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Load Flow Solution Using Simplified Newton-Raphson Method

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Abstract

The power flow analysis is of great importance in planning and designing for the future expansion of power systems as well as in determining the best operation of existing systems. There exist two widely-used numerical methods (the Gauss–Seidel: GS and the Newton–Raphson: NR) to solve this problem and therefore referred to as the GS and the NR power-flow solution methods, respectively. Although the standard Newton-Raphson (NR) method is the most powerful algorithm for the power flow analysis in electric power systems, the calculation of Jacobian matrix derivatives involves high computational time. The proposed method presents a simplified Newton-Raphson power flow solution method to simplify overall equation complexity and computation time. The simplified Newton-Raphson method employs nonlinear current mismatch equations instead of the commonly used power mismatch equations. Numerical results are presented with 5-bus test system and IEEE 30-bus test system and compared with standards NR method.

1. Introduction

The main function of electric power systems is to deliver electric energy to its loads sufficiently, efficiently and economically. The steady-state performances of an interconnected power system during normal operation can be analyzed based on nonlinear nodal analysis to form power flow equations and must be solved by some efficient iterative methods [1–9]. Power flow analysis is commonly used as a part of power system operation and planning. Since AC power-flow solution methods were first developed over half a century ago, there exist two widely-used numerical methods (the

Gauss–Seidel (GS) and the Newton–Raphson (NR) to solve this problem and therefore referred to as the GS and the NR power-flow solution methods, respectively. As broadly known, the NR method has been successfully developed and accepted as the most powerful algorithm for the power flow analysis in electric power systems. In large-scale power systems containing several hundred or up to thousand buses, the standard NR method gives a slow execution time due to a large updated Jacobian matrix that needs to be recalculated and factorized at each iteration [10, 11]. Consequently, the de-coupled and fast de-coupled power flow versions [12, 13] were released. Hence, the power-flow solution can be obtained faster. This method is very useful in practical power system analyses, e.g. contingency analysis, on-line power flow control, etc. [4, 14].

Having a long history of development gives power flow algorithms a vast number and a various kind of applications. Enhancing the algorithm efficiency of power flow calculation has been carried out in many different approaches. Network partition technique can separate a whole power system into subsystems, therefore power flow solution of the complete system can be obtained by direct coupling of solutions from separate subsystems [15] based on the GS method. This concept is very useful to parallelize power flow algorithms in order to implement a parallel and sequential power flow performing on a computer cluster whether the GS method or some other numerical methods such as successive over-relaxation (SOR) method is used as the main solver [16, 17]. In some point of view, an initial guest solution of power flow calculation is one of key factors that cause slow computation.

In [18, 19], an initial linear solution based on the decoupling principle of real and reactive power decomposition was utilized as the starting point to the power flow calculation. In addition, there are some modified versions of the NR power flow method to handle ill-conditioned power systems [20, 21]. The calculation algorithm has been continually developed by several researchers across the world. A complex form of the power flow calculation was introduced for a three phase unsymmetrical power-flow solution [22, 23]. Power-flow solutions based on a local search method were claimed [24] to be robust and be applicable to those cases in which conventional power flow method failed. Due to advancement of FACTS technology, the power flow equations were modified and rewritten into current-injected forms for the incorporation of FACTS devices and any kind of control strategy [25, 26]. Moreover, the

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study of power-flow solution methods for particular applications, e.g. economic dispatch [27], optimal power flow [28], FACTS devices [29, 30], and AC/DC power systems [31], was reported.

Over several decades, electrical power systems have been characterized using the nodal analysis to solve for a set of voltage solutions. In general, electrical demands are defined in constant power. This leads to non-linearity of nodal voltage equations. To date, the standard NR power flow method is one of the most powerful algorithms, which has long history of development, and is widely used to develop commercial power-flow solution software. Although the standard NR power flow method is very efficient and commonly used for the power flow calculation in several power system textbooks [1–9], to formulate iterative Jacobian updating matrix equations requires complicated formulae and long expressions. In this paper, the iterative NR method is still employed as the main solution framework. The essential difference is that the proposed algorithm is to find roots of the current mismatch equations instead of those of the power mismatch equations. This approach can simplify a very long and complicated mathematical formula to a very simplistic and short mathematical expression. With this simplification, reduction of the overall execution time is expected. To achieve this goal, expressions to obtain elements of Jacobian updating matrix formulae must be derived.

In this paper presents the formulation of the proposed NR power flow problem. Derivation of the Jacobian updating matrix elements is included and the floating-point operation counting to evaluate its computational effort. Numerical examples are selected to observe the effectiveness of the proposed method.

2. Formulation of proposed simplified power flow solution

The power flow problem is a zero-finding problem to determine voltage solutions of nonlinear power mismatch equations [1]. If alternative nonlinear current mismatch equations are selected and used as functions of estimating roots. Given that an n-bus power system, which bus number 1 is assigned to be a slack bus of constant voltage magnitude and zero phase angle. Considering the i^{th} bus, current balance equations characterizing this bus can be expressed as follows. In this method, the set of nonlinear equations are formulated based on current mismatch equations. The mathematical equations for simplified Newton-Raphson method are as follows [34].

The current balance equation at bus *I is*

$$I_i = \sum_{k=1}^{n} Y_{ik} V_k$$

$$(I_{g,i} - I_{d,i}) - \sum_{k=1}^{n} Y_{ik} V_k = 0 \tag{1}$$

In practice, loads in electrical power systems are in form of powers, therefore it is convenient to rewrite eqn. (1) into a function of powers as follows.

$$F_i = \left(\frac{S_{g,i} - S_{d,i}}{V_i} \right)^* - \sum_{k=1}^{n} Y_{ik} V_k = 0 \tag{2}$$

Define $F_i = G_i + jH_i$ be the current mismatch at bus *i*,

$V_i = |V_i| \angle \delta_i$; $V_k = |V_k| \angle \delta_k$

$Y_{ij} = |Y_{ij}| \angle \theta_{ij}$ is

$S_{g,i} - S_{d,i} = S_{sch,i} = |S_{sch,i}| \angle \phi_i$

Expressing eqn. (2) in polar form,

$$\left(\frac{|S_{sch,i}| \angle \phi_i}{|V_i| \angle \delta_i} \right)^* - \sum_{k=1}^{n} |Y_{ik}| \angle \theta_{ik} |V_k| \angle \delta_k = 0 \tag{3}$$

Separating the real and imaginary parts,

$$G_i = \frac{|S_{sch,i}|}{|V_i|} \cos(-\phi_i + \delta_i) - \sum_{k=1}^{n} |Y_{ik}| |V_k| \cos(\theta_{ik} + \delta_k) = 0 \tag{4}$$

$$H_i = \frac{|S_{sch,i}|}{|V_i|} \sin(-\phi_i + \delta_i) - \sum_{k=1}^{n} |Y_{ik}| |V_k| \sin(\theta_{ik} + \delta_k) = 0 \tag{5}$$

Eqns. (4) and (5) constitute a set of nonlinear algebraic equations in terms of the independent variables, voltage magnitude in per unit and phase angle in radians. Expanding eqns. (4) and (5) in Taylor's series about the initial estimates and neglecting all higher order terms results a set of linear equations. In short form, it can be written as

$$\begin{bmatrix} G \\ H \end{bmatrix} = \begin{bmatrix} \frac{\partial G}{\partial \delta} & \frac{\partial G}{\partial |V|} \\ \frac{\partial H}{\partial \delta} & \frac{\partial H}{\partial |V|} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}$$

$$\begin{bmatrix} G \\ H \end{bmatrix} = \begin{bmatrix} A_1 \, A_2 \\ A_3 \, A_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \tag{6}$$

The elements of sub matrices A_1, A_2, A_3, and $A4$ can be derived in the similar manner as jacobian matrix of the standard NR method, which are the partial derivates of eqns. (4) and (5) with respect to δ and $|V|$.

The equations are summarized as, the diagonal and the off-diagonal elements of A_1 are

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$$\frac{\partial G_i}{\partial \delta_i} = \frac{|S_{sch,i}|}{|V_i|}\sin(-\phi_i + \delta_i) - |V_i||Y_{ii}|\sin(\theta_{ii} + \delta_i) \quad (7)$$

$$\frac{\partial G_i}{\partial \delta_k} = -|V_k||Y_{ik}|\sin(\theta_{ik} + \delta_k) \quad (8)$$

The diagonal and the off-diagonal elements of A_2 are

$$\frac{\partial G_i}{\partial |V_i|} = \frac{|S_{sch,i}|}{|V_i^2|}\cos(-\phi_i + \delta_i) + |Y_{ii}|\cos(\theta_{ii} + \delta_i) \quad (9)$$

$$\frac{\partial G_i}{\partial |V_k|} = |Y_{ik}|\cos(\theta_{ik} + \delta_k) \quad (10)$$

The diagonal and the off-diagonal elements of A_3 are

$$\frac{\partial H_i}{\partial \delta_i} = -\frac{|S_{sch,i}|}{|V_i|}\cos(-\phi_i + \delta_i) + |V_i||Y_{ii}|\cos(\theta_{ii} + \delta_i) \quad (11)$$

$$\frac{\partial H_i}{\partial \delta_k} = |V_k||Y_{ik}|\cos(\theta_{ik} + \delta_k) \quad (12)$$

The diagonal and the off-diagonal elements of A_4 are

$$\frac{\partial H_i}{\partial |V_i|} = \frac{|S_{sch,i}|}{|V_i^2|}\sin(-\phi_i + \delta_i) + |Y_{ii}|\sin(\theta_{ii} + \delta_i) \quad (13)$$

$$\frac{\partial H_i}{\partial |V_k|} = |Y_{ik}|\sin(\theta_{ik} + \delta_k) \quad (14)$$

The new estimates for bus voltages are

$$\delta_i^{(t+1)} = \delta_i^{(t)} + \Delta\delta_i^{(t)} \quad (15)$$

$$|V_i^{(t+1)}| = |V_i^{(t)}| + \Delta|V_i^{(t)}| \quad (16)$$

The process is continued until the current mismatch $M_i^{(t)}$ and $N_i^{(t)}$ are less than the specified accuracy, i.e.,

$$|G_i^{(t)} \; \& \; H_i^{(t)}| \le \varepsilon \quad (17)$$

To compare the effectiveness of the proposed NR method against the standard NR method, expressions of the Jacobian matrix elements of A_1, A_2, A_3 and A_4, the calculated real and imaginary current matrix elements of G and H, and the calculated real and reactive power matrix elements of P_{cal} and Q_{cal} need to be evaluated using the floating point operation.

3. FLOPs Evolution

The execution time of the power flow calculation depends on the amount of floating-point operations (FLOPs) [34, 35]. Assume that other steps of the two NR methods are exactly the same, therefore the Jacobian updating step dominates the overall execution time. In general, the time consumed to perform multiplication and division is about the same, but is larger than addition and subtraction. Hence, the operation counting of addition FLOPs is negligible. Throughout this paper, FLOPs always means the multiplication FLOPs for short and it is employed to evaluate the computational effort of the proposed algorithm. The amount of FLOPs required by each method to formulate Jacobian matrices is summarized in Table 1 and where O(n) means "terms of order n".

Table 1 Number of FLOPs

Sub-matrix		Number of FLOPs	
		Standard NR	**Proposed NR**
J_1	Diagonal	3×(n-1)	4×(n-2)
	Off-diagonal	$3n^2$+O(n)	2×(n-2)
	Total	$3n^2$+O(n)	6×(n-2)
J_2	Diagonal	2×(n-1)+3	4×(n-2)
	Off-diagonal	$2n^2$+O(n)	(n-2)
	Total	$2n^2$+O(n)	5×(n-2)
J_3	Diagonal	3×(n-1)	4×(n-2)
	Off-diagonal	$3n^2$+O(n)	2×(n-2)
	Total	$3n^2$+O(n)	6×(n-2)
J_4	Diagonal	2× (n-1)+3	4×(n-2)
	Off-diagonal	$2n^2$+O(n)	(n-2)
	Total	$2n^2$+O(n)	5×(n-2)
Overall		$10n^2$+O(n)	22×(n-2)

As a total number of buses n gets larger, the number of FLOPs grows quadratically in the standard NR method. Interestingly, the FLOP number required by the proposed NR method is linearly proportional to the total number of buses n. Fig. 1 shows the amount of FLOPs required by the two methods.

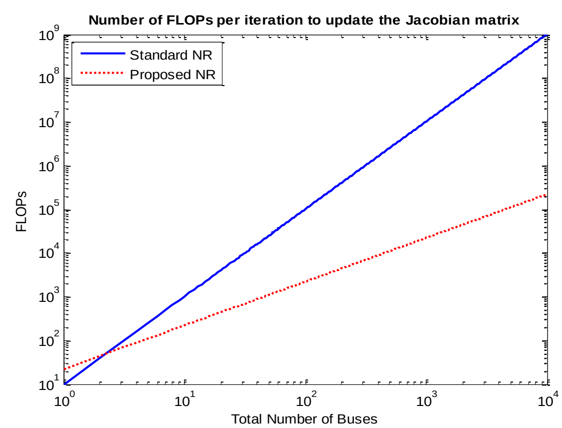

Fig. 1 Number of FLOPS per iteration to update the Jacobian Matrix

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4. Results and Analysis

The effectiveness of the simplified Newton–Raphson power flow method was tested against 5-bus [2] and 30-bus [1] IEEE test systems. Each individual test was performed by using Intel i5 Processer in which the power flow programs were coded in MATLAB [35]. From the computer simulation, the voltage solution of each test case was calculated. Both NR power flow methods used here took 1×10^{-6} per-unit as the termination criteria for the maximum allowable voltage tolerance.

The 5-bus power systems voltages and line losses are calculated using proposed simplified NR method. The obtained results are compared with the solution of existing standard NR method, has shown in the Table 2 and Table 3 respectively and observed that the results are nearly matched. The 30-bus systems voltages of standard NR method and simplified NR method are given in Table 4.

Table 2 Voltages for the 5-Bus System

Bus No	Standard NR Voltage		Proposed NR Voltage	
	|V| p.u.	Angle Deg.	|V| p.u.	Angle Deg.
1	1.0600	0.0000	1.0600	0.0000
2	1.0000	-2.0612	1.0000	-2.0502
3	0.9872	-4.6367	0.9872	-4.6286
4	0.9841	-4.9570	0.9841	-4.9483
5	0.9717	-5.7649	0.9717	-5.7547

Table 3 Line flows and line losses details for the 5-Bus System

From Bus	To Bus	Standard NR Line Losses		Proposed NR Line Losses	
		MW	MVAr	MW	MVAr
1	2	2.486	1.087	2.479	1.065
1	3	1.518	-0.692	1.515	-0.701
2	3	0.360	-2.871	0.360	-2.869
2	4	0.461	-2.554	0.462	-2.552
2	5	1.215	0.729	1.215	0.730
3	4	0.040	-1.823	0.040	-1.823
4	5	0.043	-4.652	0.043	-4.653
	Total	6.122	-10.777	6.114	-10.803

For 5-bus system the power mismatch is 9.82099e-010 and number of iterations is 4 in standard NR method where as in current mismatch is 4.17729e-007 and number of iterations is 5 in proposed NR method. For 30-bus system the power mismatch is 4.6806e-008 and number of iterations is 4 in standard NR method whereas in current mismatch is 1.16066e-007 and number of iterations is 8 in proposed NR method. The fig. 2 and fig. 3 shows the power and current mismatches with respect to standard NR method and proposed simplified NR method

Table 4 Voltages for the 30-Bus System

Bus No	Standard NR Voltage		Proposed NR Voltage	
	|V| p.u.	Angle Deg.	|V| p.u.	Angle Deg.
1	1.0600	0.0000	1.0600	0.0000
2	1.0430	-5.3504	1.0430	-5.0522
3	1.0205	-7.5309	1.0210	-7.1807
4	1.0115	-9.2830	1.0120	-8.8467
5	1.0100	-14.1684	1.0100	-13.4622
6	1.0100	-11.0625	1.0103	-10.5093
7	1.0022	-12.8651	1.0024	-12.2493
8	1.0100	-11.8154	1.0100	-11.1283
9	1.0499	-14.1031	1.0501	-13.5661
10	1.0432	-15.6944	1.0434	-15.1660
11	1.0820	-14.1031	1.0820	-13.5661
12	1.0565	-14.9577	1.0566	-14.4731
13	1.0710	-14.9577	1.0710	-14.4731
14	1.0415	-15.8500	1.0416	-15.3593
15	1.0367	-15.9373	1.0369	-15.4407
16	1.0432	-15.5301	1.0434	-15.0270
17	1.0382	-15.8606	1.0384	-15.3399
18	1.0268	-16.5480	1.0270	-16.0400
19	1.0241	-16.7193	1.0243	-16.2044
20	1.0281	-16.5205	1.0283	-16.0022
21	1.0308	-16.1386	1.0310	-15.6101
22	1.0313	-16.1243	1.0316	-15.5959
23	1.0259	-16.3233	1.0261	-15.8134
25	1.0199	-16.4931	1.0201	-15.9651
26	1.0162	-16.0730	1.0164	-15.5237
27	0.9985	-16.4936	0.9987	-15.9441
28	1.0066	-11.6871	1.0067	-11.1059
29	1.0025	-16.7844	1.0029	-16.2214
30	0.9911	-17.6688	0.9914	-17.1052

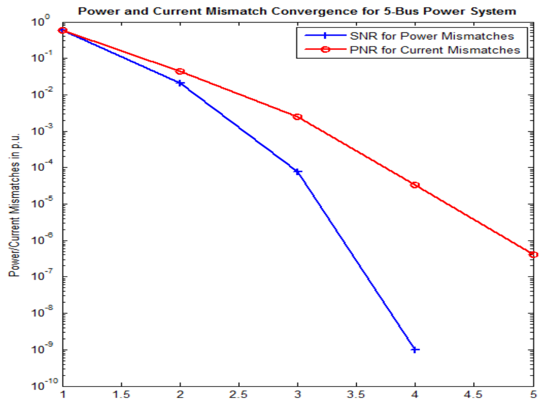

Fig. 2 Power and Current Mismatches for 5-Bus Power System

Table 4.7 is the summary of the effectiveness of the proposed method by giving the required iteration and calculation time in comparison with those of the standard NR method. It notes that, in Table 5, SNR and PNR denote the standard NR method and the proposed NR method, respectively.

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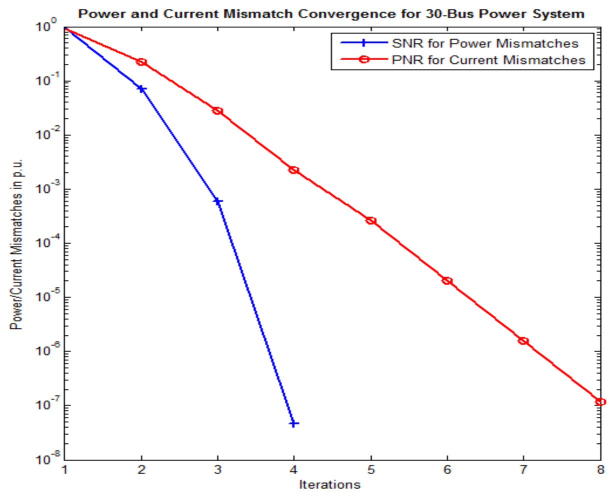

Fig. 3 Power and Current Mismatches for 30-Bus Power System

Table 5 Simulation result for required iteration and time of computation

Test system	Method	Required Iterations	Execution time (s)	Calculating time ratio
5-Bus	SNR	4	0.0231	1.5197
	PNR	5	0.0152	-
30-Bus	SNR	4	0.1686	1.3255
	PNR	8	0.1272	-

From the Table 5, the PNR method spends shorter calculation times for all test cases even the though the test cases iteration high in PNR method compared with SNR. Undoubtedly, the PNR method is faster for these two test cases. Since the PNR method takes less requirement of re-calculation in its Jacobian matrix per iteration, the calculation time ratios for these three test cases are remarkably larger with a factor of 1.5197 and 1.3255 respectively.

5. Conclusions

Power flow calculation is one of the most essential parts in electric power system operation in order to analyze, simulate, design and control the steady-state system performances properly. Although there exist several powerful power flow solvers based on the standard NR method, their problem formulation gives complication due to the need to calculate derivatives in the Jacobian matrix. The proposed method uses nonlinear current mismatch equations instead of the commonly-used power mismatches to simplify overall equation complexity. With performance evaluation found in session 3, a total number of operations required by the proposed NR method is linearly proportional to the size of the Jacobian matrix, while that of the standard NR method is quadratic. This means that the calculation time of the standard NR method increases more rapidly as a total bus number increases than that of the proposed NR method does. From this advantage, the calculation time consumed by the proposed NR method is expected to be less than that of the standard one. This can leads to improvement of power-flow software development in fast computational speed and less memory usage.

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