Kelvin – Helmholtz Instability Of Superposed Non – Newtonian Viscoelastic Fluid.

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Abstract

An analysis on the stability of incompressible superposed Non-Newtonian fluids is considered. Kelvin-Helmholtz instability is selected for this purpose. Rivilin-Ericksen elastico-viscous superposed fluid is considered. Normal mode analysis is applied. The dispersion relation is solved analytically. The stability of the system is studied for exponentially varying density, viscosity and viscoelasticity.

Key words *- Non-Newtonian fluid, Rivilin-Ericksen Elastico-viscous superposed fluid, Kelvin-Helmholtz instability.*

1. Introduction

A Non-Newtonian fluid is a fluid in which viscosity changes with the applied shear force. Therefore, Non-Newtonian fluids may not have a well defined viscosity. The role of Non-Newtonian fluid dynamics is important as it relates to plastic manufacture, performance of lubricants, clay suspensions, drilling muds, paints, processing of food and moment of biological fluids which contain higher molecular weight components. The convective stability of a general viscoelastic fluid heated from below is analyzed by Sokolov, R. I. Tanner [1], S. Rosenblat [5] and J. Martinez-Mardones [6]. The instability of the plane interface separating two uniform superposed streaming fluids, under varying assumption of hydrodynamics, has been discussed in the celebrated monograph by Chandrasekhar [3]. With the growing importance of non – newtonian viscoelastic fluids in modern technology and industries and the investigation on such fluids are desirable. Rivlin-Ericksen fluid is one such viscoelastic fluid. Johri [2] has discussed the viscoelastic Rivlin-Ericksen incompressible fluid under time dependent pressure gradient. Sharma and Kumar [7] have studied the thermal instability of a layer of Rivlin-Ericksen elastico-viscous fluid acted on by a uniform rotation and that rotation has a stabilizing effect and introduces oscillatory modes in the system. The instability of the plane interface between two uniform superposed and streaming fluids through porous medium has been investigated by Sharma and Spanos [4]. Sharma *et al.* [8] have studied the thermosolutal convection in Rivlin-Ericksen fluid in a porous medium in the presence of uniform vertical magnetic fluid. G. C. Rana *et. al* [12] investigated the stability of incompressible Rivlin-Ericksen Elastico-Viscous Superposed fluids under rotation in porous medium. Instability of superposed streaming fluids through a porous medium. Aiyub Khan *et. al* [10] have studied the instability of superposed streaming fluids through a porous medium. Neild [11] gave note on the onset of convection in a layer of a porous medium saturated by a Non-Newtonian Nanofluid of Power-Law Type. Pardeep Kumar and Roshan Lal [9] have studied the stability of two superposed viscous-viscoelastic fluids**.** When two superposed fluids flow one over the other with a relative horizontal velocity, the instability of the plane interface between two fluids, when it occurs in this instance, is known as 'Kelvin-Helmholtz instability'. The present paper focus on the Kelvin – Helmholtz Instability of Superposed Non – Newtonian Viscoelastic Fluid.

2. Mathematical Formulation

An incompressible elastico-viscous Rivlin-Ericksen in fluid which there is a horizontal steaming in the x-direction with velocity $U(z)$ is considered. The equilibrium of this initial state is analysed by supposing that the system is slightly disturbed and then following its further evolution.

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Let $p, \rho, g, \nu, \nu', \vec{q}(U(z), 0, 0)$ denote, respectively, the pressure, density, acceleration due to gravity, kinematic viscosity, kinematic viscoelasticity and velocity of Rivlin-Ericksen viscoelastic fluid.

The governing equations are given by equation of motion, continuity and incompressibility for the Rivlin-Ericksen elastico-viscous fluid as follows

$$\rho\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}\,.\nabla)\vec{q}\right] = -\nabla p + \rho\vec{g}$$

$$-\rho\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{q} \qquad (2.1)$$

$$\nabla.\vec{q} = 0 \qquad (2.2)$$

$$\frac{\partial \rho}{\partial t} + (\vec{q}.\nabla)\rho = 0 \qquad (2.3)$$

Let $\delta p, \delta\rho$ and $\vec{u}(u, v, w)$ denote the perturbations in pressure p, density ρ and velocity $\vec{q}(U(z), 0, 0)$ respectively.

Then, the linearized perturbation equations of fluid layer become

$$\rho\left[\frac{\partial \vec{u}}{\partial t} + (\vec{q}.\nabla)\vec{u} + (\vec{u}.\nabla)\vec{q}\hat{\imath}\right] = -\nabla\delta p + \vec{g}\delta\rho - \rho\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{u} \quad (2.4)$$

$$\nabla.\vec{u} = 0 \qquad (2.5)$$

$$\left[\frac{\partial}{\partial t} + (\vec{q}.\nabla)\right]\delta p = -w\frac{d\rho}{dz} \qquad (2.6)$$

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y and t is of given by

$$\exp[i(k_x x + k_y y + nt)] \qquad (2.7)$$

where n is the growth rate, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and k_x, k_y are horizontal wave numbers.

Substituting for δp, Eq.(2.4) with the help of Eqs.(2.5), (2.6) and expression (2.7) simplifies to

$$[i\,\rho(n + k_x U) + \rho(\nu + in\nu')]\vec{u} + \rho w(DU)\hat{\imath} = -\nabla\delta p + i\vec{g}\frac{w(D\rho)}{n + k_x U} \qquad (2.8)$$

where $\hat{\imath}$ is unit vector in the x-direction and $D = d/dz$.

Writing the three component equations of (2.8) and eliminating u, v and δp with the help of (2.5), we obtain

$$D\{[i\rho(n + k_x U) + \rho(\nu + in\nu')]Dw - ik_x\rho(DU)w\} - [i\rho(n + k_x U) + \rho(\nu + in\nu')]w = igk^2(D\rho)\frac{w}{n + k_x U} \qquad (2.9)$$

3. Superposed Uniform Fluids

Consider the case when two superposed streaming fluids of uniform densities ρ_1 and ρ_2, uniform viscosities μ_1 and μ_2 and uniform viscoelsticities μ_1' and μ_2' are separated by a horizontal boundary layer at $z = 0$. The superscript 1 and 2 distinguish the lower and the upper fluid respectively.

The density ρ_2 of the upper fluid is taken to be less than the density ρ_1 of lower fluid so that, in the absence of the streaming, the configuration is stable. Let the two fluids be streaming with constant velocities U_1 and U_2. Then in each of the two regions of constant ρ, μ, μ' and U, Eq. (2.9) becomes

$$(D^2 - k^2)w = 0 \qquad (3.1)$$

The boundary conditions to be satisfied here are:

(A)

$$\frac{w}{n + k_x U} \qquad (3.2)$$

must be continuous at an interface, since U is discontinuous at $z = 0$.

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(B) Integrating Eq.(2.9) between $0-\eta$ and $0+\eta$ and passing to be limit $\eta = 0$, we obtain, in view of (3.2), the jump condition for $z = 0$ is,

$$\Delta_0\{[i\rho(n+k_xU)+\rho(v+inv')]Dw - ik_x\rho(DU)w\} = igk^2\Delta_0(\rho)\frac{w}{n+k_xU} \quad (3.3)$$

while the equation valid everywhere else ($z \neq 0$) is

$$D\{[i\rho(n+k_xU)+\rho(v+inv')]Dw - ik_x\rho(DU)w\} - k^2[i\rho(n+k_xU) + \rho(v+inv')]w = igk^2(D\rho)\frac{w}{n+k_xU} \quad (3.4)$$

Here $\Delta_0 f = f(z_0+0) - f(z_0-0)$ is the jump which a quantity experiences at the interface $z = z_0$; and the superscript 0 distinguish the value a quantity, known to be continuous at an interface, takes at the interface $z = z_0$.

The general solution of Eq.(3.1) is a linear combination of the integrals e^{+kz} and e^{-kz}. Since $w/(n+k_xU)$ must be continuous on the surface $z = 0$ and w cannot increase exponentially on either side of the interface, the solution appropriate for two regions are

$$w_1 = A(n+k_xU_1)e^{+kz}, \qquad (z<0) \qquad (3.5)$$

$$w_2 = A(n+k_xU_2)e^{-kz}, \qquad (z>0) \qquad (3.6)$$

Applying the boundary condition (3.3) to the solutions (3.5) – (3.6), we obtain the dispersion relation

$$[1+(\alpha_1v_1'+\alpha_2v_2')]n^2 + [2k_x(\alpha_1U_1+\alpha_2U_2) + k_x(\alpha_1v_1'U_1+\alpha_2v_2'U_2) - \alpha_1v_1+\alpha_2v_2]n + k^2(\alpha_1U_1^2+\alpha_2U_2^2) - ik_x(\alpha_1U_1v_1+\alpha_2U_2v_2) - gk(\alpha_1-\alpha_2) = 0 \qquad (3.7)$$

where

$$\alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1+\rho_2}, \; v_{1,2} = \frac{\mu_{1,2}}{\rho_{1,2}}, \; v_{1,2}' = \frac{\mu_{1,2}'}{\rho_{1,2}}.$$

$v_1\left(=\frac{\mu_1}{\rho_1}\right)$, $v_1'\left(=\frac{\mu_1'}{\rho_1}\right)$, $v_2\left(=\frac{\mu_2}{\rho_2}\right)$ and $v_2'\left(=\frac{\mu_2'}{\rho_2}\right)$ are the kinematic viscosities and kinematic viscoelasticities of the lower and upper fluid respectively.

Equation (3.7) yields

$$in = -[+(\alpha_1v_1+\alpha_2v_2) + 2ik_x(\alpha_1U_1+\alpha_2U_2) + ik_x(\alpha_1v_1'U_1+\alpha_2v_2'U_2)] \pm \{(\alpha_1v_1+\alpha_2v_2)^2 - 4ik_x\alpha_1\alpha_2(v_1-v_2)(U_1-U_2) + 4k_x^2\alpha_1\alpha_2(U_1v_2' - U_2v_1')(U_1-U_2) - 2ik_x[(v_1\alpha_1^2v_1'U_1 + v_2\alpha_2^2v_2'U_2]) + \alpha_1\alpha_2(U_1v_2'v_1+U_2v_1'v_2) + \alpha_1\alpha_2(U_1-U_2)(v_1v_2'-v_1'v_2) + \left(k_x(\alpha_1v_1'U_1 + \alpha_2v_2'U_2)\right)^2 + 4k_x^2\,\alpha_1\,\alpha_2\,(U_1-U_2)^2 - 4gk(\alpha_1-\alpha_2)[1+(\alpha_1v_1'+\alpha_2v_2')]\}^{1/2} \qquad (3.8)$$

Some cases of interest are now considered.

(C) *When $k_x = 0$, equation (3.8) yields*

$$in = -(\alpha_1v_1+\alpha_2v_2) \pm \{(\alpha_1v_1+\alpha_2v_2)^2 - 4kg(\alpha_1-\alpha_2)[1+(\alpha_1v_1'+\alpha_2v_2')]\}^{1/2} \qquad (3.9)$$

Here we assume kinematic viscosities v_1, v_2 and kinematic viscoelasticities v_1', v_2' of the two fluids to be equal, that is $v_1 = v_2 = v$, $v_1' = v_2' = v'$. However, any of the essential features of the problem are not obscured by this simplifying assumption. Eq.(3.9), then, becomes

$$in = -v \pm [v^2 - 4gk(\alpha_1-\alpha_2)\{1+v'\}]^{1/2} \qquad (3.10)$$

- **Unstable case**

For the potentially unstable configuration ($\rho_2 > \rho_1$), it is evident from Eq.(3.10) that one of values of in is positive which means that the perturbations growth with time and so the system is unstable.

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- **Stable case**

For the potentially stable configuration $(\rho_2 < \rho_1)$, Eq.(3.10) yields that both values of in are either real, negative or complex conjugates with negative real parts implying stability of the system.

It is interesting to note from above that for the special case when perturbation in the direction of streaming are ignored $(k_x = 0)$, the system is unstable for potentially unstable configuration and system is stable for potentially stable configuration and not depending upon kinematic viscoelasticity.

(D) In every other direction, instability occurs when

$$\frac{\alpha_1 \alpha_2 k_x^2}{\varepsilon^2}(U_1 - U_2)^2 > gk\,(\alpha_1 - \alpha_2) \qquad (3.11)$$

Thus for a given difference in velocity $(U_1 - U_2)$ and for a given direction of the wave-vector $\vec{k}$, instability occurs for all wave numbers

$$k > \left[\frac{g(\alpha_1 - \alpha_2)}{\alpha_1 \alpha_2 (U_1 - U_2)^2 \cos^2\theta}\right] \qquad (3.12)$$

where θ is the angle between the direction of $\vec{k}(k_x, k_y, 0)$ and $\vec{U}(U, 0, 0)$, that is, $k_x = k\cos\theta$. Hence, for a given velocity differences$(U_1 - U_2)$, instability occurs for the least wave number when $\vec{k}$ is in the direction of $\vec{U}$ and this minimum wave number; k_{min} , is given by

$$k_{min} = \left[\frac{g(\alpha_1 - \alpha_2)}{\alpha_1 \alpha_2 (U_1 - U_2)^2}\right] \qquad (3.13)$$

For $k > k_{min}$, the system is unstable.

4. Conclusion

It has been observed that for $\rho_1 < \rho_2$, the system is found unstable and for $\rho_1 > \rho_2$ the system is stable.

5. References

[1] M. Sokolov, R. I. Tanner, convective stability of general viscoelastic fluid heated below, Phys. Fluids 15 (1986) 613-623.

[2] Johri A. K., Acta Ciencia Indica, 24(1976), 377.

[3] Chandrasekhar S., Hydrodynamic and Hydromagnetic Stability, Dover Publication, New York, 1981.

[4] Sharma R. C. and Spanos T.J.T., Canadian J. Phys., (60)(1982), 1391.

[5] S. Rosenblat, Thermal convection in a viscoelastic liquid, J. Non-Newtonian Fluid Mech. 21 (1986) 534- 539.

[6] J. Martinez-Mardones, C. Perz-Garcia, Linear instability in viscoelastic fluid convection, J. Phys. Condens. Matter 2 (1990) 1281-1290.

[7] Sharma R. C. and Kumar P. Z., Naturforsch, 51a(1996), 881.

[8] Sharma R. C., Sunil and Chand S., Appl. Mech. Engg., 3(1) (1998), 1.

[9] Pardeep Kumar and Roshan Lal, J. Fluids Eng., 2007, Vol. 129, Issue 1, 116.

[10] Aiyub Khan *et al.*, Turkish J. Eng. Env. Sci. 34, 59 – 68 (2010).

[11] D. A. Nield, A note on the Onset of Convection in a Layer of a Porous Medium Saturated by Non-Newtonian Nanofluid of Power-Law Type, Transp. Porous Med. (2010) DOI 10.1007/s11242-010-9671-z.

[12] Rana G. C., Sharma. V and Sanjeev Kumar, J. Comp & Math Sci. Vol.2(2), 316-321, (2011).

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