Mathematical Model for Dynamic Positioning of Offshore Vessels

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Abstract

Dynamic Positioning requires a mathematical model to account for the wind, current and wave forces acting on a vessel and to calculate the forces to be applied by the vessel thrusters in order to position the vessel at the required co-ordinates. One such model has been presented, which calculates the thrusters forces based on minimum power utilized in operating the thrusters.

Keywords: Dynamic positioning, FPSO, thrusters, force balancing, power minimization

Introduction

Dynamic positioning (DP) is a rapidly maturing technology, having been born of necessity as a result of the increasing demands of the rapidly expanding oil and gas exploration industry in the 1960s and early 1970s. It is a computer controlled system to automatically maintain a vessel's position and heading by using its own propellers and thrusters[1]. Position reference sensors, combined with wind sensors, motion sensors and gyro compasses, provide information to the computer pertaining to the vessel's position and the magnitude and direction of environmental forces affecting its position[2]. The computer program contains a mathematical model of the vessel that includes information pertaining to the wind and current drag of the vessel and the location of the thrusters. This knowledge, combined with the sensor information, allows the computer to calculate the required steering angle and thruster output for each thruster.

The major uses of dynamic positioning are found in FPSOs. A floating production, storage and offloading (FPSO) unit is a floating vessel used by the offshore oil and gas industry for the processing of hydrocarbons and for storage of oil[3]. An FPSO vessel is designed to receive hydrocarbons produced from nearby platforms or subsea template, process them, and store oil until it can be offloaded onto a tanker or, less frequently, transported through a pipeline.

Materials and Methods

First, the co-ordinate axis system for the vessel is fixed. X is positive towards bow, Y is positive towards port and Z is positive upwards, as seen in Figure 1. The relative environment angle, α, is positive anti-clockwise starting from the stern. Moments are positive anti-clockwise.

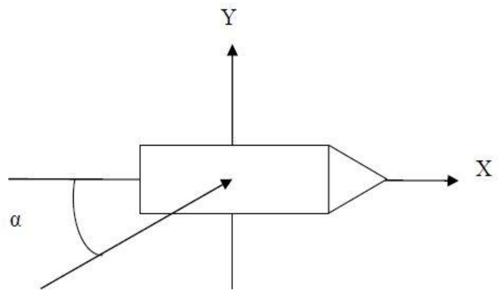

Figure 1: Axis system for a vessel

Second, we calculate the required net forces and moment to be provided by the thrusters[4]:

Force in X-direction:

$F_x = F_{xdemand} – F_{xwind} – F_{xcurrent} - F_{xwave}$ (1)

$F_y = F_{ydemand} – F_{ywind} – F_{ycurrent} – F_{ywave}$ (2)

$M = M_{demand} – M_{wind} – M_{current} - M_{wave}$ (3)

where, F_i = Thruster force required along the i-axis

$F_{idemand}$ = Resultant force along the i-axis

F_{iwind} = Wind force acting along the i-axis

$F_{icurrent}$ = Current force acting along the i-axis

F_{iwave} = Wave force acting along the i-axis

M = Thruster moment required

M_{demand} = Resultant moment on vessel

M_{wind} = Moment due to wind

$M_{current}$ = Moment due to current

M_{wave} = Moment due to waves

Now, we number all thrusters from 1 to n, and denote their co-ordinates as (X_i, Y_i); $i \in \{1, \dots n\}$.
The sum of all thruster forces must equal the net thruster force, leading to:

$F_x = \Sigma(F_x)_i$ (4)

$F_y = \Sigma(F_y)_i$ (5)

$M = \Sigma(-y_i.(F_x)_i + x_i(F_y)_i)$ (6)

We observe that each of the equations (4), (5), (6) have a degree of freedom one less than the number of variables. Hence, we arbitrarily choose (Fx)n, (Fy)n and (Fy)n-1 to be the dependent variables.

From (4) we obtain:

$(F_x)_n = F_x - \Sigma(F_x)_i$ (7)

From (5) we obtain:

$(F_y)_n = F_y - \Sigma(F_y)_i$ (8)

From (6), (7) and (8) we obtain:

$(F_y)_{n-1} = a + \Sigma b(i)(F_x)_i + \Sigma c(i)(F_y)_i$ (9)

Where, $a = M + (y_nF_x – x_nF_y)/(x_{n-1} – x_n)$

$b(i) = (y_i - y_n)/(x_{n-1} - x_n)$

$c(i) = (x_n - x_i)/(x_{n-1} - x_n)$

Substituting back in (8), we get:

$(F_y)_n = F_y - a + \Sigma b(i)(F_x)_i + \Sigma(1+c(i))(F_y)_i$ (10)

Now, a thruster configuration is to be found in which the total power is minimized. The formula for the total power function is:

$P((F_x)_1, \dots(F_x)_n, (F_y)_1, \dots (F_y)_n) = \Sigma((F_x)_i^2 + (F_y)_i^2)^{1/2}$

For minimizing this function, the partial derivatives of P with respect to the forces $(Fx)_i$ and $(Fy)_j$ need to be set to zero, where $i \in \{1, \dots n-1\}$ and $j \in \{1, \dots n-2\}$.

Utilizing the fact that $\partial(F_x)_i / \partial(F_x)_j = 0$ and $\partial(F_y)_i / \partial(F_y)_j = 0$ for $1 <= i < = n-2$ and $j \neq i$, we can obtain all the values for the thruster forces, using non-linear methods such as BFGS, Newton-Raphson methods.

Conclusion

The model described is successful in calculating all thruster forces which minimize the value of the power, subject to the net required thrusters' forces and moment for keeping the vessel at a fixed position.

References

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