Location Optimization of Sandwiched Piezoelectric Shear Actuator for Maximum Deflection using ANSYS

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***Abstract*—The commonly used configuration of bonding the extension actuators at the top and the bottom of a substrate layer to induce the flexural vibrations suffers the inherent disadvantage of de-bonding and high strain due to the location of the piezoelectric patches being far away from the neutral layer. The sandwiched piezoelectric shear actuator found to address this issue adequately. The specific location of piezoelectric patch for different mode of excitation plays a vital role in maximizing the deflection obtained.**

This paper presents the finite element modeling for location optimization of cantilever piezoelectric sandwich beam excited by shear actuation. The sandwich beam is formed by sandwiching the piezoelectric patch of a certain length and rest by foam between the two metal substrates. The finite element software ANSYS is used for modeling, meshing and analysis. The piezoelectric patch is polarized along the axial direction, while the sinusoidal electric field is applied along the thickness to induce shear vibrations. The optimum location of the piezo patch for the maximum tip deflection/central deflection of the beam is determined for the first four resonant frequencies. A comparison is also performed with the cantilever beam utilizing extension actuators. The methodology and the numerical results presented herein shall be helpful in developing the new applications based on sandwiched shear actuators.

Keywords—Sandwich beam, Shear actuation, ANSYS, Location optimization.

I.INTRODUCTION

When stress is applied on some of the materials they produce electric charge generally voltage and vice versa, this phenomenon is called piezoelectricity. The materials may be some specific crystals, ceramics and biological variants like bones, DNA, proteins. In crystal physics the evolution of piezoelectricity as a part of research was introduced by brothers Jacques Curie (1856–1941) and Pierre Curie (1859–1906) [1, 2]. The observed effect was named as piezoelectricity by Hankel [3]. On the basis of fundamental thermodynamic principle Lippman [4] has provided the existence of the piezoelectric converse effect just after the year of discovery of direct piezoelectric effect and the converse effect was verified by the curie brothers in the end of 1881. The use of piezoelectric materials was limited until World War I and at that time for detecting submarines with the use of echolocation, quartz was used in sonar for ultrasound sources as a resonator. These materials are now extensively used in form of actuator and sensor for various purposes. The evolution of the easily manufactured piezoelectric ceramics with amazing performance naturally created a big and intense development of piezoelectric devices.

For obtaining a high displacements, piezoelectric materials generally actuators are stacked in some layers. This stacking provides high displacements but also results in the failure of the system as stacked layers will produce large displacements and accordingly the stress will also increase and will cause stress concentration at the interfaces. As a result crack will form and propagate. End results are failure of actuator due to debonding. Another reason for actuator debonding in case of extension actuator is stated by Benjeddou et al.[5]; that the surface mounted actuators working on d_{31} piezoelectric constant produces boundary concentrated forces and moments in the structure.

The disadvantage of actuator debonding is resolved by the use of shear actuator [5]. The sandwich shear actuators working on d_{15} piezoelectric constant, produces distributed moments in the structure. Hence the chances of the actuators debonding are very rare for the shear actuation mechanism. I t is also common known fact that for almost all the piezoceramic materials the value of shear coupling coefficient d_{15} is much higher as compared to d_{31} and d_{33}.

Later Benjeddou *et al.* [6] also demonstrated that for stiff structure and thick piezoelectric actuators shear actuation mechanism is better. In [7] the theoretical and numerical comparison of extension and shear actuated mechanism for dynamic and static control of beams was presented. Sun and Zhang [8] constructed an adaptive sandwich structure using piezoelectric material in shear mode. They also performed comparative study of sandwich structure and surface mounted actuation structure by the help of finite element analysis. They concluded that sandwich structure given many advantages as compared to conventional surface mounted actuation structures. Zhang and Sun [9] constructed a new adaptive sandwich structure using piezoelectric material in shear mode. Variational principle was used to derive the Governing equation of proposed beam. Finite element analysis was used to verify theoretical formulation. Khdeir and Aldraihem [10] investigated the piezoelectric actuation performance of the smart beams in shear and extension mode. The beam models were based on first order and higher order beam deflection theories. They studied the effect of actuator location and length on the deflection shape of the beam and concluded that large deflection in case of shear patches are near the support and for extension patch it is at center. Raja et al. [11] derived FE formulation using quasistatic equation of piezoelectricity for modeling two different types of actuator in composite sandwich beam. They concluded that for same control effort shear actuator is more efficient than extension actuator for controlling vibration. Vel and Baillargeon [12] presented an exact analysis and active vibration reduction of laminated composite plate with piezoelectric shear actuators and sensors embedded in it. Positive position feedback and velocity feedback was implemented for suppression of active vibrations. They concluded with significant reduction in tip acceleration of beams and setting time. Baillargeon and Vel [13] assessed experimentally and numerically the vibration suppression of smart structure with piezoelectric actuator and sensor. Vibration suppression was achieved by using positive position feedback and strain rate feedback. Wang and Quek [14] provided the basic mechanics model of sandwich beam embedded with piezoelectric layer for flexural analysis. In the formulation Maxwell equations was used to obtain the distribution of piezoelectric potential.

The present paper aims with the determination of optimum location of piezoelectric shear actuator which is sandwiched in between metal substrate for maximum deflection on cantilever beam. The analysis will be performed on the Finite Element software ANSYS. The previous results will be verified for ensuring the accuracy of the method used in the present analysis. The results of extension actuator are also considered in order to provide a better comparison between the extension and the shear actuators. To the authors' best knowledge, the location optimization of sandwiched piezoelectric shear actuator for maximum deflection using ANSYS has not been reported in literature before.

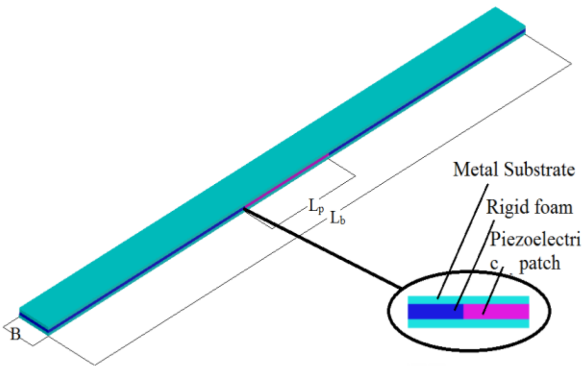


Fig. 1. Geometry of shear actuated piezoelectric sandwich beam.

II. GEOMETRIC SPECIFICATIONS

Fig. 1 shows a shear actuated sandwiched piezoelectric beam. The typical dimensions of the beam and piezo patch utilized in the work are given in Table I.

TABLE 1. GEOMETRIC SPECIFICATION OF PIEZOELECTRIC SANDWICHED BEAM.

Component	Length (mm)	Width (mm)	Height (mm)
Each substrate layer	400	25	0.5
PZT patch	70	25	1
Foam	330	25	1

III.CONSTITUTIVE EQUATIONS

The constitutive equations tells about how a material is stressed when subjected to strain and for electrical sense it tells that when a dielectric material is subjected to electrical voltage then how charges moves in it. The piezoelectricity is defined mathematically as the interaction between material stress [T], strain [S], charge density displacement [D] and electrical field [E]. Generally a relation given by Hook's law is

$$S = s.T \qquad (1)$$

For dielectrical material constitutive equation is given as

$$D = \varepsilon.E \qquad (2)$$

For piezoelectric material the coupled field equations are:
In Strain-Charge form as

$$S = s_E.T + d^t.E \qquad (3)$$

$$D = d.T + \varepsilon_T.E \qquad (4)$$

and in Stress-Charge form as

$$T = c_E.S - e^t.E \qquad (5)$$

$$D = e.S + \varepsilon_S.E \qquad (6)$$

Where

- s compliance coefficients matrix
- ε electric permittivity matrix
- d Strain-Charge form piezoelectric coupling coefficients matrix
- c stiffness coefficients matrix
- s_E tells that the measured compliance data is under at least a constant and a zero electric field preferably.
- ε_T tells that the measured permittivity data is under at least a constant and a zero electric field preferably.
- t matrix transpose.

IV. MATERIAL PROPERTIES

The properties of the material utilized in this paper are given here under:

A. Metal Beam [16]

Young's modulus: E=2.089e^{11} (N/m^2); Poisson's ratio=0.3; Density=7800(Kg/m^3)

B. Piezoceramic

Flexibility compliance matrix (m^2/N):

$$\begin{bmatrix} 0.15e-10 & -0.45e-11 & -0.45e-11 & 0 & 0 & 0 \\ -0.45e-11 & 0.15e-10 & -0.45e-11 & 0 & 0 & 0 \\ -0.5e-11 & -0.5e-11 & 0.19e-10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.39e-10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.39e-10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.39e-10 \end{bmatrix}$$

Density=7790 (Kg/m^3);

Relative permittivity
ε_{11} = 1980
ε_{22} = 2400
ε_{33} = 1980
Piezoelectric strain matrix [d] (C/N):

$$\begin{bmatrix} & x & y & z \\ x & 0 & 0 & -2.1e-10 \\ y & 0 & 0 & -2.1e-10 \\ z & 0 & 0 & 5e-10 \\ xy & 0 & 5.8e-10 & 0 \\ yz & 5.8e-10 & 0 & 0 \\ zx & 0 & 0 & 0 \end{bmatrix}$$

C. Rigid Foam [5]
Young's modulus E=35.3e^6 (N/m^2); Shear modulus G=12.76 e^9(N/m^2); Density=32 (Kg/m^3)

V. FINITE ELEMENT MODELING

For modeling a 3-D geometry of beam and foam, 20 noded brick element SOLID186 with three degrees of freedom is used and for the piezoelectric patch SOLID226 element with four degrees of freedom (including electric) is used. After modeling tetrahedron free mesh with 10 (coarse) refinement is used for meshing. The meshed model of cantilever beam with mid location of piezo patch at a distance of 200 mm from the left end is shown in Fig. 2.

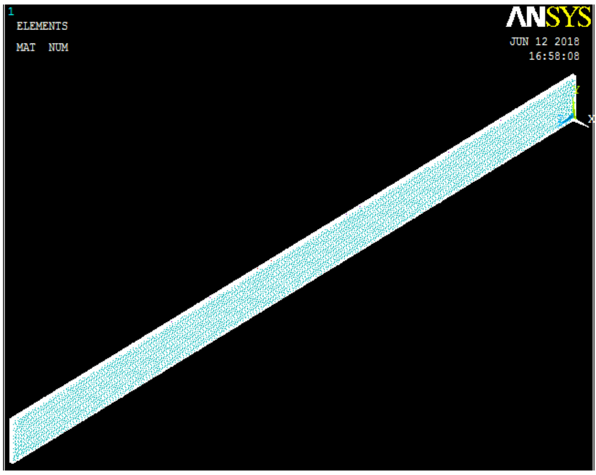


Fig 2. Meshed model

VI. VALIDATION

A comparison with the previous works on sandwich structure is carried out to validate the model.

A. Modal Analysis for Frequency Comparison

Benjeddou et al. [5] performed the modal analysis to get the first five bending natural frequency of a shear actuated sandwich cantilever beam. In order to validate the present study the natural frequencies are compared by taking the beam with same dimensions as of [5] i.e., length L=50 mm, height h=2 mm and with height of foam and patch t=0.5 mm with patch length a=20 mm and center distance of patch from the fixed end as Dc=11 mm. A similar beam is shown in Fig. 3, where the metal substrate portion is shown as aluminum (sky-blue), rigid foam (blue) and PZT-5H actuator (pink). The results of modal analysis are presented in Table 2. It can be seen that the results of the present model matches with the Ref. [5] fairly well.

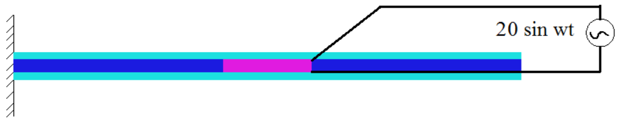


Fig. 3. Schematic diagram for harmonic excitation.

TABLE 2. NATURAL BENDING FREQUENCIES FOR SHEAR ACTUATED CANTILEVER BEAM.

Mode no.	Benjeddou et al. [5] (Hz)	Present study (Hz)	% error
1.	989	967.29	2.195
2.	3916	3854.4	1.573
3.	8374	7983.6	4.662
4.	17416	16389	5.896
5.	26025	24640	5.321

A. Static Analysis for Deflection Comparison

Zhang and Sun [9] did the static analysis to get the transverse displacement of a shear actuated sandwich cantilever beam. In order to validate the present study the transverse displacements at multiple axial locations of beam are determined and compared by taking the beam dimensions as length L=100 mm, height t=8 mm and height of patch tc=2 mm. Here, patch occupies the whole beam length and sandwiched between two metal substrate layers. The metal substrate portion is taken as aluminum with PZT-5H actuator. The voltage applied to the piezoelectric core for shear actuation has a value of V=20. The results are presented in Table 3. Again a very good match between the present study and the ref. [9] can be observed here.

TABLE 3. STATIC ANALYSIS FOR SHEAR ACTUATED CANTILEVER BEAM.

Axial distance x-direction(m)	Zhang and Sun [9] (m)	Present study (m)	% error
0	$0.001e^{-7}$	0	1
0.02	$0.21\ e^{-7}$	$0.2\ e^{-7}$	4.76
0.04	$0.43\ e^{-7}$	$0.44\ e^{-7}$	-2.32
0.06	$0.7\ e^{-7}$	$0.67\ e^{-7}$	4.28
0.08	$0.93\ e^{-7}$	$0.89\ e^{-7}$	4.3
0.1	$1.18\ e^{-7}$	$1.14\ e^{-7}$	3.38

VII.CONVERGENCE STUDY

To ensure the accuracy a convergence study is done for the present work where the convergence is achieved by using h refinement where the mesh size is defined on a scale of 1 to 10 where 1 stands for super fine and 10 stands for extra coarse. The convergence results in terms of natural frequency are tabulated in Table 4.

The results obtained provides an information that there is very less or in some cases no impact on natural frequency so extra coarse mesh size i.e. 10 is used.

TABLE 4. CONVERGENCE RESULTS WHEN PIEZOPATCH PLACED AT 200 MM FROM FIXED END BY THE USE OF H REFINEMENT.

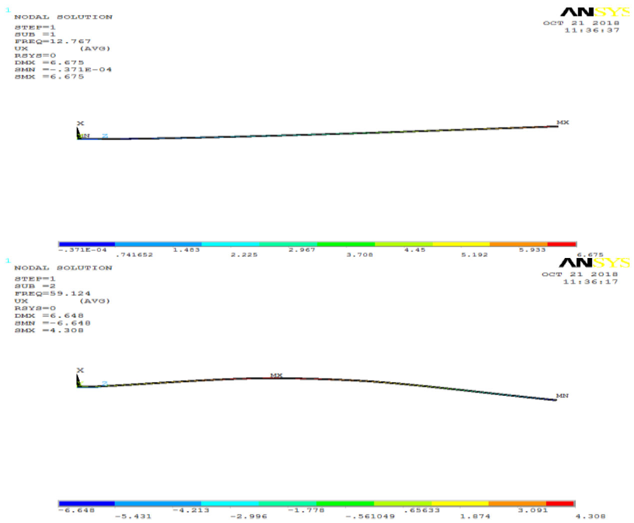
Mode	Refinement index	Natural frequency (Hz)
I	10	12.76
	8	12.766
	6	12.764
II	10	59.12
	8	59.117
	6	59.1055
III	10	16593
	8	165.904
	6	165.838
IV	10	250.26
	8	250.174
	6	249.987

VIII. MODAL ANALYSIS

To determine the location of piezo-patch for maximum deflection at each resonant frequency it is necessary to determine the natural frequencies corresponding to the first few bending modes of the actuator for all the nine location of piezo patch which are obtained by dividing the beam into nine equal segments. The values of the first four natural frequencies on a cantilever piezoelectric sandwich beam for various locations are given in Table 5
Fig. 4 shows the first four mode shapes at a particular location of 200 mm that is mid location of piezo from fixed end.

TABLE 5. NATURAL FREQUENCIES(HZ) FOR CANTILEVER BEAM WITH PIEZOPATCH.

Location	Midpoint location of piezo from fixed end	First mode	Second mode	Third mode	Fourth mode
I	35 mm	13.49	74.47	175.01	281.26
II	76.25 mm	13.46	73.13	150.14	244.17
III	117.5 mm	13.34	64.23	142.47	263.05
IV	158.75 mm	13.11	59.32	159.78	257.06
V	200 mm	12.76	59.12	165.93	250.26
VI	241.25 mm	12.31	62.49	149.83	278.02
VII	282.5 mm	11.78	66.82	144.85	255.34
VIII	323.75 mm	11.22	66.91	155.42	245.78
IX	365 mm	10.64	62.26	150.89	247.74



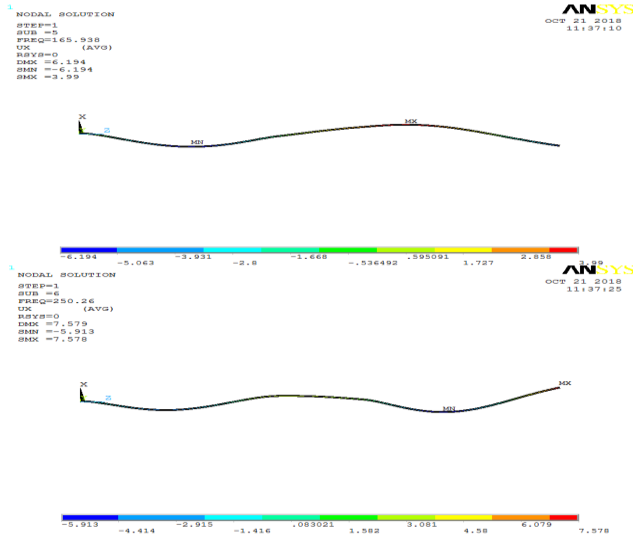


Fig. 4. Mode shapes of cantilever sandwich beam with piezopatch at 5th position i.e. 200mm from fixed end.

IX.NUMERICAL RESULTS

To obtain the numerical results the modal and the harmonic analysis are performed for nine different locations of piezo patch.
After getting the natural frequencies for each location harmonic analysis is done in order to get the tip deflection at first four frequencies of every location. For harmonic analysis the value of applied voltage taken is V=20 volts and the proportional damping with value of mass matrix multiplier α=0.15 and stiffness matrix multiplier β = $3.165e^{-6}$ is used as given in Parashar et al. [15]. In order to get the optimal location of piezo patch for maximum tip deflection at every mode various numerical experiments are performed. The values of natural frequencies and tip deflection which are calculated for all nine position of piezo patches are listed in Table 6.

TABLE 6. NATURAL FREQUENCY AND TIP DEFLECTION FOR EACH SELECTED LOCATION OF PIEZOPATCHES ON CANTILEVER SANDWICH BEAM.

Location	Mid location of piezo from fixed end	Mode	Natural frequency (Hz)	Tip deflection (mm)
I	35 mm	First	13.49	0.42
		Second	74.47	0.275
		Third	175.01	0.071
		Fourth	281.26	0.0336
II	76.25 mm	First	13.46	0.449
		Second	73.13	0.3
		Third	150.14	0.054
		Fourth	244.17	0.00014
III	117.5 mm	First	13.34	0.475
		Second	64.23	0.205
		Third	142.47	0.021
		Fourth	263.05	0.0205
IV	158.75 mm	First	13.11	0.44
		Second	59.32	0.1125
		Third	159.78	0.06
		Fourth	257.06	0.0165
V	200 mm	First	12.76	0.37
		Second	59.12	0.0085
		Third	165.93	0.0485
		Fourth	250.26	0.0141

VI	241.25 mm	First	12.31	0.33
		Second	62.49	0.096
		Third	149.83	0.026
		Fourth	278.02	0.0246
VII	282.5 mm	First	11.78	0.23
		Second	66.82	0.155
		Third	144.85	0.009
		Fourth	255.34	0.0104
VIII	323.75 mm	First	11.22	0.19
		Second	66.91	0.125
		Third	155.42	0.0475
		Fourth	245.78	0.0097
IX	365 mm	First	10.64	0.12
		Second	62.26	0.075
		Third	150.89	0.0255
		Fourth	247.74	0.0129

For location optimization the key concern is to determine the location of node and antinode for each mode. Table 7 shows the natural frequency and maximum tip deflection

TABLE 7. NATURAL FREQUENCY AND TIP DEFLECTION FOR EACH MODE OF CANTILEVER SANDWICH BEAM FOR NODE AND ANTINODE POSITION.

Mode	Mid location of piezo from fixed end	Natural frequency (Hz)	Tip deflection (mm)
II	191.5 mm(AN)	58.8312	0.03125
	311.31 mm(N)	67.517	0.137
III	131.5 mm(AN)	147.155	0.029
	206.66 mm(N)	163.754	0.045
	276.6 mm(AN)	144.307	0.0015
	344.242 mm(N)	155.876	0.0374
IV	103.5 mm(AN)	255.244	0.0156
	154.5 mm(N)	260.0014	0.0184
	207.4 mm(AN)	255.89	0.0215
	260.5 mm(N)	270.2	0.0153
	311.5 mm(AN)	244.70	0.000225
	360.45 mm(N)	249.04	0.0141

The graphs shown in Fig 5 shows the variation in maximum tip deflection corresponding to the mid location of piezo from fixed end for all four modes. The graph in Fig. 5 represents the present shear actuated beam and graph in Fig. 6 represent the extension actuated beam. The triangle on the curve represents the antinode location whereas the inverted triangle represents the node location. Initially all nine values of tip deflection are plotted against the mid location of piezo patch from fixed end and then the best fitting curve is plotted. Further the values of tip deflection for antinodes and nodes are superimposed over these graphs.

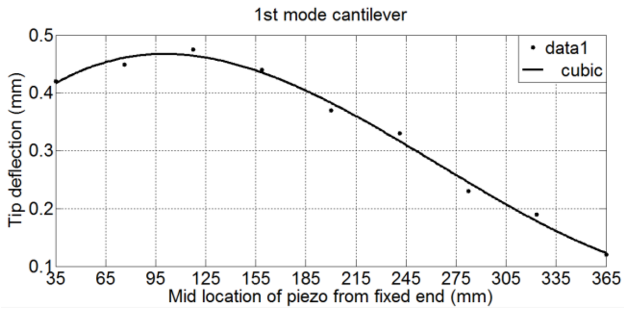


Fig. (a). 1st mode for shear actuated cantilever beam.

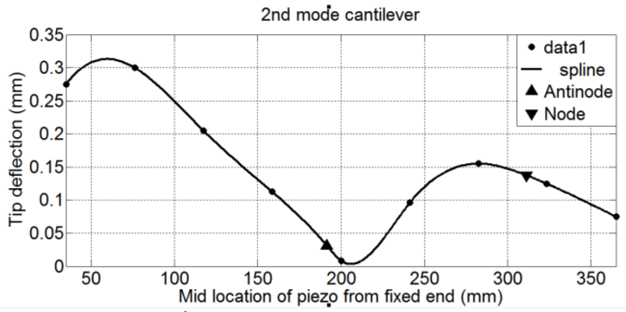


Fig. (b). 2nd mode for shear actuated cantilever beam.

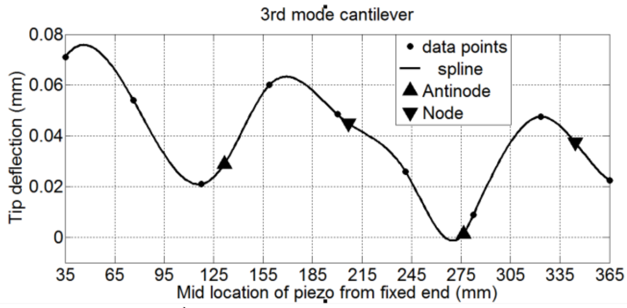


Fig. (c). 3rd mode for shear actuated cantilever beam.

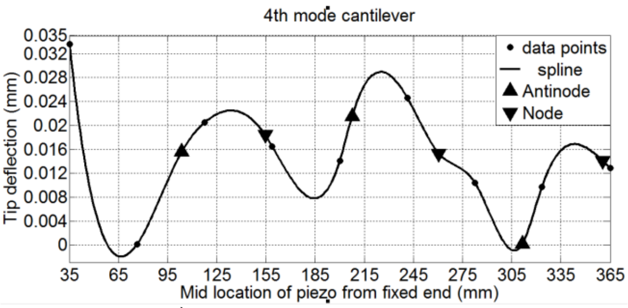


Fig. (d). 4th mode for shear actuated cantilever beam.

Fig. 5. Graph between tip deflection of beam and mid position of piezopatch for all four modes of shear actuated cantilever sandwich beam.

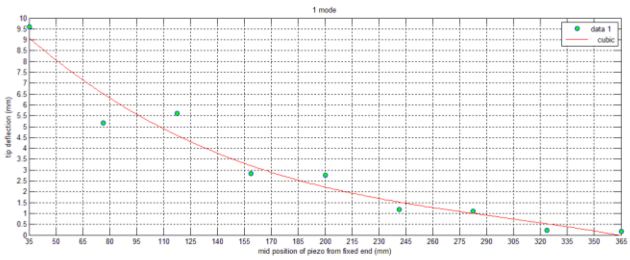


Fig. (a). 1st mode for extension actuated cantilever beam [16].

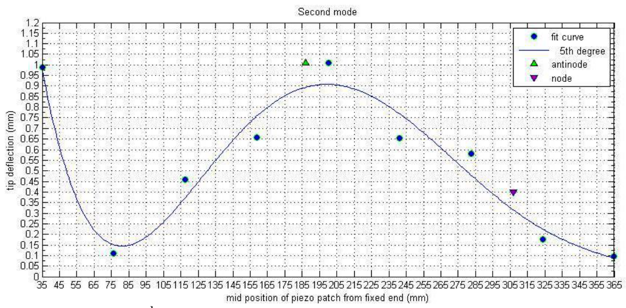


Fig. (b). 2nd mode for extension actuated cantilever beam [16].

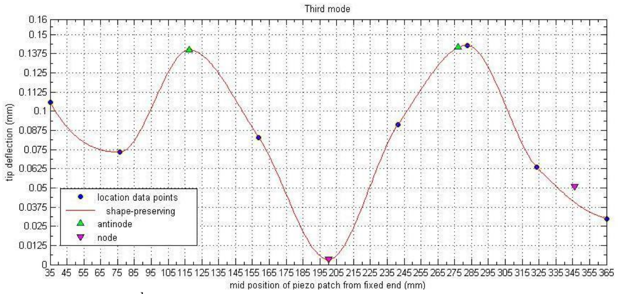


Fig. (c). 3rd mode for extension actuated cantilever beam [16].

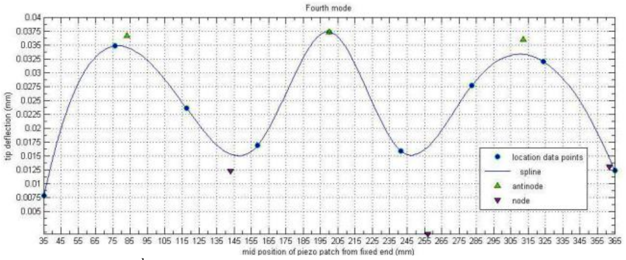


Fig. (d). 4th mode for extension actuated cantilever beam [16].

Fig. 6. Graph between tip deflection of beam and mid position of piezopatch for all four modes of extension actuated cantilever beam [16].

A common conclusion made here is that all shear actuated graph shows almost the same trend that the location of one of the antinode is having a minimum value of tip deflection and maximum deflection is seen nearer to the first location for first, second and third mode and apart from this the node location is having more tip deflection than antinode location but less than maximum deflection.
But in case of extension actuation the node location is showing the minimum deflection and antinode location is showing the maximum deflection for any mode.

X. CONCLUSION

In the present work the optimum location of the piezo patch is determined to provide maximum tip deflection of a cantilever sandwiched beam. Finite element software ANSYS is utilized to study the piezoelectric actuator behavior for the shear mode of vibration. The three dimensional geometric model is produced and meshed in ANSYS. To get the maximum tip deflection at resonance condition for beam, a 20 volt sinusoidal electric field is assumed to be applied.
For cantilever sandwiched beam the minimum deflection for every mode is given by the approximate location of one of the antinode whereas approximate node location gives the second maximum deflection values. Unlike the case of extension actuator [16] where the node location shows a minimum value of deflection and antinode shows the maximum deflection. Here in for shear actuator the antinodes are always the point of minimum deflection.
The present work can be extended to study other boundary conditions of the beam.

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