Mathematical Modeling of Leak Detection Pipelines Conveying Fluids

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Abstract: The leakage from pipelines in various industrial applications is the cause of environmental problems and the loss of conveyed fluid, It is therefore essential to detect the location of unintended leakage resulting from cracks, holes and pores. This requires the knowledge of pipe geometrical configuration and nature of fluid inside it. Mathematical models to simulate the leakage problem are needed to provide suitable predictions necessary to help developing technical methods for the detection of any leakage problem. Pipeline leak detection techniques and systems are based on computational leak methods implemented on several pipeline systems. The leak detection includes volume mass balance, pressure monitoring, transient flow detection methods. The transient leakage detection has become one of the major applications of transient simulation techniques aiming to detect and locate pipeline leakage efficiently. In the present work a method of leakage from pipelines having cracks is introduced. It is based on the continuous introduction of sinusoidal pressure waves of small amplitudes at the entrance of the pipeline. The variation of the amplitude of the pressure gradient and the shear stress on the inside surface of the pipeline in the streamwise flow direction are to be numerically evaluated. The effect of the geometrical parameters concerning the pipe and crack along with the flow parameters namely the Strouhal and Reynolds numbers are examined to provide information necessary for the technique to be implemented. Since this transient simulation is of a numerical kind accurate solution of the unsteady partial differential equations of the flow system is very much required. It is the aim of the present work to undertake this task.

Keywords Modeling- leakage- detection- fluids- incompressible flow-*pipelines*

1. INTRODUCTION

Baghdadi and Mansy [1] presented a novel method for leak detection in pipelines. The method is based on a unidimensional flow analysis. The theoretical findings have been verified experimentally for two different hole geometries (circular and rectangular). The comparison set between theory and experiment confirms the physical realism of the mathematical model.

Turner [2] considered a pipeline network in which gas pressure and temperature were measured at every

measurement location, and mass flows were measured at some of these locations including all inlets and outlets to the network. Leak detection is the process of determining whether a leak is present in a particular section. Once a leak is detected, the time it appeared, its size and its location are determined. Turner and Mudford [3] described a method of detecting leaks in gas pipelines by use of flow, pressure and temperature measurements that are usually made in long-distance natural gas pipelines for other reasons.

Sharp and Campbell [4]applied a technique which involves the injection of a sound pulse into the object under investigation and recording of the resultant reflections. Analysis of the reflections gives information about the bore profile and input impedance of the object.

Silva et al [5] presented on-line computational technique used in the analysis of hydraulic transients caused by leakage, in order to detect and locate pipeline ruptures. Pressure transients were obtained in two PVC pipelines 3/4"(19mm) diameter by using four pressure transducers connected to a PC computer equipped with A/D and D/A converters.

Miller et al [6] constructed a reference standard for setting up and evaluating Acoustic Emission (AE) equipment to be used in pipeline leak detection.

Fukushima et al [7] put forward a leak detection method based on a dynamic simulation with the wave equations is presented. An industrial application to one of the longest gas pipeline is also presented with its performance information.

Verde [8] solved the multi-leak detection problem using only sensors of flow and pressure at the extremes of the duct, and using the analytical redundancy given of these measurements.

Covas and Ramos [9] focused on leakage detection and location in pipe networks based on a recent and novel approach, known as inverse transient analysis. The main idea behind this methodology is the identification of leaks location in pipe networks using observed pressure data, collected during the occurrence of transient events, and the minimization of the difference between observed and calculated parameters.

Wang et al [10] made attempts to detect leaks in pipelines which contribute to damping of transient events. That fact leads to a method of finding location and magnitude of leaks. Because the problem of transient flow in pipes is nearly linear, the solution of the governing equations can be expressed in terms of a Fourier series.

Mpesha et al [11] presented a new procedure utilizing transient state pressures to detect leakage in piping systems. Transient flow, produced by opening or closing a valve, is analyzed in the time domain by the method of characteristics and the results are transformed into the frequency domain by the fast Fourier transform.

Liu et al [12]presented particle filters are sequential Monte Carlo methods based on point mass (or "particle") representations of probability densities, which can be applied to estimate states in nonlinear and non-Gaussian systems without linearization.

Xu et al [13]described how the belief rule based expert systems can be trained and used for pipeline leak detection. Pipeline operations under different conditions are modeled by a belief rule base using expert knowledge, which is then trained and fine-tuned using pipeline operating data, and validated by testing data.

Covas, Ramos and Almeida [14]Focused on leakage detection in pipe systems by means of the standing wave difference method (SWDM) used for cable fault location in electrical engineering.

Huang et al [15] proposed a hybrid configuration of Mach-Zehnder and Sagnac interferometer as sensing frame. In this interferometer, there are two light paths that have the same optical length but travel different sequence paths. Because the propagation lights of the two light paths pass through the leaking point at different times, the resulting phase signals differ respectively.

Paivar, Salahshoor and Hourfar [16] proposed a neural decision making approach to oil pipeline leak localization. The one main methods, model based fault detection is used (to find leaks quantity and location-making) to form a novel fault diagnosis scheme.

Koppel, Ainola and Puust [17] Proposed a mathematical model for the determination of unregistered consumption and leakage using the heads and flows at the inlet and at the outlet of the main or at some nodes of the network.

Abhulimen and Susu [18] developed a novel model for detecting leaks in complex pipeline network systems. The model was derived from the theory of Liapunov stability criteria. A leak is detected if the resulting eigenvalues from the deviation flow matrix have values less than a predetermined value.

Pedro et al [19] proposed the impulse response function as a means of leak detection in pressurized fluid pipelines. The experimental extraction of the impulse response function indicates that frequency content of the injected transient signal must be taken into account to minimize distortion.

Chuanhu, Guizeng and Hao [20]proposed one of the most concerned performance indices of leak detection and location systems is the smallest detectable leakage flow rate (SDLFR). Based on the physical model of pipeline, mathematical description for the amplitude change of negative pressure wave (NPW) and its attenuation traveling along the pipeline have been deduced

Ekuakille, Vendramin and Trotta [21] studied a spectral analysis response, used for leak detection, the generally based on fast Fourier transform (FFT).

Zhou et al [22] A belief rule base inference methodology using the evidential reasoning approach (RIMER) has been developed recently, where a new belief rule base (BRB) they proposed to extend traditional if-then rules and can capture more complicated causal relationships using different types of information with uncertainties, but these models are trained off-line and it is very expensive to train and re-train them.

Olunloyo and Ajofoyinbo [23]proposed a model for real time leakage detection in pipelines based on integration of a detection subsystem and Global Positioning System (GPS) receiver.

Shuqing et al [24] Proposed a new time-frequency analysis method of Hilbert- Huang Transform applied in Pipeline leak detection. Choosing the negative pressure wave signal from dynamic pressure transmitter as research object, the dynamic pressure signal is decomposed by the empirical mode decomposition, and we obtain the proper mode functions that satisfy the condition.

González et al [25] focused on the modeling and simulation of a gas distribution pipeline network with a special emphasis on gas ducts. Gas ducts are the most important components of such kind of systems since they define the major dynamic characteristics.

Wan and Qiu [26] introduced an optical fiber early-warning system based on Mach-Zehnder in order to monitor the normal operation of pipelines. Three single-mode fiber in the cable which is buried along the pipeline probe vibration signal on the ground, in which the two was not in the same casing as a sensing fiber, the formation of intervention arm, the other one as the transmission fiber, the return of the interference signal.

Zhou et al [27]put forward a recursive algorithm based on the Bayesian reasoning approach is proposed to update a belief rule based (BRB) expert system for pipeline leak detection and leak size estimation.

Reddy et al [28] Proposed dynamic simulation models that can be used along with flow and pressure measurements, for on-line leak detection and identification in gas pipeline networks. They proposed method uses the available pressure and flow rate measurements, sampled at regular intervals and is based on an efficient state estimation technique, developed by the authors (Reddy.

Reddy et al [29] evaluated the performance of the a proposed leak detection and identification methodology, using experiments with compressed air on a laboratory scale network.

Liang and Zhang [30] presented a novel pipeline leak detection scheme based on gradient and slope turns rejection (GSTR). Instead of monitoring the pipeline under constant working pressure, GSTR introduces a new testing method which obtains data during the transient periods of different working pressures.

Meng et al [31] summarized in detail the technologies of recognizing and extracting wave characteristics, which is to distinguish leaking and disturbing signals from time and frequency domain.

Mandal, Chan and Tiwari [32] proposed a novel leak detection scheme based on rough set theory and support vector machine (SVM) was to overcome the problem of false leak detection. In their approach, 'rough set theory' is explored to reduce the length of experimental data as well as generate rules further;

Jing and Zhi-Hong [33] studied pipelines leakage factors, based on Grey relational analysis (GRA) to analyze and

evaluate all the factors and draw an order of factors influencing on pipeline leakage.

II. THEARETICAL ANALYSIS

Fig. 1 shows the cracked pipe geometry and the coordinate system. It is to be noted that the crack is modeled here as circumferential slot in order to facilitate the theoretical analysis as well as the numerical treatment using the finite difference approximation



Fig.(1) The cracked pipe geometry and the coordinate system

Where,

 V_m is the Mean flow velocity, v_r is the Radial velocity v_z is the Axial velocity , W is the Crack size,

L- The length of the pipeline, R-pipe's radius.

A. Governing equations

$$\frac{\partial^2 \phi_r}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_r}{\partial r} - (\frac{V_z}{\upsilon}) \frac{\partial \phi_r}{\partial z} + \frac{\partial^2 \phi_r}{\partial z^2} + (\frac{1}{r^2}) \phi_r + (\frac{\omega}{\upsilon}) \phi_i = -\frac{1}{r^3} \frac{\partial^2 \psi_r}{\partial z^2} \quad (1)$$

$$\frac{\partial^2 \phi_i}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_i}{\partial r} - (\frac{V_z}{\upsilon}) \frac{\partial \phi_i}{\partial z} + \frac{\partial^2 \phi_i}{\partial z^2} + (\frac{1}{r^2}) \phi_i - (\frac{\omega}{\upsilon}) \phi_r = -\frac{1}{r^3} \frac{\partial^2 \psi_r}{\partial z^2} \quad (2)$$

$$\frac{\partial^2 \psi_r}{\partial r^2} - \frac{1}{r} \frac{\partial \psi_r}{\partial r} + \frac{\partial^2 \psi_r}{\partial z^2} = -r\phi_r$$
(3)

$$\frac{\partial^2 \psi_i}{\partial r^2} - \frac{1}{r} \frac{\partial \psi_i}{\partial r} + \frac{\partial^2 \psi_i}{\partial z^2} = -r\phi_i \tag{4}$$

B – Dimensionless form

Let:

$$\mathbf{r}^* = \frac{\mathbf{r}}{\mathbf{R}} \quad , \quad \mathbf{z}^* = \frac{\mathbf{z}}{\mathbf{L}} \quad , \quad \mathbf{W}^* = \frac{\mathbf{W}}{\mathbf{L}} \quad , \quad \mathbf{t}^* = \omega \mathbf{t}$$
$$\mathbf{v}_r^* = \frac{\mathbf{v}_r}{\mathbf{V}_m} \quad , \quad \mathbf{v}_z^* = \frac{\mathbf{v}_z}{\mathbf{V}_m}, \quad \mathbf{v}_r^{\prime *} = \frac{\mathbf{v}_r^{\prime}}{\mathbf{V}_m}$$

$$v_z'^* = \frac{v_z'}{V_m} = \frac{1}{V_m} \left(\frac{1}{r} \frac{\partial \psi'}{\partial r}\right) = \frac{1}{V_m} \left(\frac{1}{r^* R^2} \frac{\partial \psi^*}{\partial r^*}\right) ,$$

$$\psi^* = \frac{\psi'}{V_m R^2} , v_r'^* = \frac{1}{r^*} \frac{\partial \psi^*}{\partial r^*} \left(\frac{1}{V_m R^2}\right), \phi^* = \frac{\phi' R}{V_m}$$

Equation (1), (2), (3) and (4) in dimensionless form may be written as,

$$\frac{\partial^{2} \phi_{r}^{*}}{\partial r^{*2}} + \frac{1}{r^{*}} \frac{\partial \phi_{r}^{*}}{\partial r^{*}} + \left(\frac{R}{L}\right)^{2} \frac{\partial^{2} \phi_{r}^{*}}{\partial z^{*2}} - \left(\frac{1}{r^{*2}}\right) \phi_{r}^{*} - \frac{1}{r^{*3}} \frac{\partial^{2} \psi_{r}^{*}}{\partial z^{*2}} \left(\frac{R}{L}\right)^{2} \left[\frac{1}{R_{e}}\right]$$

$$- W_{z}^{*} \frac{\partial \phi_{r}^{*}}{\partial z^{*}} \left(\frac{R}{L}\right) + S \phi_{i}^{*} = 0$$
(5)

And,

$$\frac{\partial^{2} \phi_{i}}{\partial r^{*2}} + \frac{1}{r^{*}} \frac{\partial \phi_{i}^{*}}{\partial r} + (\frac{R}{L})^{2} \frac{\partial^{2} \phi_{i}^{*}}{\partial z^{*2}} - (\frac{1}{r^{*2}}) \phi_{i}^{*} - \frac{1}{r^{*3}} \frac{\partial^{2} \psi_{i}^{*}}{\partial z^{*2}} (\frac{R}{L})^{2} \bigg] \frac{1}{R_{e}}$$
(6)
$$W_{z}^{*} \frac{\partial \phi_{i}^{*}}{\partial z^{*}} (\frac{R}{L}) - S \phi_{r}^{*} = 0$$

$$\frac{\partial^{2} \psi_{r}^{*}}{\partial r^{*2}} - \frac{1}{r^{*}} \frac{\partial \psi_{r}^{*}}{\partial r^{*}} + (\frac{R}{L})^{2} \frac{\partial^{2} \psi_{r}^{*}}{\partial z^{*2}} = -r^{*} \phi_{r}^{*}$$
(7)
$$\frac{\partial^{2} \psi_{i}^{*}}{\partial r^{*2}} - \frac{1}{r^{*}} \frac{\partial \psi_{i}^{*}}{\partial r^{*}} + (\frac{R}{L})^{2} \frac{\partial^{2} \psi_{i}^{*}}{\partial z^{*2}} = -r^{*} \phi_{i}^{*}$$
(8)

III. RESULTS AND DISCUSSION

Fig.2 shows the amplitude of the stream function (ψ_r^*) of the perturbed sinusoidal flow in the pipe in the absence of a crack. There exists a small value of phase shift from the input sinusoidal signal applied at the pipe entrance as seen from Fig 3. It is clear from the results that the signal has rapidly developed from the entrance of the pipe as can be inferred from the parallel streamlines along the axis of the pipe. This rapid development is true from the pipe centre to its wall. This obviously shows that there is no attenuation of the signal along the pipe.



Fig.(2) Amplitude of the flow stream function in the pipe without crack



Fig.(3) Phase shift of the flow stream function in the pipe without crack

On the other hand Fig. 4 Shows the amplitude of the stream function (ψ_i^*) in the presence of a crack positioned axially at the middle of the pipe length. It is clear that the presence of the crack has affected the development of the signal not only close to crack but even at the entrance of the pipe. This is attributed to the loss of momentum from the crack out flowing from the pipe wall which to be substituted at the pipe entrance.



Fig.(4) Amplitude of the flow stream function in the pipe with crack.

It can also be seen from Fig. 5 that the phase shift has also been affected by the presence of the crack despite its small value. It is be noted in this respect that the case just considered are for Reynolds number (Re=1) and Strouhal number (S=1). The effect of (Re) and(S) on the behavior of the signal is more or less qualitatively the same.



Fig. 5 phase shift of the flow stream function in the pipe with crack.

Is this respect that the axial position of the crack does not qualitatively affect the behavior of the signal as can be seen from Figs. 6 and 7 for positions (z*=0.1) and (z*=0.8) close the pipe entrance and exit respectively. This is true for all values of Re and S considered in this work.





c- R_e=100

Fig .6 a,b,c Variation ot amplitude of pressure gradient with Reynolds number (R=0.1m , R/L=0.01 , Vm=0.1m/s , U*=1 , S=1 , W=1mm , $q{=}0.001\ Q)$

For the pressure gradient phase shift it is seen that the increases in Re increase its absolute value considerably although it is relatively small. This can be seen from Figs (7).







Fig. 7 a,b,c Variation of phase shift of pressure gradient with Reynolds number (R=0.1m , R/L=0.01 , Vm=0.1m/s , U*=1 , S=1 , W=1mm , q=0.001 Q)

A -Pressure gradient and shear stress on the pipe wall in the presence of a crack:

Fig. 8 shows the effect of Re on the distribution of the wall shear stress amplitude along the pipe. Similar to the behavior observed for the pressure gradient it is the case for wall shear stress as far as the form of the distribution and the indication of the presence of the crack are concerned. It is quite clear from the figures that the value of the shear stresses amplitude varies slightly, that is less than 10%, as Re increase from 1 to 100 with S=1.





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Fig. 8 a, b ,c Variation of amplitude of shear stress with Reynolds number (R=0.1m , R/L=0.01 , Vm=0.1m/s , U*=1 , S=1 , W=1mm , q=0.001 O)

From Fig. 9 it is seen that the absolute value of the phase shift of the shear stress increases considerably with the increase in Re.





Fig.9 a, b c Variation of phase shift of shear stress with Reynolds number(R=0.1m , R/L=0.01 , Vm=0.1m/s , U*=1 , S=1 , W=1mm , q=0.001 Q)

Figs. (10) Show the effect of Strouhal number S on the amplitude of the pressure gradient along the pipe wall while keeping Re=1. The results forms are indicative of the pressure of the crack irrespective of the value of S.

On the other hand the increase in S from 1 to 100 shows no appreciable change in the pressure gradient (of order about 10%).



b- S=50



Fig. 10 a,b,c Variation of amplitude of pressure gradient with Strouhal number(R=0.1m , L/R=0.01 , Vm=0.1m/s , U*=1 , Re=1 , W=1mm , $q{=}0.001~Q)$

In a similar fashion the absolute value of the phase shift of the pressure gradient increases considerably with the increase of Strouhal number S although the value is relatively small (see Fig.11).







Figs.12 show the variation of the amplitude of the wall shear stress along the pipe with the Strouhal number (S). The behavior is quite similar to the effect of (Re) in that the shear stress varies by the order of only (10%).





b- S=50



c- S=100

Fig. 12 a,b,c Variation of amplitude of shear stress with Strouhal number(R=0.1m , R/L=0.01 , Vm=0.1m/s , U*=1 , Re=1 , W=1mm , q=0.001 Q)

Exactly the same observation as for the effect of Re applies to the phase shift of the shear stress and its relation to Strouhal number as can be seen from Figs.(13).

This behavior is very well expected by the inspection of the equations governing the perturbed flow originating from the velocity sinusoidal variations at the pipe entrance. As is clear from Eqns.(5) and (6) after multiplying both equations by Re that the dependence of the solution can be put effectively in terms of ReS.





Fig. 13 a,b,c Variation of phase shift of shear stress with Strouhal number(R=0.1m , R/L=0.01 , Vm=0.1m/s , U*=1 , Re=1 , W=1mm , q=0.001 Q)

The results have also been tested for the effect of the crack size (W) and the percentage amount of leak on the behavior and magnitude of the flow resulting from the velocity input signal.

It is found that the of the size of the crack and the amount of leak both have no sensible effect on the flow signal as can be seen figs.14. The aspect ratio (R/L) is also found to have no appreciable effect on the behavior and magnitude of the disturbing signal. Although he was the effect of this ratio clearly shows the value of the pressure at the entrance to the pipe as in Fig.15 where we note increasing pressure to increase the value of (R/L)



Fig. 14 a,b,c Variation of amplitude of pressure gradient with crack size (W) (R=0.1m , Re=1 , L/R=0.01, Vm=0.1m/s , $U^{*}{=}1$, $S{=}1$, $q{=}0.001\ Q$)





b- R/L=2000



Fig. 15 a,b,c Variation of amplitude of pressure gradient with Ratio (R/L)(R=0.1m , Vm=0.1m/s , U^*=1 , Re=1 , S=1 , W=1mm , q=0.001 \$Q\$

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III- CONCLUSIONS

The following conclusions may be drawn from the foregoing analysis and discussions:

- The presence of a crack in the pipe affects the development of the disturbing signal irrespective of the axial position of the crack. This shows how effective the signal is in detecting the presence of a crack.
- Reynolds number is seen to have no sensible effect on the detection of the crack whether on the pressure gradient and the shear stress along the pipe wall.
- Strouhal number has also no sensible effect on the detection of the crack as far as the pressure gradient and wall shear are concerned.
- The aspect ratio of the pipe, the size of the crack and the percentage amount of leak from it are not to affect the disturbing velocity signal used.
- The velocity sinusoidal signal applied at the entrance of the pipe is shown to be successful in detecting the presence of pipe crack.

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