International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 4 Issue 05, May-2015

Level Set based Topology Optimization and its Application for Structures

Suresh C. S. Garia
Department of Mechanical & Industrial Engineering,
Indian Institute of Technology,
Roorkee, India

Dr. M. M. Joglekar
Department of Mechanical & Industrial Engineering,
Indian Institute of Technology,
Roorkee, India

***Abstract*—Topology optimization has played a key role in structural optimization. It has been basically introduced by Bendsoe and Kikuchi in 1988. It is very important to use the material wisely to reduce the weight of the structure without compromising the strength and other desired properties. Now, the optimization concept has grown in many directions and can be done with material distribution methods, Heuristic (experience) based methods, and Boundary based method. This article presents an introduction to level set method (Boundary based method) and some of its application in optimization of 2D structures and shows the convergence behaviour of the objective function. Minimum compliance is taken as the objective function which is subjected to volume constraint for some static structures.**

Keywords—Optimization, structural topology, Level set method, Matlab, Beam

INTRODUCTION

In today's world, Optimality has become a driving a force for individuals(engineers,scientist, and mathematicians) and companies for research, better product and services forsociety. Optimization can be described as a best solution among the many availablealternatives under a given set of predefined conditions. Better product, less overall cost, serviceability, maintenance has made optimization an essential part of every research andindustry. Optimization not only application to a physical component, but can be broadly used in an assembly of components, products, plans as well as in management services. Theadvancement in computation has made this process very popular in recent years to minimize the time and to speed up the process of design and development.

Topology optimization has become an integral part of design and development of structuralcomponents in recent years. The main objective of topology optimization is the distribution of material with in the components to improve the performance of the structure. This is possible because of the material distribution method for generating optimal topologies of structural elements. In classical design where material played a very important role, with the development of topological optimization, the distribution of material becomes an essential part of designing. Initially this concept was used for mechanical design problems, but now it has spread its wing all around. The topologydesign is used broadly in structural problems (vibrations, buckling, stress constraints, pressure loads, compliant mechanisms, material design, support design, civil engineering applications and bio-mechanics, etc.). It can be used for linear and nonlinear kind of problems. Moreover, the new areas where it has been implemented successfully areelectro-thermalactuators, MEMS,flow problems, transducers, electromagnetic, acoustics, optics etc.

The first paper on topology was published by an Australian named Michell in 1904 tooptimize the weight of a truss. Later this theory was extended to optimize the beam system. The advancement in numerical techniques with the development of analytical and computationaltools made the optimization process very easy and fast because of their capability to handle the large amount of optimization data. With the development of computational tools, newtechniques for the optimization developed to best optimized structure.

The first general theory on topology optimization termed as optimal layout theory waspresented by Prager and Rozvany in 1977 for grid type structures. In 1988 a paper by Bendsoe and Kikuchi [1] provided a numerical method for topology optimization. They have used homogenization methodin their work. The major drawback of this method was creation of large amount of voids, and variable densities ranges from 0 to1. Therefore it was difficult to interpret the exact shape of the optimized object.

The next development was to avoid the above said problem means to provide density toeach element. This method is commonly known as a solid isotropic material with penalizationmethod. In this method, the solution was in the form of check board with intermediate densitiesi.e. the densities very between 0 to 1. Still there was no clear view of an optimized structure.To overcome the above problem, a new method known as evolutionary structural optimization approach (ESO) developed. The main motive of this method was to remove the weakelements of the object in a predefined design domain. By doing this exercise, all intermediate densities could be eliminated. The outcome of this method was an object having jagged voidsin structure. There were no smooth boundaries around the voids. This kind of structure was notacceptable as a design point of view and as well as difficult to manufacture. This was the main limitation of ESO method.

Another method proposed by Querin et al. [2] was bi-directionalevolutionaryoptimization approach (BESO). In this approach the efficient material to be added while the inefficient material is removed. Simultaneously many authors proposed very efficient methodto eliminate the check board pattern in optimization techniques specially the sharp corners incheck board pattern.

To overcome all above problems, a new method, implicit moving boundary method, also known as level set method used which was devised by Osher and Sethian[3] This method is capable of getting clear boundaries during all steps of optimization process. The major benefit of level set method is to track the motion of the boundaries. Additionally it automatically nucleates holes, merges hole with each other and with boundaries throughout the optimization process.

Mapping of an implicit and structural model of a continuously variable geometry is animportant part of the level set method. The mapping can be done by two methods via densitymethod and boundary method. In density method, Finite element inside the geometry isrepresented as solid and those outside or within a hole as empty. The limitation of this method is variation in density during the intersection of elements within the structural boundaries. This again gives an ambiguous design and cannot be predicted as desired result. However boundarybased method gives a clear prediction on an optimized structure.

Level-set-based methods (implicit moving boundary)

The interfaces propagate in variety of physical phenomenon. Ocean waves, combustion,and material boundaries are some of examples. In moving boundaries method, less obvious boundaries are also play a vital part. In boundary based method, the design variables directlycontrol the exterior as well as interior of the design domain. The level set method was devisedby Osher and Sethian (1988). It is much simpler and more versatile method for computing and analysing the motion of an interface in two or three dimensions. Initially the level set method was limitedto the tracking of propagating interfaces. Later on it has been implemented for a wide range of problems especially structural optimization. The level set models sometimes mentioned asimplicit moving boundary (IMB) models. The level set models are very versatile as they caneffortlessly represent intricate boundaries. It is having the ability of creating new holes, cansplit a bigger hole into multiple holes, or merge with other holes to form a single one.

Level set method is based on the concept of propagation of the level set surface. It is non-traditional method in which fixed grid mesh is used as a design space. It has gained thesignificant popularity as it works on the boundaries during whole optimization process. Thedesign changes are carried out as a mathematical Programming for the problem of optimization.The level set method can be categorized on the basis of level set function parameterization,geometry, and regularization techniques. The LSM can handle topological merging, nucleationand splitting of holes naturally by embedding the interface as the zero level set of a higherdimensional function. The Implicit boundary method (The level set method) improves theaccuracy near the boundaries and removes the ambiguity of intermediate densities as in densitybased approach. Level-set-based topology optimization methods allow a convenient treatmentof topological changes over explicit boundary descriptions. Additionally, the results of mostlevel-set-based topology optimization methods do not involve mesh-dependent which are oftenencountered in density-based topology optimization methods. LSMs for structural topologyand shape optimization: In structural optimization the two-phase, material-void problem is themost simple and most frequently treated case. Ifwechangelevelset function, the shape and topology of material domain changes.

Application: [3] has initially used LSM for tracking of the boundaries. LSM initially used for study of geometry, image enhancement and noise removal, combustion problems, crystal growth and shape detection. Later the application of LSM extended to structural optimization. 2D and 3D cases have been solved by many authors.Eigen value problem [4], contact problems [5], check board implementation of material [6], problems related to fluid, thermal electro-mechanical [7], electromagnetic [8], optical applications [9] shell structure ([10]) and many more are the examples of LSM implementation. Aircraft wing structure, bridges,medical instrument, Aircraft fuselage, bulkheads etc., dams, composite beam structure, wind excited tall buildings are few of them.

Mathematical formulation

Minimum compliance: method is a simplest method of formulation of a design problem. The general design problem can be formulated as Maintaining the Integrity of the Specifications

Objective function c(x), subject to a volume constraint $V(x) \leq 0$ and N other constraints $gi \leq 0$, $i = 1 . . . N$.
The distribution ofmaterial is described by the density variable $\rho(x)$ that will be in a 0-1 configuration.
Mathematically, it can be written as [11]

Min: f(x)
Subject to: $gi(x) \leq 0 \; i = 1...p$ (1) $\qquad hj(x) = 0, j = 1...q$

In our case the objective function is minimization of compliance which is subjected to volume constraint.

$$\min c(x) = F^T U = U^T K U = \Sigma u_e^T k_e u_e$$
$$\text{Subject To: } V(x) = V(req) \qquad (2)$$

Where F represents global force, K represents global stiffness and U represents the global displacement.
In general, the real life applications have many constraints, but in this case only a linear volume constraint has been taken for simplicity. The volume constraint improves the general convergence.If D is design domain, ψ is material part of the domain and (D/ψ) represents the void, then the level set function can be defined as [12][14][21-22]

$$\begin{aligned} \phi(X) > r &\Leftrightarrow X \in \psi(\text{material}) \\ \phi(X) = r &\Leftrightarrow X \in \Gamma(\text{interface}) \qquad (3) \\ \phi(X) < r &\Leftrightarrow X \in (D/\psi)(\text{void}) \end{aligned}$$

The level set function can be redefined on the basis of grid points in the design domain. Ifs_e is the center of element e, then the modified level set function can be defined as[23]

$$\Phi(s_e) \begin{cases} < 0 \text{if} x_e = 1) \\ > 0 \text{if} x_e = 0) \end{cases} \qquad (4)$$

Hamilton Jacobi equation can be used to update the level set function.

$$\partial v/\partial \tau + |\nabla \phi| v = 0 \qquad (5)$$

The above equation is applicable to a one dimensional problem. In this case the level setfunction changes along the surface which do not allow new void creation. [20][25]

$$\partial \phi/\partial \tau + |\nabla \phi| v_n + \eta z = 0 \qquad (6)$$

.

For two dimensional problems, two new parameters added. Where η is the positive parameterand z is used for creating new voids in the structure. v_n is the normal velocity of the interfaceand represents the geometrical motion of the boundary of the structure. It is the derivative ofthe shape sensitivity of the Lagrangian.The modified Hamilton Jacobi equation can be used to update the modified level set function.Courant-Friedrichs-Lewy (CFL) condition is used to check the convergence.

$$C = u * \Delta t/\Delta x \le C_{max} \qquad (7)$$

Where Δ x is the minimum grid points distance and Δ t is the time step. Usually the value of C_{max} is taken as unity.

Evaluation of velocity v and parameter z

The velocity field and the term z can calculated by using the shape and topology sensitivities ofLagrangian function. The Lagrangian can be a function of both compliance as well as volume with some constants in a quadratic form. [23]

$$L = c(x) + \alpha^r (V^\#) + 1/\beta^r * (V^\#)^2 \qquad (8)$$

Where$V^\#$=V(x)-V(req).

The two parameters $\alpha$and β changeswith each iteration.

In the next step, there is a need to find out the shape

sensitivities of compliance and volume. Both the term will be used to find out the velocity of the front.The shape sensitivity ($\delta c/\delta\psi|_e$)of the compliance objectiveis negative of the strain energy density function whereas the shape sensitivity($\delta V/\delta\psi|_e$) of volume is one [21]

Thus the normal velocity is given by

$$v|_e = -\delta L/\delta\psi = \delta c/\delta\psi + \alpha^r \delta(V^\#)/\delta\psi + 1/2\beta^r \delta[V^\#]^2/\delta\psi \qquad (9)$$

This equation can be written in the form of

$$v|_e = -u_e^T k_e u_e - \alpha^r - 1/\beta^r * [V^\#] \qquad (10)$$

The term z which is responsible for new voids can be taken as

$$z = -\text{sgn}(\phi)\delta_T L$$

$$z \left\{ \begin{array}{ll} \delta_T L & if\ \phi < 0 \\ 0 & if\ \phi \ge 0 \end{array} \right. \qquad (11)$$

The next step is to find out the topological sensitivities of the Lagrangian, compliance and the volume V(x).[6]Found that the topological sensitivity of volume is –π whereas the

the help of [24]

$$\delta_T c(x) = \{\pi(\lambda + 2\mu)\}/\{\mu(\lambda + \mu)\}\{(4u_T ku + (\lambda - \mu)u^T(ku)\} \qquad (12)$$

δ_TL is the topological sensitivity of Lagrangian L,$\lambda$and$\mu$are the Lame's constant.By using these results we can get the value of the source term z.

RESULT AND DISCUSSIONS

Application in structures: This section shows the implementation of the level set method for partially loaded simply supported beam and rolling support,cantilever beam.

Case 1

Topology optimization of asimply supported 2D beam, fixed at both ends by using minimumcompliance. The static load applied at each node is unit. The initial configuration of the beamis rectangular without any hole. Fig. 1 shows a simple structure with supports and partially distributed load.

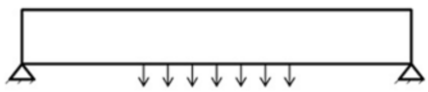

Fig: 1Simple structure with boundary conditions

The target value of reduction is set as 70%. During the optimization process, the material gradually moves out from the beam till it reaches to its target value.

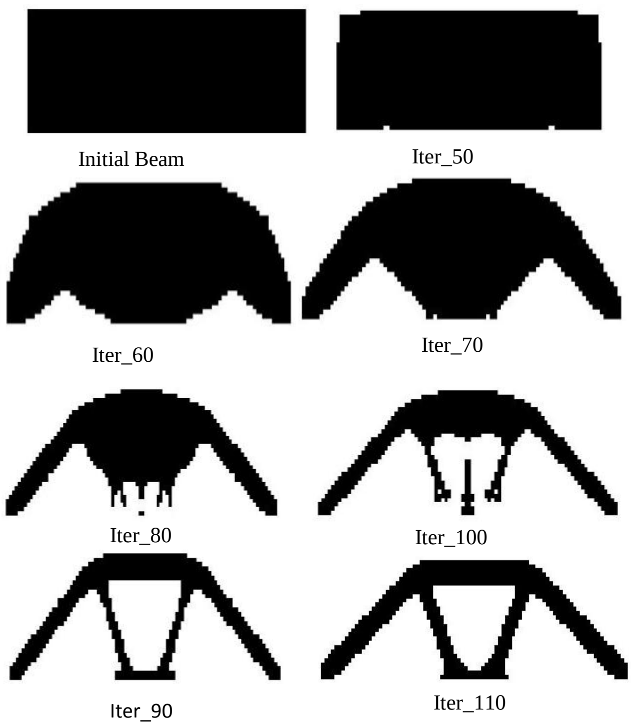

Fig: 2 Optimization results

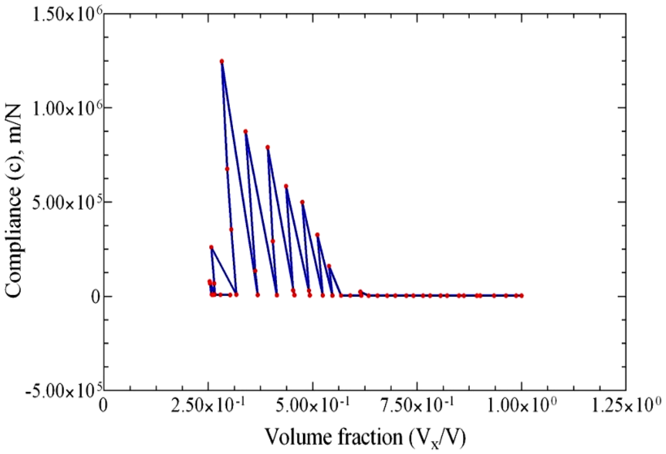

Fig. 3 : Change in volume with iteration number

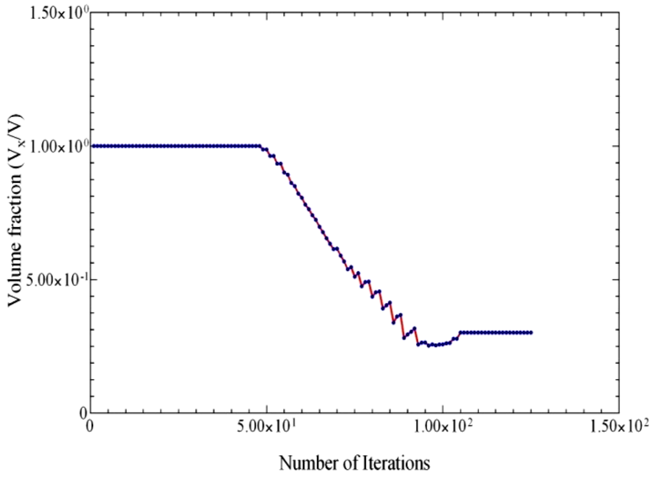

Fig. 4 : Change in volume with iteration number

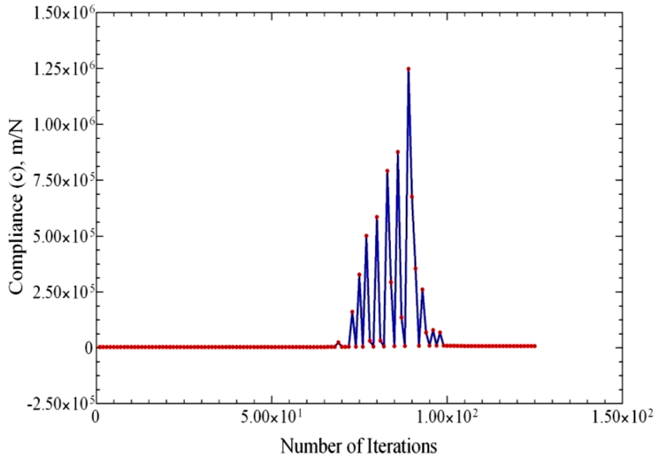

Fig. 5 : Change in compliance with iteration number

Fig. 2 shows the optimization of a simple beam. Initially the beam was in a solid (black) form. As the optimization process moves on, gradually the material comes out from the beam until the volume constraint met. In the above process, subtraction and addition of material both can takes place. This can be seen from iteration 90 and 110. Fig. 3 shows avariation in compliance with volume. In Fig.4 the variation in volume can be seen with iteration number.Fig.5 shows as the number of iteration increases, there is a random variation in the compliance. Laterit stables.

Case2

Topology optimization of a simply supported 2D beam, having one end rolling and other fixed by using minimum compliance. The static load applied at each node is unit. The initial configuration of the beam is a rectangular one without any hole. Fig. 6 shows geometry of Michell beam

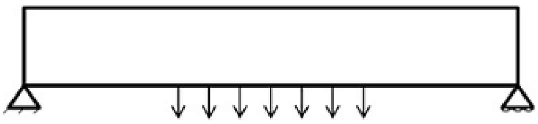

Fig. 6 :Michell beam with boundary conditions

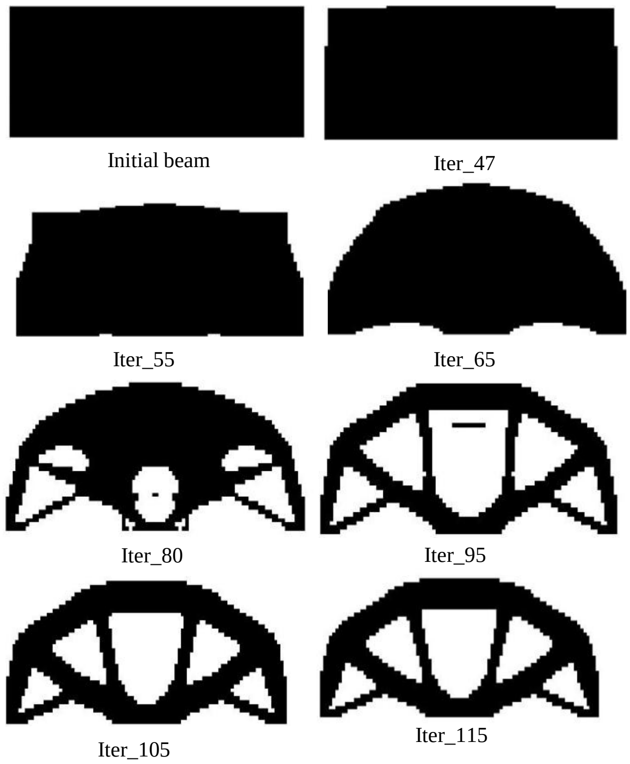

Fig. 7 : Iterative process

The target value of reduction is set as 70%. During the optimization process, the material gradually moves out from the beam till it reaches to its target value.

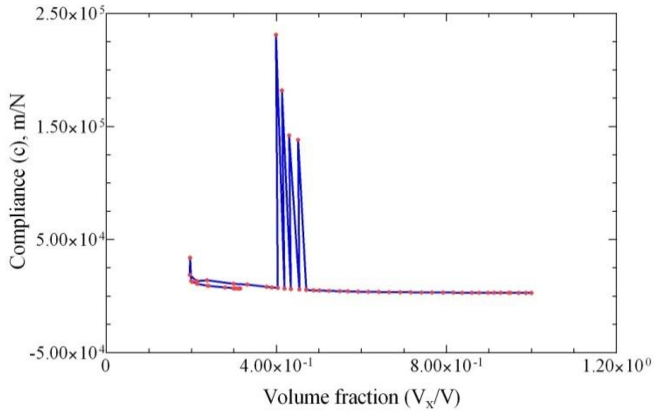

Fig. 8 : Variation in compliance with Volume

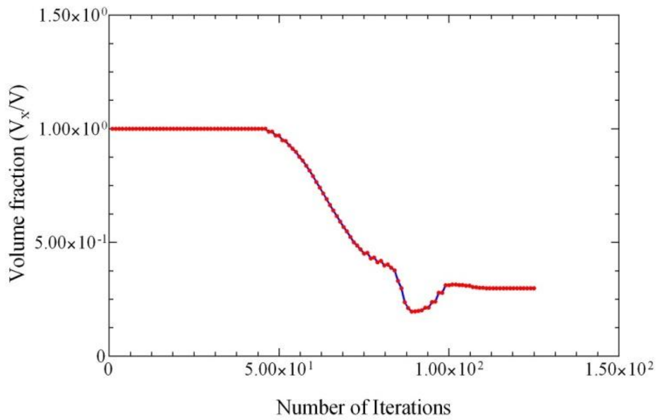

Fig. 9: Change in volume with iteration numbers

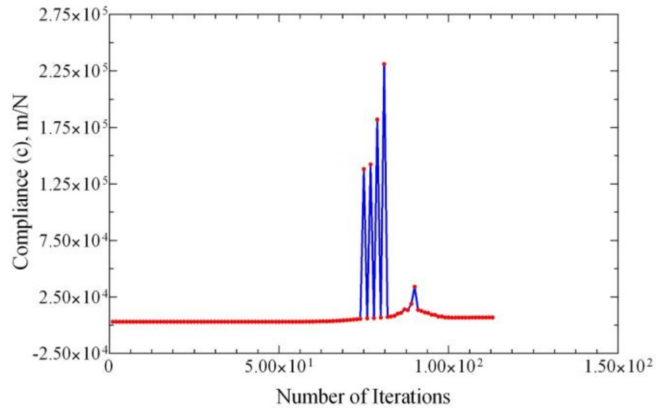

Fig. 10: Change in volume with iteration numbers

Fig. 7 shows the process of optimization of Michell beam for partially distributed load. Fig. 8, Fig. 9, Fig.10 shows the variation in compliance with volume, variation in volume with iteration number and variation in compliance with iteration number.

Case 3

The third case is a study of cantilever beam. The applied load is a partially distributed load. The target volume reduction in this case is taken as 60% of the total material.

Fig. 11: Cantilever beam with boundary conditions

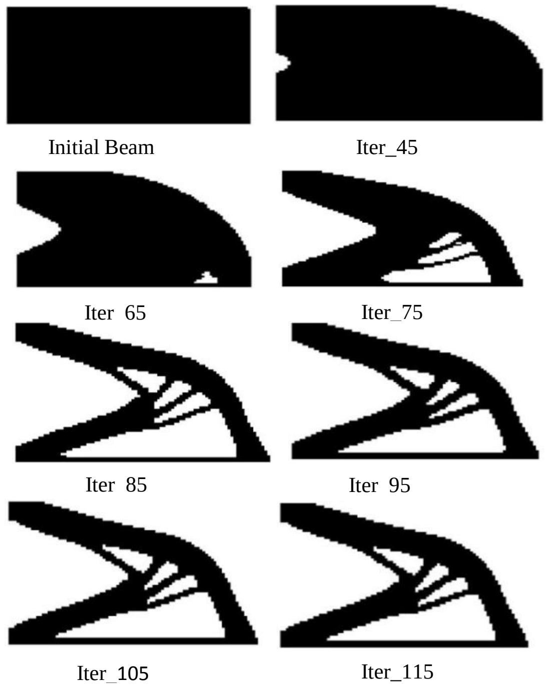

Fig. 12 : Iterative process

Figure 12 shows the optimization of a cantilever beam. It also starts with a solid beam. As the optimization process progresses, new hole creation starts. The material gradually goes off from the beam to get the targeted value of the material

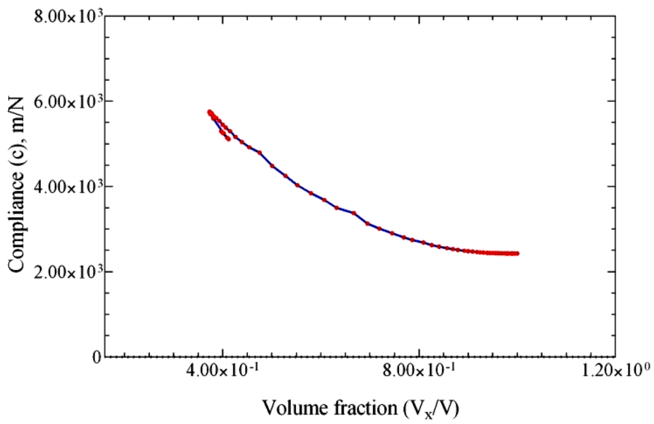

Fig. 13: Change in compliance with volume

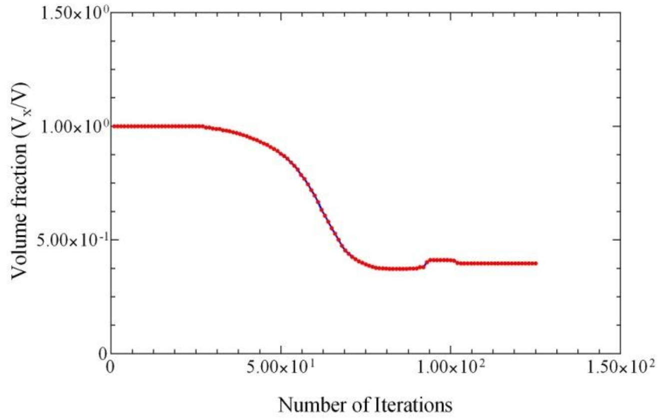

Fig.14 : Change in volume with iteration numbers

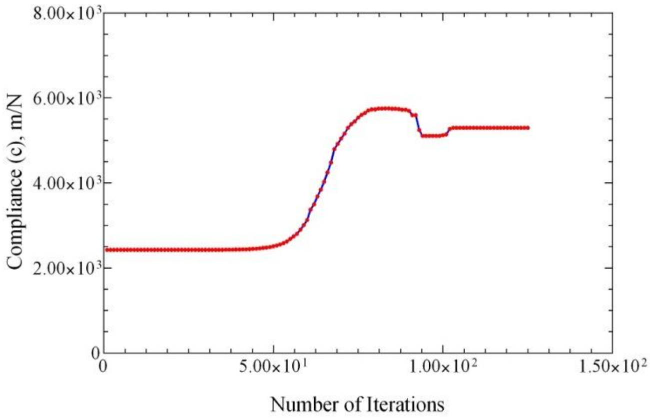

Fig. 15 : Change in compliance in iteration numbers

Fig. 13shows the variation in compliance with volume, as the iteration number goes up, the compliance also increases with volume reduction and then converges to a targeted value of volume.Fig. 14 Change in volume with iteration number. Fig. 15 shows the variation in compliance with the change in volume. It clearly shows that the solid beam has the maximum stiffness. It reduces up to a certain limit, then again optimizes for better stiffness.

CONCLUSION

The objective of this paper is to provide a brief introduction about the level set method and its application for simple structures. The major benefit of LSM is that, there is no need of pre-existing holes in the structure. It can create new voids, can merge small voids and can split bigger voids. The final solution will be 0/1 configuration without any intermediate densities. Secondly it works with moving boundaries. The level set method transforms the objective and the constraint in the form of speed of propagation of the boundary. The movements of the interfaces are governed by the Hamilton-Jacoby equation.LSM works only with the moving boundary, there is no need to pay any kind of attention inside or outside of the boundary.

Though ample amount of work has been done on application of Level Set method for structural topology optimization in recent years but still a lot of research work has to be done in this field. Till date the level set method is used for simple cases in structure optimization, which are based on density field. The physical application of this method is limited to few simple cases. For better and faster results, this method can be coupled with some other optimization method.

ACKNOWLEDGEMENT

The authors are truly thankful to Mr.P. K. Jain, Head of Department, Mechanical and Industrial Engineering Department for their valuable and significant suggestions which substantially improved the manuscript.

REFERENCES

[1] Martin Philip Bendsøe,Noboru Kikuchi, Generating optimal topologies in structural design using a homogenization method, *Computer Methods in Applied Mechanics and Engineering, Volume 71, Issue 2, Pages 197–224,* November 1988,

[2] O.M. Querin, G.P. Steven, Y.M. Xie, Evolutionary structural optimization (ESO) using a bidirectional algorithm,*Engineering Computations, Vol. 15 Iss 8 pp. 1031 – 1048,* 1998

[3] Osher, S., and Sethian, J.A., Fronts Propagating with Curvature Dependent Speed: Algorithms Based on Hamilton-Jacobi Formulations, *Journal of Computational Physics, 79, pp.12-49*, 1988

[4] Stanley J. Osher and FadilSantosa, Level Set Methods for Optimization ProblemsInvolving Geometry and Constraints. Frequencies of a Two-DensityInhomogeneous Drum, *Journal of Computational Physics 171, 272–288,* 2001

[5] Myslinski, Level set method for optimization of contact problems, *Engineering Analysis with Boundary Elements 32 986– 994*, 2008

[6] G. Allaire et al., Structural optimization using topological and shape sensitivity via a level set method,*Control and Cybernetics vol. 3*, 2005

[7] Zhen Luo, Liyong Tong,Haitao Ma, Shape and topology optimization for electro-thermo-mechanicalmicroactuators using level set methods, *Journal of Computational Physics 228 3173–3181*, 2009

[8] Shim H, Ho VTT, Wang S, Tortorelli DA., Topological shape, optimization of electromagnetic problems using level set method and radial basis function., *Comput Model EngSci 37(2):175–202,* (2008)

[9] Kao CY, Osher S, Yablonovitch E., Maximizing band gaps in two-dimensional photonic crystals by using level set methods. *ApplPhys, B Lasers Opt 81(2):235–244,* (2005)

[10] Park KS, Youn SK.,Topology optimization of shell structures using adaptive inner-front (AIF) level set method. *Struct Multidisc Optim 36(1):43–58,* (2008)

[11] M.P. Bendsoe, o. Sigmund, Topology optimization: Theory, methods and applications, *Springer, Berlin*,2003.

[12] N. P. van Dijk, K. Maute, M.Langelaar, F.vanKeulen, Level-set methods for structural topology optimization: a review . *Struct Multidisc Optim,* 2013

[13] Steffen Johnsen, Structural Topology Optimization. MS Thesis, *Norwegian University of Science and Technology*,2013

[14] Ole Sigmund,KurtMaute, Topology optimization approaches:A comparative review,. *StructMultidisciplOptim*21, 2013

[15] G. I. N. Rozvany., A critical review of established methods of structural topology optimization, *Structural and Multidisciplinary Optimization,January, Volume 37, Issue 3, pp217-237*, 2009

[16] Ole Sigmund Kurt Maute, A discrete level-set topology optimization. *Structural and Multidisciplinary Optimization, Volume 41, Issue 3, pp 453-464,*April 2010

[17] J. A. Sethian, Level set methods and fast marching methods *Cambridge University Press,* -1999

[18] J.A. Sethian, Theory, Algorithms, and Applications of Level Set Methods for PropagatingInterfaces *University of California, Berkeley,*1995

[19] Vivien J. Challis, A discrete level-set topology optimization codewritten in Matlab, Struct Multidisc Optim 41:453–464, 2010

[20] Burger M, Hackl B, Ring W., Incorporating topologicalderivatives into level set methods. J ComputPhys 194:344–362, 2004

[21] G. Allaire et al., Structural optimization using sensitivity analysis and a level-set method,*Journal of Computational Physics 194*,363–393, 2004

[22] Allaire G, Jouve F., Shape and topology optimizationof the robust compliance via the level set method, *ESAIM COCV 14(1):43–70,* 2008

[23] JunzhaoLuo, Zhen Luo, Semi-implicit level set method for structural shape and topology optimization, *Journal of Computational PhysicsVolume 227, Issue 11, Pages 5561–5581,* 2008

[24] G. Allaire et al., Structural optimization using topological and shape sensitivity via a level set method,*Control and Cybernetics vol. 34 No. 1,* 2005

[25] Lin He, Incorporating topological derivatives into shape derivatives based level set methods, *Journal of Computational Physics 225 891–909,* 2007

TABLE1: Variation in compliance and volume with iteration number

Sr. No.	Iteration Number	(case 1)		(case 2)		(case 3)	
		Compliance (m/N)	V_x/V	Compliance (m/N)	V_x/V	Compliance (m/N)	V_x/V
1	1	3325.8855	1	3009.1718	1	2430.4078	1
2	27	3325.8855	1	3009.1718	1	2430.4078	1
3	28	3325.8855	1	3009.1718	1	2430.4718	0.994
4	45	3325.8855	1	3009.1718	1	2456.8571	0.924
5	46	3325.8855	1	3009.1718	1	2462.9463	0.918
6	47	3325.8855	1	3009.9064	0.987	2473.2483	0.909
7	48	3325.8855	1	3009.9064	0.987	2483.0267	0.9
8	49	3326.0004	0.987	3012.928	0.97	2494.6585	0.892
9	50	3326.0004	0.987	3012.928	0.97	2513.5596	0.879
10	51	3329.2236	0.963	3020.3874	0.95	2531.9765	0.869
11	52	3329.2236	0.963	3020.3962	0.946	2553.1156	0.857
12	53	3339.1675	0.934	3036.5112	0.927	2588.6334	0.841
13	54	3339.1675	0.934	3042.1663	0.912	2626.5872	0.826
14	62	3578.6611	0.764	3368.1505	0.741	3501.7507	0.632
15	71	4514.0167	0.59	4743.0792	0.524	5163.7482	0.426
16	72	4788.7108	0.568	5019.1833	0.501	5297.072	0.415
17	73	161193.7156	0.539	5264.63	0.487	5386.5637	0.406
18	74	4958.8547	0.547	5585.8191	0.47	5450.9251	0.4
19	75	327469.571	0.511	138247.0879	0.451	5538.9944	0.393
20	85	6958.4508	0.414	10319.8081	0.331	5748.725	0.373
21	86	876048.1131	0.339	11003.985	0.299	5748.725	0.373
22	87	136011.861	0.362	14141.8649	0.237	5738.56	0.373
23	88	7904.7421	0.368	13430.3681	0.212	5726.5346	0.374
24	89	1247969.147	0.282	18945.2337	0.196	5726.5346	0.374
25	95	10024.7732	0.265	9247.7606	0.238	5111.9533	0.412
26	96	79392.0043	0.253	9097.9335	0.24	5111.9533	0.412
27	97	9744.5134	0.258	7731.5676	0.279	5111.9533	0.412
28	98	68535.2299	0.254	7718.2944	0.279	5111.9533	0.412
29	99	9661.0897	0.257	6858.764	0.312	5111.9533	0.412
30	100	9612.3744	0.258	6858.764	0.312	5126.6501	0.411
31	101	9306.8337	0.262	6774.7996	0.315	5145.1525	0.409
32	110	7906.3072	0.302	6909.6188	0.301	5300.0327	0.397
33	111	7906.3072	0.302	6935.178	0.299	5300.0327	0.397
34	112	7906.3072	0.302	6935.178	0.299	5300.0327	0.397
35	113	7906.3072	0.302	6935.178	0.299	5300.0327	0.397
36	114	7906.3072	0.302	6935.178	0.299	5300.0327	0.397