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Kinematic Modelling and Analysis of a 5 Axis Articulated Robot Arm Model VRT-502

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***Abstract*— Kinematics is the study of motion without regard to the forces that create it. Generally, kinematics for robotic manipulators includes two problems, forward kinematics problem and inverse kinematics problem. Deriving both forward and inverse kinematics is an important step in robot modelling. The forward kinematics problem is defined as the transformation from the joint space to the Cartesian space. An analytical solution using Denavit-Hartenberg (D-H) representation is obtained, to describe the position and orientation of the VRT-502 robot model end-effector. The modelling is necessary before applying control techniques to guarantee the execution of any task according to a desired input with minimum error.**

Index Terms— Direct Kinematics, Denavit-Hartenberg,VRT-502, Homogeneous transformation matrices

I.INTRODUCTION

A robot is a reprogrammable multi-functional manipulator designed to move material, parts, tools, or specialized devices through programmed motions for the performance of a variety of tasks [3]. Kinematics is the study of motion, for example, position, velocity and acceleration without regard to the forces that create it. A manipulator generally referring to a serial manipulator can be viewed as an open loop of links connected by some joints, such as the revolute joint and prismatic joint. Robot manipulator is a collection of n-links that are connected together by joints [1]. Each one of these joints has a motor that actuates a motion to the commanded link. The motors have feedback sensors to measure the output (e.g. position, velocity, and torque) at each instant. Links and joints form a kinematic chain connected to ground from one side, and the other is free. At the end of the open side, the end-effector (e.g. gripper, welding tool, or another tool) is connected, which helps the manipulator to do some tasks such as welding, or handle materials [2]. Robot manipulator is named according to number of DOF, which refers to the number of joints. As an example, if a robot manipulator has 5 joints, it means the robot has 5 DOF. In physical applications, it is important to describe the position of the end effectors of the robot manipulator in global coordinates. In transforming, the coordinates of the end effectors from the local position to the global position, the robot movements are represented by a series of movements of rigid links. Each link defines a proper transformation matrix relating the position of the current link to the previous one. Robot manipulator whose all joints are prismatic is known as a Cartesian manipulator while the robot whose joints are revolute is called an articulated manipulator. Kinematics is the motion geometry of the robot manipulator from the reference position to the desired position with no regard to forces or other factors that influence robot motion [3]. In other words, the kinematics deals with the movement of the robot manipulator with respect to fixed frame as a function of time. The fixed frame in robot represents the base and all other movements are measured with respect to base as reference. It is important in practical applications such as trajectory planning and control purposes. Generally, to control any robot manipulator the core of the controller is a description of kinematic analysis, this is done by using a common method in industrial and academic research, namely Denavit-Hartenberg method [1], [2] and [3].

The paper is organized as follows. Section 2 presents the robotic arm model and its joint and link parameters. Hardware details about the model are introduced in Section 3. Section 4 presents the link diagram and kinematic model. Kinematic model derived is analysed and results are obtained in Section 5. Finally, the paper is concluded in Section 6.

II. DESIGN DETAILS

The robot under consideration is VRT-502 model, which is a 5 DOF stationary articulated robot arm having base, shoulder, elbow, tool pitch and tool roll and consisting of only rotary joints [1]. There are 5 joints and therefore 5 axes. Out of the 5 axis, 3 are major axis (base, shoulder and elbow- to position the wrist) and 2 are minor axis (tool roll and tool pitch-to orient the gripper in the direction of the object).

Since n=5; 20 kinematic parameters are to be obtained and 6 unit frames are to be attached to the various joints as shown in the link co-ordinate diagram in Fig 2.

Fig 1: VRT-502 model

The vector of joint variables is given by [1]

$$q = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\}^T.$$

The vector of joint distances are given by [1]

$$d = \{d_1, d_2, d_3, d_4, d_5\}^T = \{25, 0, 0, 0, 0\}^T \text{ cm.}$$

The vector of link lengths are given by [1]

$$a = \{a_1, a_2, a_3, a_4, a_5\}^T = \{0, 23, 22, 8, 15\}^T \text{cm.}$$

The vector of link twist angles are given by [1]

$$\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}^T = \{-90^0, 0, 0, 90^0, 0\}^T.$$

L0 to L5 : Six unit frames.
a5 : Tool length
q1 to q5 : Joint variables (q = θ)
p : Tool-tip
d1 : Height of shoulder from base
1, 2, 3, 4, 5 : Rotary joints
a2, a3, a4 : Link lengths

The robot is a five axis articulated robot having a non-spherical wrist, i.e., the pitch and the roll axes meets at different points. The five axes are**:** five DOF - Base, Shoulder, Elbow, Pitch, Roll and no Tool Yaw. Base motor is mounted vertically on a horizontal plane. Shoulder, elbow, tool pitch motors are mounted horizontally on the base. The tool roll and grip motors are mounted at the wrist joint

II.HARDWARE DETAILS

The robot model under consideration can carry a payload of 2kg.It has a jointed arm type configuration. The base joint at Joint 1 is responsible for the rotational angular motion around the *z*-axis in *x* and *y* plane. The range of motion for Joint 1 is from 0° to +320°. The second link is connected between Joint 1 and Joint 2 (shoulder joint). The angle limitation for Joint 2 is between 0° to +80°. Joint 2 is then attached to Joint 3 (elbow joint) by a third link. This joint has a boundary rotation at 0° to +85°. The final link where the end-effector is located is attached to this joint called the Joint 4 (wrist joint) possess a roll revolution between 0° to +300°. At this end, the wrist joint at Joint 5 has a 0° to 40° limit for pitch revolution. The joints are actuated by stepper motors. The gripper at the tip is a pneumatic based miniature cylinder

III. LINK DIAGRAM AND KINEMATIC MODELLING

Given the joint variable vector q (θ for rotary joint) and the Geometrical Link Parameters (GLP - physical dimensions of the robot arm **:** constant for a given robot), finding the position p of the tip of the gripper and the orientation R of the gripper of the robot arm w.r.t. base of the robot from the reference position is called as direct kinematics .To solve the DKP means to find the p and R of the tool w.r.t. base [1].In analysing a robot mechanism, we often create a frame diagram that graphically shows the relationships between the DH-frames of the robot.

To find the position and orientation of the robot arm means, we have to find a matrix called as the arm matrix, i.e., the composite homogeneous coordinate transformation matrix, which is a (4 × 4) matrix [2].The 1st three columns gives the three possible orientations (Yaw, Pitch, Roll) of the gripper and the last column gives the position of the tip of the gripper 'p', thus solving the DK problem. If we give this matrix as input to the robot, the robot will go and stop in that particular position and in that particular orientation [1].

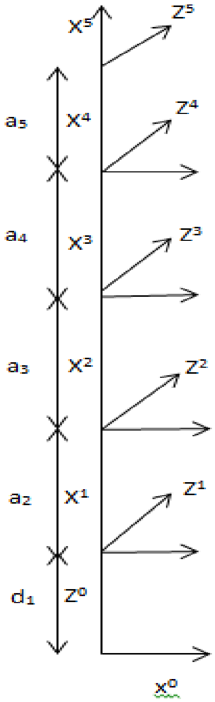

Fig 2: Link coordinate diagram of robot arm

TABLE 1. KINEMATIC PARAMETER TABLE OF THE DEVELOPED ROBOT

Axis	Type	θ_k	d_k	a_k	α_k	SHP
1	Base	θ_1	d_1	0	$-\pi/2$	$-\pi/2$
2	Shoulder	θ_2	0	a_2	0	0
3	Elbow	θ_3	0	a_3	0	0
4	Tool Roll	θ_4	0	a_4	0	0
5	Tool Pitch	θ_5	0	a_5	0	0

A. *Arm Matrix*

Once a set of link coordinates is assigned using the D-H algorithm, the next step is to transform from coordinate frame k to coordinate frame k-1 using a homogeneous coordinate transformation matrix. The composite transformation matrix that transforms the tool coordinates into the base coordinates is called the arm matrix. In this case, there are 5 coordinate frames attached with the 5 joints. The frames are partitioned at the wrist, thus there will be two wrist partitioned factors of the arm matrix.

$$T_{base}^{tool}(q) = T_{base}^{should} T_{should}^{elbow} T_{elbow}^{roll} T_{roll}^{pitch} T_{pitch}^{tip} \tag{1}$$

$$T_0^5(q) = [\, T_0^1 \; T_1^2 T_2^3 \,] \; [\, T_3^4 \; T_4^5 \,] \tag{2}$$

B. Computation of the First Wrist Partitioned Arm Matrix

$$T_0^3 = [\, T_0^1 \; T_1^2 T_2^3 \,] \tag{3}$$

$$T_0^1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & a_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{base}^{wrist} = T_0^1 T_1^2 T_2^3 \tag{4}$$

$$T_{base}^{wrist} = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & -s_1 & c_1(a_2 c_2 + a_3 c_{23}) \\ s_1 c_{23} & -s_1 s_{23} & c_1 & s_1(a_2 c_2 + a_3 c_{23}) \\ -s_{23} & -c_{23} & 0 & d_1 - a_2 s_2 - a_3 s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This matrix T^3_0 gives position and orientation of the wrist coordinate frame L_3 w.r.t. the base frame L_0. To check whether the matrix obtained is correct or not, evaluate it at the Soft Home Position [SHP]; i.e., put q = $[\, 0 , -90° , 0 \,]^T$ in the computed T^3_0 matrix, we get ;

$$T^3_0\,(\text{home}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & d_1 + a_2 + a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

C. Computation of the Second Wrist Partitioned Arm Matrix

$$T_{wrist}^{tool} = T_3^{\,4} T_4^{\,5} \tag{6}$$

$$T_3^{\,4} = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & a_4\cos\theta_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & a_4\sin\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^{5} = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & a_5\cos\theta_5 \\ \sin\theta_5 & \cos\theta_5 & 0 & a_5\sin\theta_5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{wrist}^{tool} = \begin{bmatrix} C_{45} & -S_{45} & 0 & a_5 C_{45} + a_4 C_4 \\ S_{45} & -C_{45} & 0 & a_5 S_{45} + a_4 S_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

This matrix T_{wrist}^{tool} gives position and orientation of the tool coordinate frame L_5 w.r.t. the wrist frame L_3. To check whether the matrix obtained is correct or not, evaluate it at the Soft Home Position [SHP]; i.e., put q = $[\, 0 , 0 , 0 \,]^T$ in the computed T_{wrist}^{tool} matrix, we get ;

$$T_{wrist}^{tool}\,(\text{home}) = \begin{bmatrix} 1 & 0 & 0 & a_4 + a_5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

E. Computation of the Final Arm Matrix

$$T_{base}^{tool} = T_{base}^{wrist} * T_{wrist}^{tool} \tag{9}$$

$$T_{base}^{tool} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (10)$$

where ,

$n_x = c_1 c_{2345}$

$n_y = s_1 c_{2345}$

$n_z = -s_{234} c_5 - c_{234} s_5$

$o_x = -c_1 s_{2345}$

$o_y = -s_1 s_{2345}$

$o_z = s_{234} s_5 - c_{234} c_5$

$a_x = s_1$

$a_y = c_1$

$a_z = 0$

$p_x = c_1 (a_2 c_2 + a_3 c_{23} + a_4 c_{234} + a_5 c_{2345})$

$p_y = s_1 (a_2 c_2 + a_3 c_{23} + a_4 c_{234} + a_5 c_{2345})$

$p_z = d_1 - a_2 s_2 - a_3 s_{23} - a_4 s_{234} - a_5 s_{2345}$

This matrix T_{base}^{tool} gives position and orientation of the tool coordinate frame L_5 w.r.t. the base frame L_0. To check whether the matrix obtained is correct or not, evaluate it at the Soft Home Position [SHP]; i.e., put q = [0 , –90° , 0 ,0,0]^T in the computed T_{base}^{tool} matrix. This arm matrix T_{base}^{tool} given by Eqn (10) is the output of direct kinematics of the designed five axes articulated robot arm, thus giving the position and orientation of the gripper w.r.t. base. The 1st three columns gives the orientation of the frame L5 w.r.t. base, while the last column gives the position of the tip of the gripper 'p' w.r.t. base, thus obtaining a unique direct kinematic model.

$$T_{base}^{tool} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & d_1 + a_2 + a_3 + a_4 + a_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (11)$$

V. Analysis of the Kinematic Model

The selection of a single polynomial for the entire joint path depends on the number of constraints imposed and the type of motion desired. The minimum numbers of constraints required for a smooth motion between two points are initial position, initial velocity, final position, and final velocity. The initial and final velocity is taken as 0. With four constraints, a third degree polynomial with four co-efficient can be used, that is

$$q(t) = a_0 + a_1 * t + a_2 * t^2 + a_3 * t^3 \qquad (12)$$

A. Base and Tool Roll Joint

The base and tool roll joint is moved from zero degrees to 300 degrees in 5 sec.

$$t_0 = 0; t_f = 5\sec$$
$$\theta_0 = 0; \theta_f = 300^o$$

B. Shoulder and Elbow Joint

The shoulder and elbow joint is moved from zero degrees to 70 degrees in 5 sec.

C. Tool Pitch Joint

The tool pitch joint is moved from zero degrees to 40 degrees in 5 sec.

The variation of the end effector position (px,py,pz) when the angles are varied is shown below:

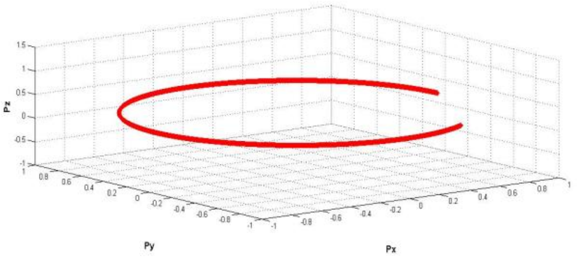

Fig.3 Variation of End-Effector Position Vector when Base Joint Angle is varied while others are Zero.

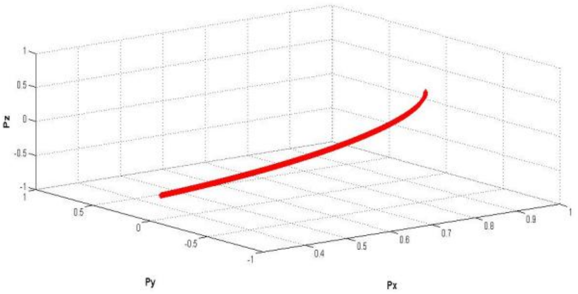

Fig.4 Variation of End-Effector Position Vector when Shoulder Joint Angle is varied while others are Zero.

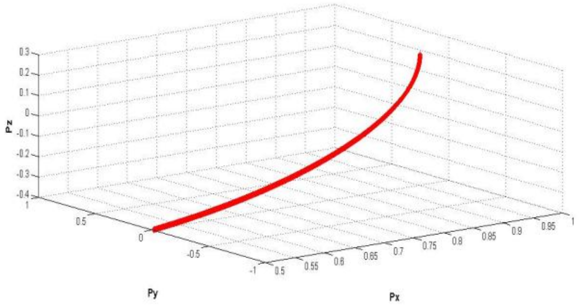

Fig.5 Variation of End-Effector Position Vector when Elbow Joint Angle is varied while others are Zero.

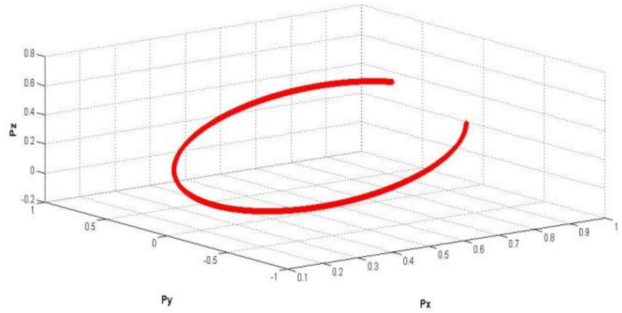

Fig.6 Variation of End-Effector Position Vector when Tool Roll Joint Angle is varied while others are Zero.

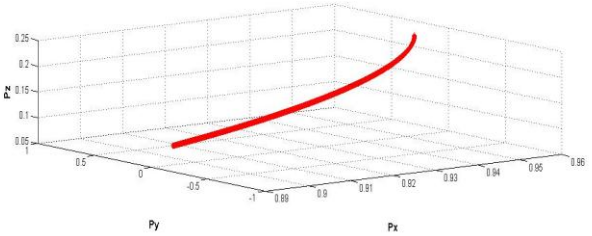

Fig.7 Variation of End-Effector Position Vector when Tool Pitch Joint Angle is varied while others are Zero.

The variation of the end effector position (px, py,pz) when all the joint angles are varied simultaneously through the defined trajectories is shown in Fig(8)

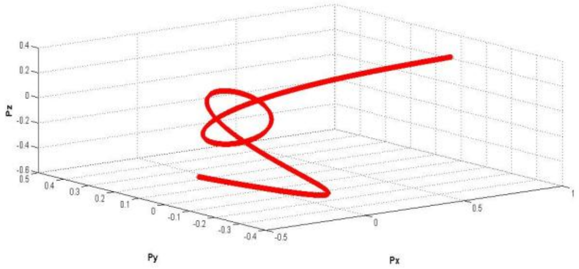

Fig.8 Variation of End-Effector Position Vector when all Joint Angles are varied simultaneously.

V. CONCLUSION

In this work, the mathematical formulation of complete kinematics of 5 degrees of freedom robotic arm is presented.Theoretical analysis of the forward kinematics was done in matlab to determine the end effector's position and orientation. As a future work, inverse kinematic models will be developed which is necessary to determine the joint variable as the desired tool position and orientation is used to formulate of the manipulation tasks.

VI. ACKNOWLEDGMENT

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