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Mathematical Modeling and Sensitivity Analysis of Mounting Forces in Quick Connectors

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Abstract
This study investigates the mounting force dynamics of quick connectors used in automotive engine pipe connections. The research focuses on the two primary contributors to mounting force: the rubber O-ring and the metal locking spring. Using beam theory and Lindley's formula as a foundation, we develop a comprehensive mathematical model to quantify the mounting force generated by the locking spring and the O-ring, respectively. We employ variance-based sensitivity analysis, specifically the Sobol method, to evaluate the relative influence of various properties and characteristics on the overall mounting force. The results are based on the coding of a model using the Python language. Our findings reveal that the second-order Sobol indices are very low. Each variable influences the output independently. These insights provide a deeper understanding of quick connector mechanics and offer a quantitative basis for improving connector design.

Keywords— **Quick connector, O-ring, locking spring, sensitivity analysis, Sobol indices, beam theory**

1 INTRODUCTION

1.1 Background on quick connectors in automotive applications

Quick connectors (see Figure 1) play a crucial role in modern automotive engines by providing rapid, secure, and leak-free connections for fluid and gas lines [7]. They are essential for efficient assembly in production lines, reducing manufacturing time and labor costs compared to traditional fastening methods. Quick connectors ensure reliable sealing under various operating conditions, including high temperatures, vibrations, and pressure fluctuations typical in engine environments [13] [8]. These connectors allow for easy maintenance and replacement of components, improving serviceability and reducing downtime during repairs. Quick connectors enhance safety by minimizing the risk of improper connections, which could lead to fluid leaks or system failures. Unlike clamps, which require tools and more time to install or remove, quick connectors can be engaged or disengaged by hand, often with a simple push-to-connect mechanism. While clamps can accommodate a wider range of tube sizes and materials with a single design, quick connectors often require specific matching components but offer superior reliability and ease of use in automotive applications.

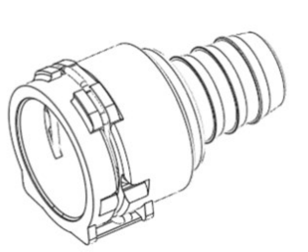

Figure 1: View of a quick connector

1.2 Research objectives

For manual assembly, a well-designed mounting force balances the need for a secure connection with the ergonomic considerations for assembly line workers, reducing the risk of repetitive strain injuries. Understanding and controlling mounting force is thus critical for ensuring the reliability, safety, and efficiency of automotive assembly processes, as well as the long-term performance of the vehicle. The research focuses on the development of a mathematical model of assembly effort, taking material characteristics and part geometry as parameters. We will then apply a sensitivity analysis to understand the contribution importance of each parameter.

2 THEORETICAL FRAMEWORK

2.1 Integrated mounting force model

Understanding the total mounting force for a quick connector requires considering both the O-ring contribution and the locking spring force. The figure 2 below shows the main 3 components of a quick connector in an exploded view.

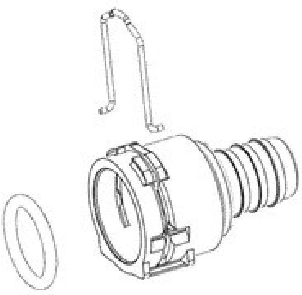

Figure 2: Exploded view of quick connector (Body, O-ring and locking spring)

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To work properly, the connection needs a male part or mating port. Spigot plays this role in the coupling with a quick connector. The spigot serves as the male part of the quick connector system. It is designed to fit into the female part, often referred to as the quick connector. It facilitates the transfer of fluids (such as fuel, coolant, or hydraulic fluid) between different parts of the vehicle's system. The spigot and quick connector in combination ensure a secure and leak-free connection, which is vital for maintaining the integrity of the fluid system. In this paper, we use the "VDA quick connector" definition. "VDA quick connector" is widely used in the automotive industry to describe a specific type of quick connector that complies with design guidelines established by the Verband der Automobilindustrie (VDA) [1], the German Association of the Automotive Industry. This specific VDA standard for quick connectors spigot profile is not publicly accessible, so we will use only the dimensions needed for this study. Since we are performing a sensitivity analysis, the result will be valid regardless of the specific dimensions (See Fig 7 and Table 1 section Spigot).

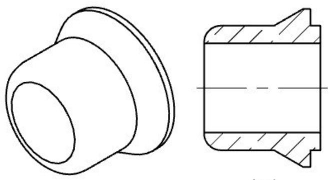

Figure 3: Isometric and cross-section view of VDA spigot

Key points on their interaction in sequential Engagement: Initially, the O-ring force dominates as it begins to compress (step 1). As insertion progresses, it begins to slide along the spigot (male part) introducing a friction force (step 2). Finally, the locking spring engages, adding its force (step 3) and reaching the max at the end of the spigot slope (step 4).

2.2 Beam theory for metal locking spring

The metal locking spring in a quick connector can be considered as a beam that bends when a force is applied. This bending behavior can be analyzed using beam theory, which provides a framework for understanding how the spring deforms and generates force. More precisely, the mounting force is generated by the interaction between the locking spring and the slope in the spigot. When the locking spring meets the inclined surface of the spigot, it deforms. This deformation generates a reaction force that contributes to the overall mounting force. The angle and surface properties of the spigot slope play a crucial role in determining the magnitude of this force. The relation between force and deflection using Castigliano's theorem [2] is described below.

$$F = \frac{3EI}{b^2(3h+b)}\delta \qquad (1)$$

where:

F = The force under deflection δ = Deflection h = First length of the spring's leg (beam) b = Second length of the spring's leg (beam)
E = Young's modulus
I = Moment of inertia

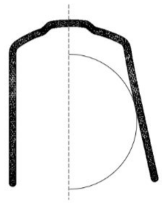

Figure 4: Locking spring: free state (left side) and max deflection (right side)

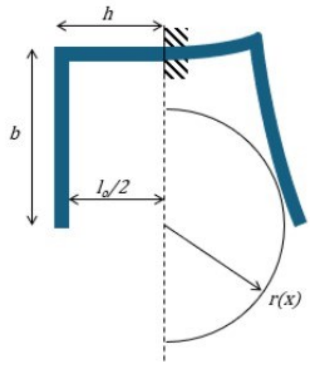

Figure 5: Locking spring's modelling with beam theory

2.3 O-ring mechanics and compression models

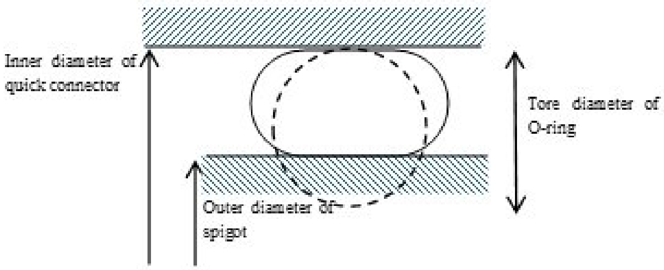

Figure 6: O-ring installation and deformation

In quick connectors, the O-ring provides the primary sealing function. The compression of this O-ring is crucial for creating an effective seal. The O-ring's contribution to the mounting

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force can be understood through the compression of the O-ring. When the quick connector is assembled, the O-ring is compressed between the spigot and the connector housing (Fig 6). This compression generates a reaction force that contributes to the overall mounting force required to secure the connection. The Lindley formula [5] [3] for the compression characteristics of laterally unrestrained rubber O-rings is indeed highly relevant to understanding the mounting force of quick connectors.

$$F = \pi D_o E d_o \left(1.25\epsilon^{1.5} + 50\epsilon^6\right) \tag{2}$$

where:

F = Total load on the O-ring
D_o = Mean diameter of the O-ring
E = Young's modulus of the rubber
d_o = O-ring cross-sectional diameter
ϵ = Fractional compression (compression ratio)

Typically, in the automotive industry and for engine cooling functions, O-rings are made from EPDM rubber (ethylene propylene diene monomer).

2.4 Frictional force during assembly

The friction coefficient between an O-ring and a plastic spigot during the mounting of a quick connector is influenced by the material properties, surface roughness, and contact pressure [10]. The O-ring, typically made of elastomer, deforms to create a seal, generating friction against the plastic spigot. Lubrication plays a vital role in reducing this friction by forming a thin film between the O-ring and the spigot, minimizing direct contact, and thus lowering the friction coefficient [9]. The friction coefficient [6], defined as the ratio of the frictional force to the normal force, is essential for predicting the performance of mounting effort. The equation for the friction coefficient μ is:

$$F = \mu N \tag{3}$$

where:

μ = friction coefficient
F = frictional force
N = normal force

3 METHODOLOGY

3.1 Mathematical model development

The axis in the direction of quick connector insertion on the spigot is called x. The first step is to figure out the spigot's outer profile called $r(x)$.

$$r(x) = \begin{cases} r_m - \left(r_e - \sqrt{r_e^2 + x^2}\right) & \text{if } x < r_e \\ r_m & \text{if } x \geq r_e \\ r_m + (x - l_{flat}) \times \tan 30^\circ & \text{if } x \geq l_{flat} \\ \frac{d_{max}}{2} & \text{if } r(x) \geq \frac{d_{max}}{2} \quad \forall x \end{cases} \quad \forall x \in [0, l_{total}] \tag{4}$$

where:
r_m = Main diameter
r_e = Entrance radius
l_{flat} = Length of flat diameter area
d_{max} = Overall maximum diameter
l_{total} = Total length of spigot

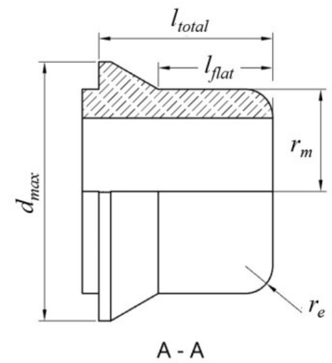

Figure 7: Cross section of VDA spigot with dimensions

Now, we propose an analytical force model created by locking spring using the equation 1 and by expanding some variables. See equation 5 for the result. The locking spring deflection is $\delta(x) = r(x) - Lo/2$. For a circular cross-section of diameter d_{ls}, the quadratic moment of area is $I = \pi d^4_{ls}/64$. The force projected in the x-axis is then:

$$F_x\ (x) = 2\frac{3E}{b^2(3h+b)}\frac{\pi d_{ls}^4}{64}\left(r(x) - \frac{L_o}{2}\right)\frac{\Delta r}{\Delta x} \tag{5}$$

It is crucial to project the force in the x-axis direction. While the locking spring force is naturally aligned along the r-axis, the operator mounts the quick connector onto the spigot in the x-axis direction. The slope and radius of the spigot shape effectively convert the force to the x-axis direction.
Finally, we propose a force model created by O-ring compression and friction. For this, we replace ϵ in the Lindley equation 2 by the following formula:

$$\epsilon = \frac{d_o - \left(\frac{ID}{2} - r(x)\right)}{d_o} \tag{6}$$

3.2 Variance-based sensitivity analysis

The Sobol method is a global sensitivity analysis technique [12], it considers the entire input space rather than just local variations around a nominal point. This is crucial for complex systems such as the one we are studying here for quick connectors, where interactions between multiple variables (material properties or geometric dimensions of components) significantly impact the outcome.
It decomposes the variance of the output (in this case, the mounting force) into contributions from each input variable and

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their interactions. This helps in identifying not only the main effects of individual variables but also the combined effects of multiple variables, providing a comprehensive understanding of the system.

The Sobol method can handle non-linear [11] and nonmonotonic relationships between input variables and the output. This is important in our applications where the relationship between material properties, geometric dimensions, and mounting force can be complex.

The mounting force model is expressed by a function f associating with random variables $X_i, i \in [1,n]$. The random variables X_i are material properties and geometric dimensions.

The random variable Y is the output of f. The i^{th} first-order sensitivity index is defined by:

$$S_i = \frac{Var\left[E[Y|X_i]\right]}{Var[Y]} \tag{7}$$

First-order Sobol Indices (S_i) represent the main effect of each input parameter on the output. They quantify the proportion of output variance caused solely by variations in a single input parameter. A high value of S_i for a parameter means it has a strong independent influence on the output, so measuring the output provides valuable information about that parameter. If S_i is low but ST_i is high, the parameter does not significantly affect the output on its own but does so through interactions with other parameters. Therefore, it still plays a significant role in influencing the quantity of interest.

The model we want to analyze depends on one parameter (x or x-axis) that is not part of the sensitivity analysis. The analysis will be performed for each position "bin" separately. So, we will get not a single number for S1 but a number along the x-axis $S1(X)$.

4 RESULTS

4.1 Spigot profile and force modelling

The first intermediate result is to check whether the spigot profile is correctly modeled by equation 4. The graph obtained (see Fig 8) verifies that the spigot profile is as expected. This data can then be used to calculate the forces involved in fitting the quick connector to the spigot.

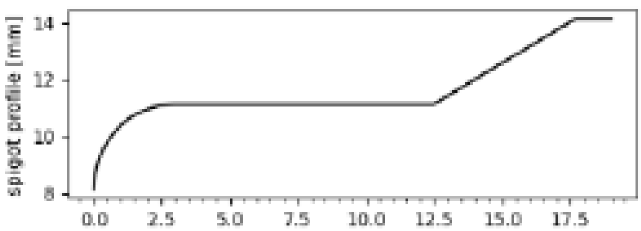

Figure 8: Spigot profile vs x [mm]

The second intermediate result is to verify the modeling of the forces and compare it to measurements taken on actual parts (see Fig 9).

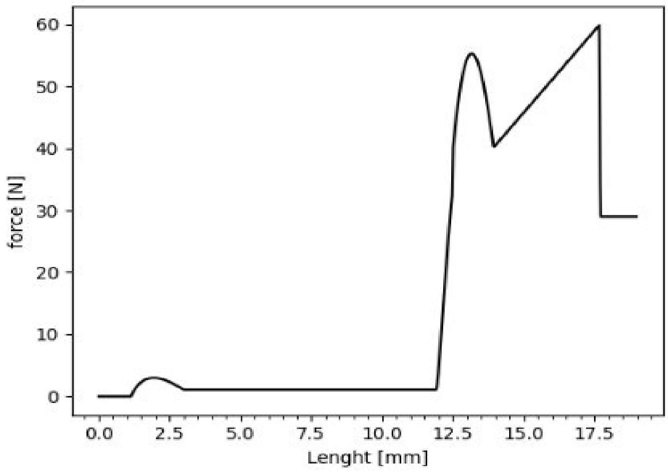

Figure 9: Force [N] vs displacement [mm]

At this stage of the study, it is interesting to compare our mathematical model of assembly effort with measurements on real parts. Figure 10 shows a set of measurements of the assembly force of a quick connector on a spigot. Both on the x-axis (displacement [mm]) and on the y-axis (force [N]), the similarity of the curves between simulated and real values (Fig 11) shows that our theoretical model provides a good representation of the reality of the physical phenomena involved.

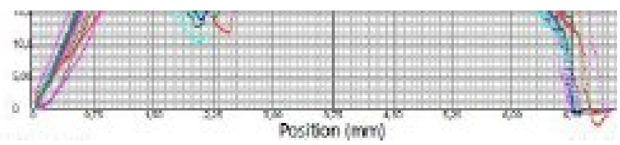

Figure 10: Measures on several real parts. Force [N] vs displacement [mm]

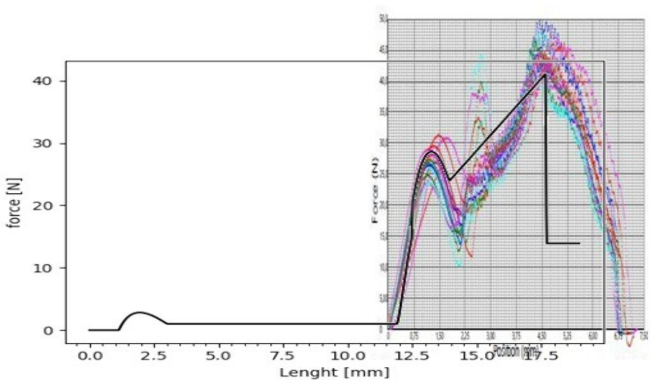

Figure 11: Superposition of theoretical and real measures

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4.2 Sensibility analysis results

The hypothesis for all parameters (dimensions and material properties) for sensitivity analysis are described in table 1. Simulation is performed over 8 parameters. Spigot dimensions and friction factor are out of simulation (no minimum and no maximum values). The number of resamples when computing confidence intervals is chosen at 8,192. The confidence interval level is 0.95. A 0.95 confidence interval in the context of a Sobol indices simulation means that there is a 95% probability that the true value of the parameter being estimated lies within the calculated interval.

Parameter	Nominal	Unit	Min	Max
Body				
Inner diameter	27.8	mm	27.7	27.9
O-ring				
Tore diameter	3.5	mm	3.4	3.6
Young modulus	1.92e6	Pa	666	777
Locking spring				
Wire diameter d	1.5	mm	1.4	1.65
Length b	1.4	mm	1.35	1.45
Length h	1.4	mm	1.35	1.45
Free length Lo	20	mm	19.9	20.1
Young modulus E	2e11	Pa	1.8e11	2.2e11
Spigot				
Main diameter	22.3	mm	-	-
Entrance radius	3	mm	-	-
Flat length	12.5	mm	-	-
Total length	19	mm	-	-
Max diameter	28.3	mm	-	-
Frict ion coefficient				
Friction coeff. (spigot / spring)	0.13		-	-
Friction coeff. (spigot / O-ring)	0.2		-	-

Table 1: List of parameters and values by component

Sensitivity analysis along the x-axis reveals a bimodal behavior. See Fig 16 at the end of this section. The first mode arises from the O-ring reaction, while the second mode is due to the locking spring behavior. Since our objective is to analyze these phenomena to reduce the mounting force, we examine the two modes separately. First, we identify the x-values corresponding to the maximum of each mode.

Then, we perform sensitivity analyses separately at each of these x-values. With this method, there are no dependency with the x-axis, we can then compute also the second-order Sobol indices to determine whether interactions between variables exist.

Python script gives the position of the first mode (i.e., O-ring reaction) at $Argmax(f(x)) = 13.25mm\ \forall x \in [0,14]$ and the position of the second mode (lock spring reaction) at $Argmax(f(x)) = 17.68mm\ \forall x \in [14,19]$. The value 14(mm) is chosen as a relevant value to separate easily both modes. Comparing total and first-order indices gives an initial idea of the interaction between parameters. In our case and in both modes, as the values are very close, there is no interaction between the parameters. See Fig 12 and Fig 13.
The high confidence level for Total indices (STi) and First-order indices (S1i) indicates that the estimates of the indices are reliable and stable. The sampling is sufficient: the number of samples used in the Sobol simulations is likely adequate. The parameter influence is clear: the model parameters have a well-defined influence on the output. There is low variability in the estimates of the indices, suggesting that the results are consistent across different runs of the analysis. The numerical methods and algorithms used are robust and well-suited for the model, minimizing computational errors and instability.

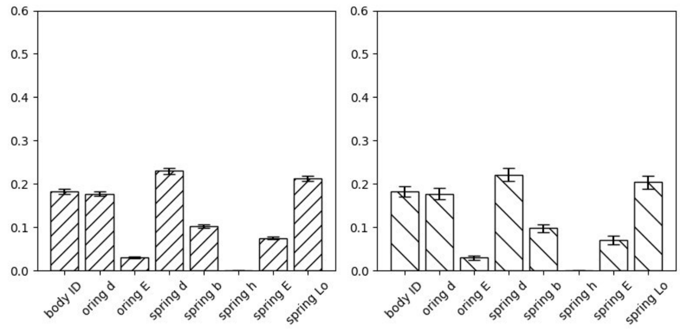

Figure 12: Total and First-Order Sobol Indices (x = 13.25)

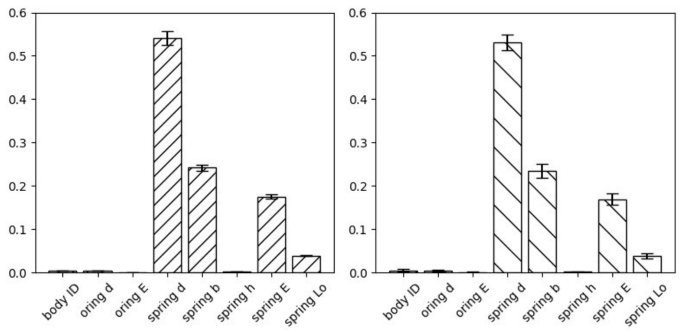

Figure 13: Total and First-Order Sobol Indices (x = 17.68)

The result for the first mode at x = 13.25mm (Fig 14) shows a S1 values fairly high for O-ring and Locking Spring. An idea of improvement will be to eliminate spatial correlation between the locking spring reaction and the O-ring. This will facilitate part design by creating spatial independence.

We decided to represent the S2 indices in the form of a correlation matrix, as [4] had done. Diagonal values are filled with S1 indices values. The analysis shows that the second-order Sobol indices are extremely low (Fig 14 and Fig 15), it means that the interactions between pairs of input variables have a negligible impact on the output variance.

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The variance in the model output is primarily due to the individual effects of the input variables (Dominance of Individual Effects), not their interactions. Each variable influences the output independently. The model behaves approximately additively, meaning the output can be represented as the sum of functions of individual variables with minimal loss of accuracy. This simplifies the understanding and interpretation of the model. Since interactions are insignificant, focusing on optimizing or studying individual variables is sufficient. There is less need to consider complex interdependencies between variables. This can be advantageous in control and prediction scenarios.

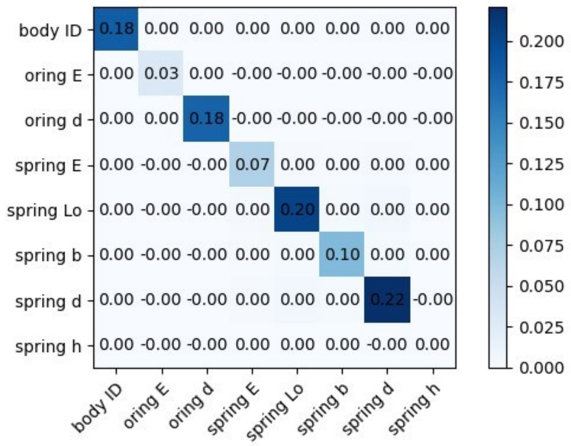

Figure 14: S1-S2 matrix for O-ring mode

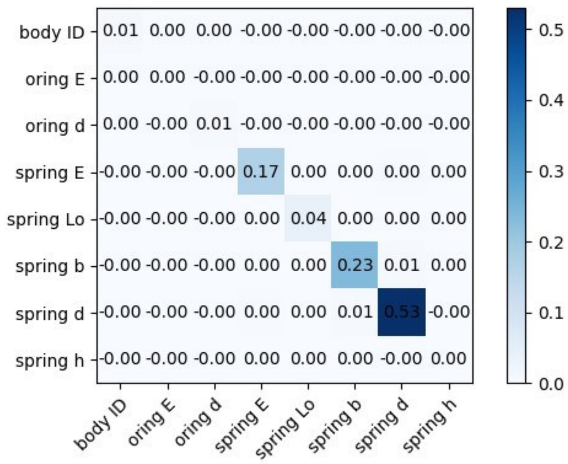

Figure 15: S1-S2 matrix for Locking Spring mode

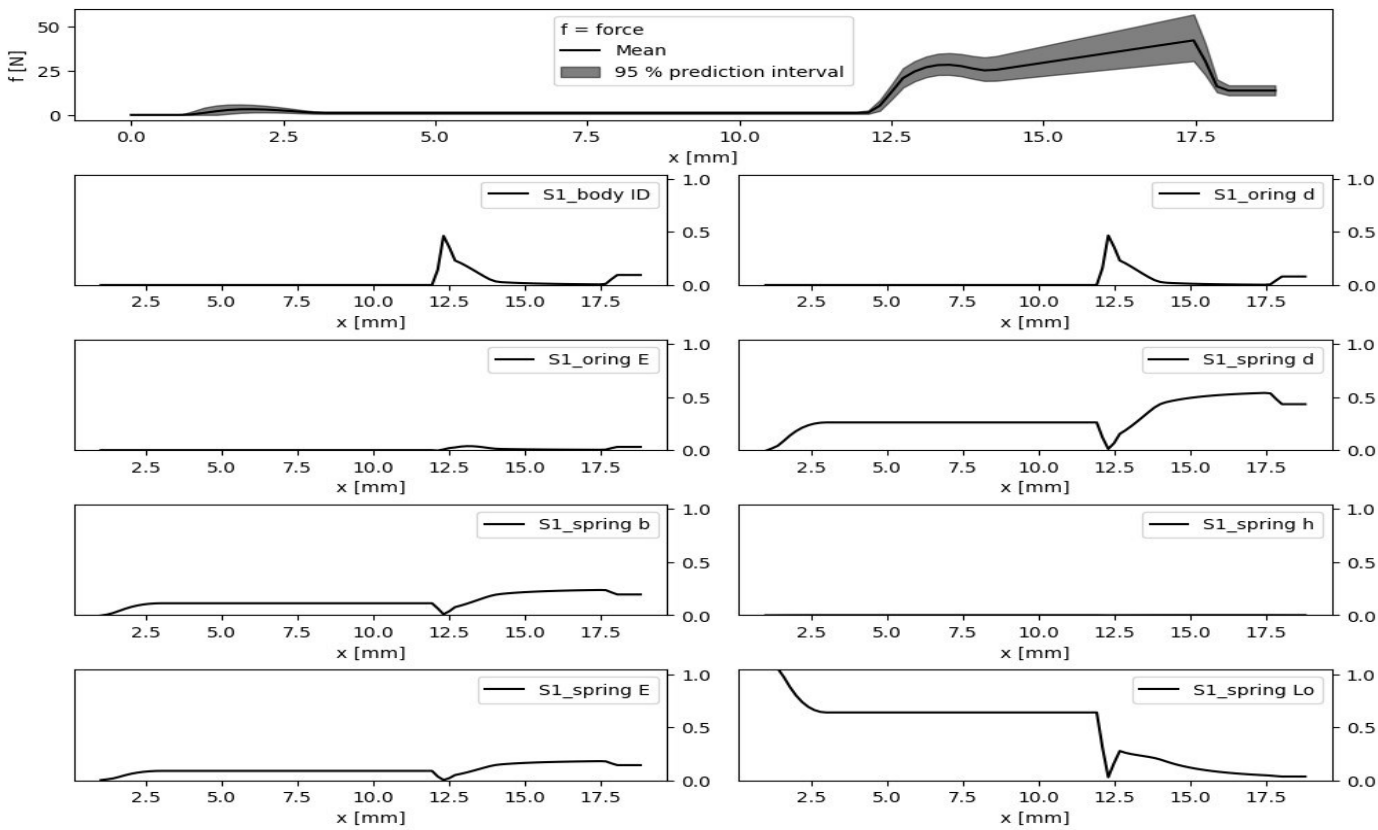

Figure 16: Sensitivity analysis along x-axis

5 DISCUSSION

5.1 Spatially Resolved Sensitivity Analysis of Assembly Forces

An analysis of the first-order Sobol indices (S_i) along the x-axis (Fig 16) offers a novel representation of how assembly forces are generated in the quick connector fitting. The magnitude of a component's S_i index at specific segments of the curve indicates its impact on those segments. Each component contributes differently along the x-axis, depending on its position and interactions with the spigot profile and other components.

The free length of the locking spring (L_o) significantly influences the initial phase of spigot insertion. However, once the locking spring reaches the slope, the impact of this parameter progressively diminishes.

The contribution of the O-ring—specifically its torus diameter and hardness—reaches its maximum at 12.2mm. This corresponds to the point where the O-ring compression is greatest along the x-axis and in the direction of x-axis. After this initial compression, the influence of the O-ring becomes associated primarily with the friction factor.

5.2 Limitations and future research directions

Using analytical formulas like Lindley's equation for the Oring force and beam theory for the locking spring force is a solid approach, but there are limitations when comparing this model to real-world scenarios. Lindley's equation assumes ideal conditions for the O-ring, such as perfect material properties and uniform deformation. Lindley's equation was also elaborated for an unrestrained O-ring, and this is not the case in Quick Connector construction. In reality, O-rings can experience non-uniform stress distribution, material imperfections, and varying environmental conditions (temperature, pressure) that affect their performance. Beam theory simplifies the behavior of the locking spring by assuming linear elasticity and uniform material properties.

However, the locking spring geometry was made simple, and real springs may exhibit non-linear behavior, plastic deformation, or fatigue over time, which are not captured by this theory. Analytical models simplify interactions, potentially overlooking factors like surface roughness, lubrication, and dynamic loading conditions that can significantly affect the mounting force. Analytical models typically focus on static conditions and may not account for dynamic effects. In real applications, these factors can alter or improve the performance of both the O-ring and the locking spring, leading to deviations from the predicted mounting force.

To improve the accuracy of our model, we might consider incorporating finite element analysis (FEA) to simulate more complex behaviors and interactions or conducting experimental validation to calibrate and refine your analytical models, but the objective was less to get an accurate result than to understand the contribution of each parameter or characteristic of O-Ring, locking spring in quick connector design.

6 CONCLUSION

To evaluate the relative influence of various properties and characteristics on the overall mounting force, we employ variance-based sensitivity analysis, specifically the Sobol method. This technique allows us to decompose the variance of the mounting force into contributions from each input parameter and their interactions. By calculating Sobol indices, we quantify the importance of each factor, including material properties, geometric dimensions, providing a robust and comprehensive understanding of their impact on connector mounting force performance. The low second-order Sobol indices indicate that your model's output is largely influenced by individual input variables, with minimal contribution from their interactions. This simplifies the model's interpretation and suggests that focusing on individual variables will be most effective for understanding and optimizing the system.

The model developed in this study helps to streamline the design process, reduce prototype iterations, and lead to more efficient and reliable pipe connections in automotive engines. Practical implications of this research include improved connector design methodologies, enhanced quality control processes, and more accurate performance predictions for automotive quick connectors.

7 ACKNOWLEDGMENT

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