highest for Rayleigh channel and least for AWGN channel. [9] exploresKalman filter based tracking, the channel prediction, and the delay optimized adaptive DFE design. In [10] a MatLab based multipath fading simulator is developed. The [11] evaluates various methods such as Pilot arrangement, Blind estimation, Kalman filter based estimation, Bayesian Cramer-Rao bounds on the OFDM model using algorithms such as LS, LMS, MMSE and ML. The Kalman filter gives optimal estimate of the channel. It however, is highly complex. To reduce the complexity it can be implemented on per-carrier basis.

II. THE WIRELESS CHANNEL

The wireless channel is a difficult medium, susceptible to noise, interference, blockage and multipath. These effects change over time due to mobility of the user.

A. TIME-VARYING NATURE OF CHANNEL

The multipath effect is a phenomenon that causes multiple versions of the transmitted signal to arrive at the receiver at different time delays. Reflecting objects and scatterers in the transmission environment generate multiple versions of the transmitted signal. Each of the paths will have different characteristics, such as amplitude, phase, arrival time, and angle of arrival. The multiple signals may constructively or destructively add up at the receiver, thus creating the rapid fluctuations in the received signal envelope. The time variations appear to be unpredictable to the user of the channel. Therefore it is reasonable to characterize the time-variant multipath channel statistically.

B. DOPPLER SHIFT

Due to the relative motion between the transmitter and the receiver, each multipath wave is subjected to a shift in frequency. The frequency shift of the received signal caused by the relative motion is called the Doppler shift. It is proportional to the speed of the mobile unit.
It is given by:

$$fd = vfc.cos\theta/c \quad (1)$$

Where, fc =transmitted frequency
V=velocity of vehicle
Θ= incident angle
C=speed of light

The Doppler shift in a multipath propagation environment spreads the bandwidth of the multipath waves within the range of fc ± fdmaxwhere fdmaxis the maximum Doppler shift, given by$f(\mathrm{d}\,max) = v * \frac{fc}{c}$

(2)

C. STATISTICAL MODEL

If the transmitted signal bandwidth is greater than the channel coherence bandwidth, the spectral components of the transmitted signal with a frequency separation larger than the coherence bandwidth are faded independently. This phenomenon is known as *frequency selective fading*. In wideband systems, the transmitted signals usually undergo frequency selective fading.

D. RAYLEIGH FADING DISTRIBUTION

The Rayleigh fading describes the statistical time varying nature of received envelope of a flat fading signal, or the envelope of an individual multipath component. The Rayleigh distribution has a probability density function (pdf) given by

$$p(r) = \begin{cases} r/\sigma^2 \exp\left(-\frac{r^2}{2\sigma^2}\right) (0 \le r \le \infty) \\ (0 \le r \le \infty) \end{cases} \quad (3)$$

Where σ is the RMS value of the received signal before envelope detection.

E. RICEAN FADING DISTRIBUTION

When a dominant stationary signal component is present among the multipath components, the fading envelope is Ricean. The random multipath components are superimposed on a dominant signal such as Line-of-sight path. At the output, the effect is of adding a dc component to random multipath. The Ricean distribution degenerates to Rayleigh distribution when the dominant component fades away. $p(r) = \begin{cases} r/\sigma^2 \exp\left(-\frac{r^2+A^2}{2\sigma^2}\right) I\left(\left(\frac{AR}{\sigma^2}\right)\right) (0 \le r \le \infty) \\ 0\ (r < 0) \end{cases}$(4)

The parameter A denotes the peak amplitude of the dominant signal. The I(.) is the modified Bessel's function of first kind and zero-order. The parameter K is defined as the ratio between deterministic signal power and the variance of multipath. It is given by$K = \frac{A^2}{2\sigma^2}$

(5)

K is also known as Ricean factor. As K increases to ∞ the Ricean distribution degenerates to Rayleigh distribution.

F. MIMO CHANNEL

The multiple-input-multiple-output (MIMO) channels exploit the random nature of radio propagation by finding independent signal paths for communication. It is based on the observation that if one radio path undergoes deep fade, other path may have strong signal. By having more than one paths, both instantaneous and average SNRs at the receiver may be improved. The main advantages of MIMO channels are: Array Gain, Diversity Gain and Spatial Multiplexing Gain.

G. CHANNEL ESTIMATION

In any communication system, the primary goal is to recover the signal transmitted at the transmitter side, at the receiver. In doing this there are a number of equalization /detection methods depending on the diversity or the spatial multiplexing system applied to the MIMO decoding techniques. Regardless of the type of the MIMO system, almost all of the equalization or detection methods require knowledge of the channel information in order to recover the transmitted signal. Therefore, the development of an efficient method of approximating the transmission channel between the transmitter and the receiver is an essential requirement of the receiver design. The methods of channel estimation are:

H. Training Sequence-based Estimation

The estimation is based on transmission of training/pilot symbols. These symbols are already known to the receiver and thus aid the training of equalizer.

I. Blind Methods

The training sequence based estimation is a simple technique. However, it requires additional bandwidth. Since in wireless systems, bandwidth is a scarce resource, a need for blind estimation methods arises. These systems utilize the data transmitted to update the equalizer coefficients. Thus, avoiding additional overhead.Blind algorithms estimate the channel based on properties of the transmitted signals for example, by averaging properties over a number of samples or by applying certain transformation at the transmitter end and corresponding operations at receiver end.

J. Semi Blind Methods

Semi-blind algorithmscan improve the performance of blind algorithms by exploiting the knowledge of bothknown symbols and properties of the transmitted signals. The objective of semi-blind channelestimation algorithms is to get better performance than blind algorithms while requiringfewer known symbols than training based channel estimation algorithms.

A Kalman filter based channel estimation technique is presented in the report. This method uses the underlying channel model and the available data based estimate, to implement the channel estimationalgorithm in the form of a Kalman filter. The Kalman filter based channel estimator leads to a significant gain in performance as compared to the data-only estimator. The Kalman filter also allows us to predict the state of the beforethe frame is actually received.

K. KALMAN FILTER

The Kalman filter is recursive predictive filter that is based on the use of state space techniques and recursive algorithms. It estimates the state of dynamic system. This dynamic state is disturbed by noise assumed to be white noise. The measurement y can be expressed in terms of the state variable x and observation noise v as the following.

$$\boldsymbol{y} = C\boldsymbol{x} + \boldsymbol{v} \qquad (6)$$

L. Technical description

Thus Kalman filter consists of two steps:

- Prediction: It refers to projecting forward the current state to obtain the estimates for next step. It is therefore known as time update step.

- Correction: It is the feedback stage that incorporates new measurements into the estimates. It is therefore known as measurement update step.

This process is repeated for each state with value from previous state as initial value. Therefore, Kalman filter is called a recursive filter.

M. Basic Components of a Kalman Filter

1. State Vector

The state vector contains variables of interest. It describes the state of dynamic system and represents its degrees of freedom. The variables of state vector cannot be measured directly and are inferred from other measurable values. The variables can be frequency, amplitude and phase.
The state vector has two values at same time:
Apriori value –predicted value before update represented by $\mathbf{x}^-$
Aposteriori value-corrected value after update represented by $\mathbf{x}^+$

2. Dynamic Model

Dynamic model describes the transformation of state over time. It is represented by a system of differential equations.In linear case it is written as

$$\mathbf{x}'(t)= \frac{dx}{dt} = A.\boldsymbol{x}(t) + B\boldsymbol{u}(t) + \boldsymbol{w}(t) \qquad (7)$$

where n(t) is white Gaussian noise with covariance matrix Q(t).

3. Observation Model

Observation model represents the relationship between states and measurements. In the linear case measurements can be described by state of linear equations, which depend on state variables. Usually observations are made at discrete steps.

$$\boldsymbol{y}(ti) = C.\boldsymbol{x}(ti) + \boldsymbol{v}(ti) \qquad (8)$$

Where $y(t_i)$ is vector of observation, C is observation matrix and w is measurement noise with covariance R(t). In the linear system C is also a constant matrix.

A. State Space Model Of Kalman Filter

A state space model consists of two equations:

$$\mathbf{X}_{t+1} = A\mathbf{X}_t + B\mathbf{U}_t + \mathbf{W}(t) \qquad (9)$$

$$\mathbf{Y}_t = C\mathbf{X}_t + D\mathbf{U}_t + \mathbf{V}(t) \qquad (10)$$

where$\mathbf{X}_t$ is a state vector of dimension n, **Y**t is the observed time series, A,B,C,D are matrices describing state and input/output conditions.

Kalman filter theory is based on a state-space approach in which a state equation models the dynamics of the signal generation process and an observation equation models the noisy and distorted observation signal. For a signal x(*t*) and noisy observation y(t), equations describing the state process model and the observation model are defined as

$$\boldsymbol{X}(t+1)=A\mathbf{X}(t)+\boldsymbol{W}(t) \qquad (11)$$

$$\boldsymbol{Y}(t)=C\mathbf{X}(t)+\boldsymbol{V}(t) \qquad (12)$$

X(t) is the *N*-dimensional signal vector, or the state parameter, at time t,
W(t) (*process noise*) is the N-dimensional uncorrelated input excitation vector of the state equation. ***W***(t) is assumed to be a normal (Gaussian) process $p(\boldsymbol{W}(k))\sim N(0, Q)$, *Q* being the N × N covariance matrix of ***W***(t) or *process noise covariance*. ***Y***(t) is the *M* dimensional noisy observation vector,
V(*k*) is the *M*-dimensional noise vector, also known as measurement noise, **V**(*t*) is assumed to have a normal distribution $p(\mathbf{V}(k))\sim N(0, R)$) and *R* is the*M* ×M covariance matrix of *n*(*k*) (*measurement noise covariance*).

We define ***X***(t|t-1) to be our *a priori* estimate (prediction) at step t-1 from the previous trajectory of *x*, and ***X***(*t*|*t*) to be our *a posteriori* state estimate at step *t* given measurement *y(t)*.***X***(*t*|t-1) is a prediction of the value of ***X****(t)* which is based on the previous values and not on the current observation at time t.***X***(*t*|*t*) on the other hand, uses the information in the current observation (the notation |tis used to emphasize that this value is an estimation of ***X***(t) based on the evidence or observation at time t). The *a priori* and *a posteriori* estimation errors are defined as:

$$\mathbf{e}^-(t)=\boldsymbol{X}(t)-\boldsymbol{X}(t|t-1) \qquad (13)$$

$$\mathbf{e}(t)=\boldsymbol{X}(t)-\boldsymbol{X}(t|t) \qquad (14)$$

The *a priori* estimate error covariance then is:

$$\boldsymbol{P}^-(t)=E\{\boldsymbol{e}^-(t)\boldsymbol{e}^{-T}(t)\} \qquad (15)$$

and the *a posteriori* estimate error covariance is:

$$\boldsymbol{P}(t)=E\{\boldsymbol{e}(t)\boldsymbol{e}^T(t)\} \qquad (16)$$

In deriving Kalman filter formulation, we begin with the goal of finding an equation that computes an a posteriori state estimate as a linear combination of an a priori estimate (prediction) and a weighted difference between an actual measurement and a measurement prediction (innovation). Hence, each estimate consists of a fraction which is predictable from the previous values and does not contain new information and a fraction that contains the new information extracted from the observation.

$$\boldsymbol{X}(t|t)=\boldsymbol{X}(t|t-1)+\boldsymbol{K}(t)(\boldsymbol{Y}(t)-C\boldsymbol{X}(t|t-I) \qquad (17)$$

The difference ***Y***(t)-*C****X***(t|t-*1*) is called the measurement innovation. The innovation reflects the discrepancy between the predicted value and the actual measurement.

The N×M matrix, **K**(t), is chosen to be the gain or blending factor that minimizes the a posteriori error covariance. This minimization can be accomplished by first substituting innovation from (17) into the (14), substituting that into error covariance equation (16), performing the indicated expectations, taking the derivative of the trace of the result with respect to $\boldsymbol{K}$, setting that result equal to zero, and then solving for $\boldsymbol{K}$. One form of the resulting **K**(t) that minimizes error covariance is given by:

$$\boldsymbol{K}(t)=\mathbf{P}^{-}(t)C^{T}(C\boldsymbol{P}^{-}(t)C^{T}+\boldsymbol{R})^{-1} \quad (18)$$

From the above equations we see that as the measurement noise covariance, R, approaches zero, the gain, K, weights the innovation more heavily.

Specifically: $\lim_{R\to 0}\boldsymbol{K}(t) = (\boldsymbol{H})^{-1}$

On the other hand as the a priori estimate error covariance $P^{-}(t)$, approaches zero the gain, K, weights the innovation less heavily.

Specifically: $\lim_{P-(k)\to 0}\boldsymbol{K}(t) = \mathbf{0}$

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groups: *time update equations (prediction)* and *measurement update equations (correction).* The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate. The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations.

B. Summary Of Kalman Filter Equations

Predict Stage

A priori estimate of the current state

$\mathbf{X}(t|t-1) = A^{*}\mathbf{X}(t-1|t-1)$ (19)

A priori estimate of the state covariance matrix

$\mathbf{P}(t|t-1) = A^{*}\mathbf{P}(t-1|t-1)^{*}A' + \mathbf{Q}$ (20)

Correct Stage

Kalman gain

$\mathbf{K}(t) = \mathbf{P}(t|t-1) * C' * (C^{*}\mathbf{P}(t|t-1) * C' + \mathbf{R})^{-1}$ (21)

Estimated observation

$\mathbf{Y}(t|t-1) = C^{*}\mathbf{X}(t|t-1) + \mathbf{R}$ (22)

Measurement residual error or innovation error Innov= $\mathbf{Y}(t) - \mathbf{Y}(t|t-1)$ (23)

A posteriori estimate of the current state $\mathbf{X}(t|t) = \mathbf{X}(t|t-1) + \mathbf{K}(t)^{*}(\mathbf{Y}(t)-\mathbf{Y}(t|t-1))$ (24)

A posteriori state covariance matrix

$\mathbf{P}(t|t) = (I - \mathbf{K}(t)^{*}C) * \mathbf{P}(t|t-1)$ (25)

III. SYSTEM MODEL

In the MIMO environment, the channels are time varying. For successful detection of transmitted symbols, the channel needs to be tracked and the equalizer coefficients updated periodically. This is achieved with the help of Kalman Filter state space equations.

$$\boldsymbol{Y}(k) = \boldsymbol{X}(k) * \boldsymbol{h}(k) + \boldsymbol{W}(k) \quad (26)$$

$$\text{And}\,\boldsymbol{h}(k) = A * \boldsymbol{h}(k-1) + \boldsymbol{v}(k) \quad (27)$$

Where **Y**(k) is the received symbol matrix, **X**(k) is the transmitted symbol matrix.

$\mathbf{h}(k)=[h_{11},h_{12},h_{21},h_{22}]^{T}$

The system model is presented as

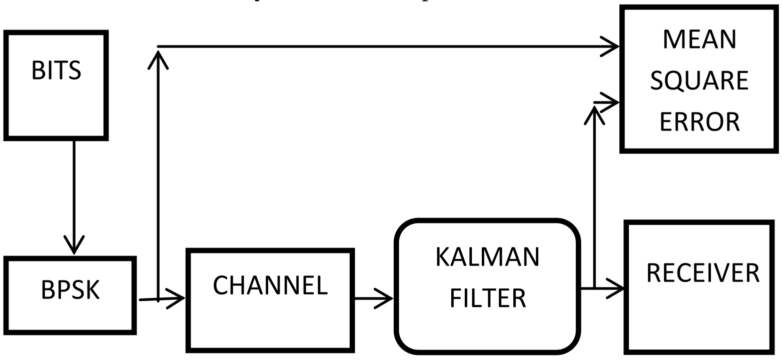

Fig.1 Block Diagram of the System

A. SIMULATION PARAMETERS

For the paper following parameters are assumed:

No. of samples/simulations=100

No. of transmitters M_t=2

No. of receivers M_r=2

Input= Random bits

State transition matrix A (of dimensionM_tX M_t) = I

Both process and measurement noise is assumed to be Gaussian noise.

The modulation is Binary Phase Shift Keying (BPSK). The binary 0 and 1 are separated by 180^{0} phase shift.

The channels under consideration are AWGN, Ricean and Rayleigh channels.

The channel tracking algorithm works as follows:

1. Initialize the variables and matrices.
2. Generate the input data
3. Modulate the data using BPSK modulator
4. Assume transmission through Rayleigh/Ricean of AWGN channel
5. Obtain the A priori estimate of the current state **h_** = A***h**
6. Obtain the A priori estimate of the state covariance matrix **M_2** = A***M_1***A' +Q
7. Compute the noise free output **y_free** and noisy output **y**
8. Compute the Kalman gain **K**(t) = **M_1**(t|t-1) * C' * (C***M_1**(t|t-1) * C' + R) $^{-1}$
9. Obtain the Estimated observation **h_hat_2**= C***h_hat_1**+ R
10. Compute the updated estimate **h_hat_2** = **h_hat_1** + **K***(**y-y_free**)
11. Measurement residual error or innovation error, innov=y-y_free
12. Compute the updated state covariance matrix **M_1** = (eye(p) - **K***V')***M_2**;
13. Demodulate the received data
14. Plot the results

B. RESULTS

The figures show the results for Rayleigh channel. With minor modifications in the code AWGN andRicean channel can be obtained. The fig.2shows the tracking capability of the Kalman filter.

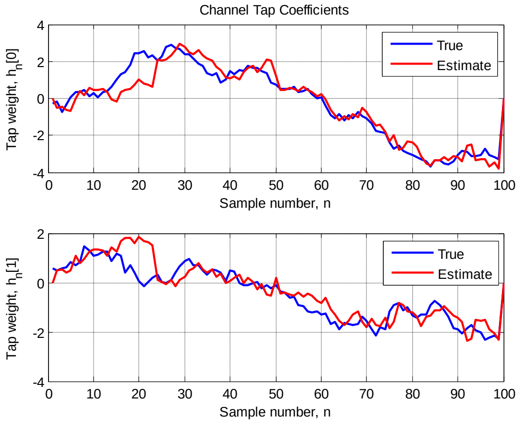

Fig.2 Channel Tap Coefficients

The mean square error of the tracker is shown in fig.3. Initially mean square error is chosen arbitrarily. It reflects our little knowledge about the channel. As seen the mean square error converges. After an initial transient the Kalman filter "lockson" to the true weights and tracks them closely. From the simulations it is evident that as the measurement error covariance approaches zero, the actual measurement is "trusted" more, while the predicted measurement is trusted less. On the other hand, as the a priori estimate error covariance approaches zero the actual measurement is trusted less, while the predicted measurement is trusted more. The SNR for both the channels is shown in fig.4.The fig.5plots the error between transmitted and received data.

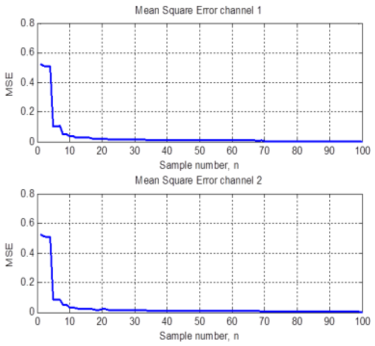

Fig.3 MSE

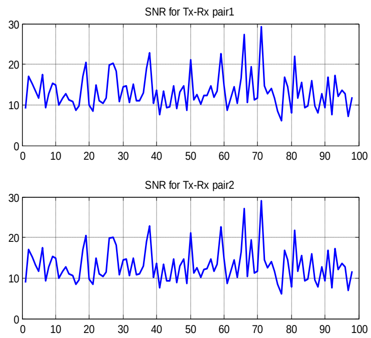

Fig.4 SNR

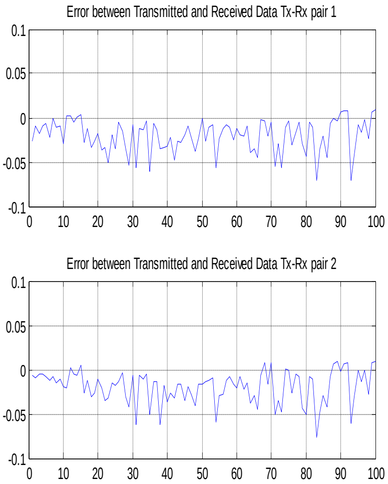

Fig.5 Error between Received and Transmitted Data

TABLE 1: COMPARISON OF SNR AND MMSE FOR AWGN, RAYLEIGH AND RICEAN ENVIRONMENT

Parameter/Channel	**AWGN**	**Ricean**	**Rayleigh**
SNR	14.4721	13.8014	13.4467
MMSE	0.0551	0.0603	0.0693

The error increases from AWGN to Ricean to Rayleigh. The SNR and MMSE reported are the mean SNR and MMSE of both the channels. In the AWGN channel, the only amplitude variation is due to the White Gaussian noise. These bits are easy to detect in such an environment since the receiver has to make decision only the basis of amplitude of the received signal i.e. whether it lies closer to binary 0 or binary 1. This results in low mean square error. The signal is at higher level than noise.In the Ricean channel along with the diffuse multipaths a strong Line of sight (LOS) path exists. The LOS component serves as a DC level. Thus, facilitating easy detection of the bits. The SNR is high due to this component. In the Rayleigh channel, exist diffuse multipaths. The transmitted signal arrives through these diffuse multipaths suffering from variable attenuation and distortion at each multipath. At receiver these multipaths combine constructively or destructively. Due to presence of the multipaths, several deep nulls can be observed in the error pattern of the Rayleigh channel. The SNR is also low as compared to other channels.

IV. CONCLUSION AND FUTURE WORK

A. CONCLUSION

The work is a study and implementation of Kalman filter for channel estimation. The Kalman filter estimator works in time domain. The simulation results prove that it is an excellent channel tracker and predictor. The results were compared for AWGN, Rayleigh and Ricean environment. The mean square error and SNR are compared for each case. The Rayleigh environment suffers from higher errors. The Kalman filter based channel equalization gives fairly good results in the time-domain. The recursive nature of the filter leads to adaptation of filter coefficients in order to minimize the mean square error.The convergence is quick. It accurately detects data in low SNR conditions as well.

B. FUTURE WORK

The given model can be applied to The Orthogonal Frequency Division Multiplexed (OFDM) system. Accurate channel estimation algorithms can be applied in OFDM systems to allow coherent detection, thereby improving system performance. The application of Kalman filter to OFDM system may however, be complex. Therefore, the different carriers of OFDM can be modeled as separate channels suffering from flat fading. Kalman filter model can be applied to these subcarriers separately and data can be aggregated. The complexity reduces as compared to the case when entire OFDM system is considered. The coding techniques such as Cyclic codes and Space Time Block Codes (STBC) can be used in conjunction with the proposed filter to improve the performance.

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