

# Literature Survey for Minimum Euclidean Distance Based Precoder For MIMO Systems

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## Abstract

In recent years, multiple transmitter and receiver antennas are employed in the wireless communications systems to adapt various demands of high speed wireless links. In order to take full advantage of the Multi Input Multi Output (MIMO) systems, precoding technique is needed at the transmitter side. Generally, the precoding algorithm may be classified as either diagonal or nondiagonal type. Many precoding techniques exist in literature. This literature survey deals with different types of precoders for MIMO systems in the existing methods.

Keywords: Channel state information, linear precoder, minimum Euclidean distance, Multi Input Multi Output (MIMO) systems.

## 1. Introduction

The Multi Input Multi Output (MIMO) systems used in a rich scattering environment for wireless communications improve the reliability or the data rate of transmissions significantly in comparison with Single Input Single Output (SISO) systems. These systems achieve large capacity and diversity gains in comparison with single transmitter and single receiver systems [1]. Through a feedback link, the channel knowledge can be made available at the transmitter and precoding techniques can be used to adapt the information signal to the channel and significantly improve the performance of MIMO communication. If full channel state information is considered at the transmitters, linear precoder can be designed according to various criteria, such as ergodic capacity, received signal to noise ratio (SNR) or error probability [2]. To integrate the quality of service criterion in the design, a weighted minimum mean squared error criterion subject to a transmit power constraint is proposed in [3]. A variety of criteria can be used for channel characteristics optimization such as, for example, minimizing the mean-square error (MSE) [3], maximizing the received signal to noise ratio (SNR) [4], or maximizing the minimum singular value [5]. Using singular values decomposition (SVD), a MIMO channel can be decomposed into several

independent subchannels for data transmission for each subcarrier [6]. Another efficient nondiagonal precoder minimizes the upper bound of pairwise error probability (PEP) when using arbitrary STBC over correlated Ricean fading channels [7]. A nondiagonal structure is given in [8] by maximizing the minimum distance of the symbols at the receiver side. An extension to 16-QAM modulation is found in [9] that confirm the optimality of the max- $d_{\min}$  criterion. A design of a max- $d_{\min}$  precoder allows transmitting more than two independent data streams, and increases the 4-QAM alphabet to 16-QAM or 64-QAM modulations. But, this precoding techniques is only suitable to quasi-stationary MIMO channels [11]. An optimal precoder was derived for two virtual subchannels and a cross-form matrix [12] to get the max- $d_{\min}$  precoder for any even number of data streams. The focus of this paper is to analyze various precoding techniques for MIMO systems.

## 2. Literature review

### 2.1. Weighted MMSE criterion

H. Sampath, P. Stoica, and A. Paulraj [3] proposed designing jointly optimum linear precoder and decoder for a MIMO channel with and without delay-spread, using the weighted minimum mean-squared error (MMSE) criterion subject to a transmit power constraint. The optimum linear precoder and decoder maximizes information rate or minimizes the sum of the output symbol estimation errors.

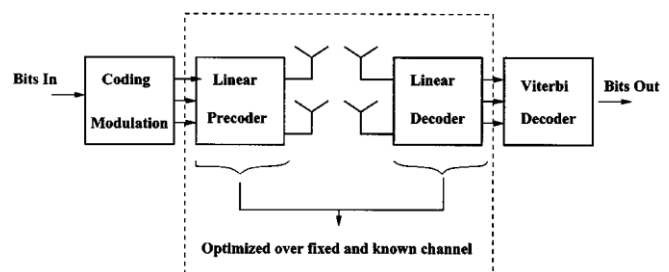


Figure. 1. MIMO communication system.

A generic MIMO communication system model is shown in Figure. 1. The input bit streams are first coded and modulated. The latter are then passed through the linear precoder which is optimized for a fixed and known channel. The precoder is a matrix with complex elements and adds redundancy to the input symbol streams to improve system performance. The precoder output is launched into the MIMO channels through  $M_T$  transmit antennas. The signals are received by  $M_R$  receive antennas and processed by linear decoder, which is optimized for the fixed and known channel. The linear decoder also operates in the complex field and removes any redundancy that has been introduced by the precoder.

This design method does not consider the coding and modulation, but instead focuses only on the design of the linear precoder and decoder. The generalized jointly optimum linear precoder and decoder minimizes any weighted sum of symbol estimation errors assuming total transmit power constraint across all transmit antennas.

The main goal of this method involves the design of precoder (F) and decoder (G) matrices. The weighted combination of symbol estimation errors is given by,  $E[e^*W^{*1/2}W^{1/2}]$  where  $e=s-(GHFs+Gn)$  is the bit error vector. In this paper, the design of equal-error and MMSE systems are also compared. These two designs have similar total average BERs. The main drawback of this system is that it is suitable for limited range of the bit streams only.

## 2.2. Space-Time Linear Precoder

A. Scaglione, P. Stoica, S. Barbarossa, G. Giannakis, and H. Sampath [5] introduced a new model for the design of transmitter space-time coding that is referred to as linear precoding. The design model is based on an optimal pair of linear transformations precoder (F) and decoder of blocks of the transmit symbols and receive samples, respectively. This operates jointly and linearly on the time and space dimensions. The design targets different criteria of optimality and constraints, assuming that the channel is known both at the transmitter and receiver ends. Channel State Information (CSI) can be acquired at the transmitter either if a feedback channel is present or when the transmitter and receiver operate in time division duplex (TDD) so that the time-invariant MIMO channel transfer functions is the same in both ways.

The specific design targets minimization of the symbol mean square error and the approximate maximization of the minimum distance between symbol hypotheses under average and peak power constraints. The first design corresponds to the MMSE criterion, whereas the second one is proved to be equivalent to the maximization of the mutual information between transmitter and receiver. The optimal

design allow us to define space-time modulation design that takes advantage of the channel state information and offers simple closed-form solutions, scalable with respect to the number of antennas, size of the coding block, and transmit average/peak power.

## 2.3. Zero Padding-MBER Precoder

Y.Ding, T. Davidson, Z. Luo, and K. Wong [19] proposed the linear precoder that minimizes the bit error rate (BER) at moderate to high signal to noise ratios for block transmission systems with zero-forcing (ZF) equalization and threshold detection. This scheme attempts to eliminate the inter-block interference (IBI) for two standard schemes that include cyclic prefix and zero padding. These two standard schemes minimum BER precoder provide substantially lower error rates than standard block transmission schemes such as orthogonal frequency division multiplexing (OFDM). For finite impulse response (FIR) channels, the equalized symbols can be simplified by appropriate choice of the block size and redundancy. During symbol transmission, IBI is eliminated by adopting either zero padded or cyclic-prefix-based transmission.

The channel is completely known and ZF equalization is employed at the receiver. The transmitted symbols are equiprobable antipodal symbols uncorrelated with each other. The noise vector is zero-mean, white and Gaussian, with covariance matrix. The advantage of this scheme is that this design scheme is flexible because it applies directly to both the zero-padding and cyclic-prefix schemes for avoiding inter-block interference and allows the block sizes to be chosen liberally. The designs improve the BER performance and also the minimum BER precoder has a simple logical form whereas the water-filling-based designs require the solution of a nonlinear optimization problem using an iterative algorithm. The optimal design obtained in this paper is for a single-user system, with ZF equalization and threshold detection, white uncoded data, white noise, uniform bit loading, and a known channel. This method does not deal with MIMO system and deals only single-input single-output systems.

## 2.4. Decision Feedback Equalization / Tomlinson- Harashima Precoding

M. Shenouda, and T. Davidson [16] introduced joint transmitter and receiver design for MIMO systems with Decision Feedback Equalization (DFE) or Tomlinson-

Harashima (TH) precoding. In this paper, a broad range of design criteria is considered that can either be expressed as Schur-convex or Schur-concave functions of the logarithm of the MSE of each data stream. Optimal design for Schur-convex objectives is that it simultaneously minimizes the total MSE, the average bit error rate and maximizes the Gaussian mutual information. These objectives are not valid for a linear transceiver. For Schur-concave objectives, the optimal DFE design results in linear equalization and the optimal TH precoding design results in linear precoding.

The design of the precoding matrix optimizes the design criteria that are expressed as a function of the individual MSE of each stream. In order to characterize the optimal precoder, two inequalities are derived in this paper. These inequalities depend on multiplicative and additive majorization. The slight advantages of the TH transceiver over the DFH transceiver can be attributed to the fact that interference subtraction at the transmitter is inherently free from error propagation.

## 2.5. Three dimensional max- $d_{\min}$ Precoder

Q.-T. Ngo, O. Berder, and P. Scalart [10] proposed the new parameterized form of the precoder for three virtual subchannels. A new precoder for MIMO transmission, which is based on the maximization of the minimum Euclidean distance between received symbols, has been introduced for BPSK and QPSK modulation following the max-SNR approach, which consists in pouring power only on the most special virtual sub-channel.

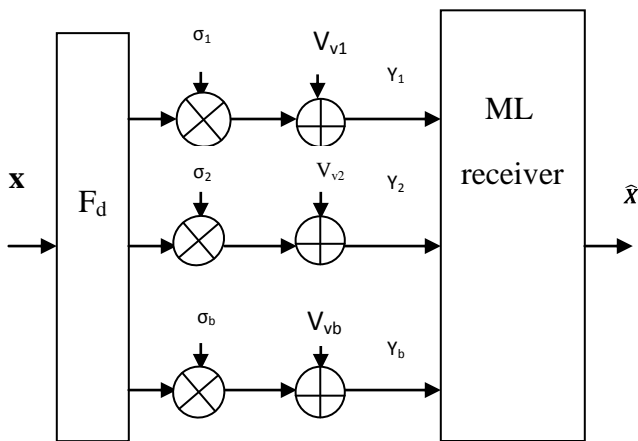


Figure. 2. MIMO block diagram with nondiagonal precoder in case of CSI

For a MIMO system with  $n_R$  receive,  $n_T$  transmit antennas and  $b$  independent data stream, the basic system model can be expressed as

$$y = GHFx + Gv \quad (1)$$

where  $H$  is the  $n_R \times n_T$  channel matrix,  $F$  is the  $n_T \times b$  precoder matrix,  $G$  is the  $b \times n_R$  decoder matrix,  $x$  is the  $b \times 1$  transmitted symbol vector, and  $v$  is the  $n_R \times 1$  additive noise vector,  $\sigma_i$  stands for every subchannel gain,  $v_v$  is the  $b \times 1$  trasformed additive noise vector.

The MIMO system of a nondiagonal precoder is shown in Figure. 2. At the receiver side, an ML detection is considered so the decoder matrix  $G_d$  has no impact on the performance and is consequently assumed to be  $G_d = I_b$ , with  $I_b$  is the identity matrix of size  $b \times b$ . By using singular value decomposition, the precoding matrix in  $F_d^{\text{dmin}}$  can be performed as

$$F_d = A \Sigma B^* \quad (2)$$

where  $A$  and  $B^*$  are  $3 \times 3$  unitary matrices, and  $\Sigma$  is a  $3 \times 3$  diagonal matrix with real positive values in decreasing order.

This precoder is suited for BER minimization in MIMO systems when maximum-likelihood detection is used at the receiver side. The performance improvement of SNR-like max- $d_{\min}$  precoder depends on the channel characteristics and is less significant if the virtual sub-channels are small dispersive.

## 2.6. New max- $d_{\min}$ Precoder

Q.-T. Ngo, O. Berde, and P. Scalart [20] introduced a new general expression of the minimum Euclidean distance based precoder for MIMO systems using rectangular QAM modulations. For two independent data streams transmission, the MIMO channel is diagonalized by using a virtual transmission and the precoding matrix is obtained by optimizing the minimum distance on both virtual subchannels. Then the optimized expression is reduced to two simple form precoders  $F_1$  and  $F_2$ . These precoding matrices provide optimized minimum distance for any dispersive channels.

The precoder  $F_1$  pours power only on the strongest virtual subchannel and the precoder  $F_2$  uses both virtual subchannels to transmit data symbols. The precoding matrix  $F_d$  can be represented as

$$F_d = \sqrt{E_s} \begin{pmatrix} \cos \psi & 0 \\ 0 & \sin \psi \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} \quad (3)$$

where  $E_s$  is the average transmit power,  $\psi$  controls the power allocation,  $\theta$  &  $\varphi$  correspond to scaling and rotation of the received constellation. Since the precoder  $F_1$  pours power only on the strongest virtual subchannel, then the precoding matrix  $F_1$  becomes

$$F_1 = \sqrt{E_s} \begin{pmatrix} \cos \theta & \sin \theta e^{i\varphi} \\ 0 & 0 \end{pmatrix} \quad (4)$$

It is found that the precoder  $F_1$  provides a slight improvement in term of  $d_{\min}$  in comparison with the beamforming design. The precoder  $F_2$  uses both virtual subchannels. The precoding matrix  $F_2$  becomes

$$F_2 = \frac{\sqrt{E_s}}{2} \begin{pmatrix} \cos \psi & 0 \\ 0 & \sin \psi \end{pmatrix} \begin{pmatrix} \sqrt{2} & 1+i \\ -\sqrt{2} & 1+i \end{pmatrix} \quad (5)$$

The precoder  $F_2$  optimizes the distance  $d_{\min}$  when there is no dispersion between both virtual subchannels. The precoder  $F_1$  provides the optimized  $d_{\min}$  for small channel angle  $\gamma$ , while precoder  $F_2$  is valid for high values of the channel angle  $\gamma$ . This precoder can also be extended for large MIMO channels.

### 3. Conclusion

Thus a detailed survey of various precoding schemes present in the literature for various modulation schemes is discussed elaborately for multiple input- multiple output (MIMO) systems.

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