International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 4 Issue 03, March-2015

Mathematical Modeling, Simulation and Control of Ball and Beam System

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***Abstract*—The ball and beam system can usually be found in most control labs since it is relatively easy to build, model and control theoretically. The system includes a ball, a beam, a motor and several sensors. The basic idea is to use the torque generated from motor to control the position of the ball on the beam. The ball rolls on the beam freely. By employing linear sensing techniques, the information from the sensor can be taken and compared with desired position values. The difference can be fed back into the controller, and then into the motor in order to gain the desired position. The mathematical model for this system is nonlinear but may be linearized around the horizontal region. This simplified linearized model, however, still represents many typical real systems, such as horizontally stabilizing an airplane during landing and in turbulent airflow. By considering real plant problems such as the sensor noise and actuator saturation, the controllers of the system become more efficient and robust. There are number of alternative controller design theories that can be used to stabilize the ball and beam system. In the system, to stabilize the ball and beam system modified PD controller is designed and system responses of modified PD controller and modified PD-PSO controller are compared.**

I. INTRODUCTION

The ball and beam system is also called 'balancing a ball on a beam'. It can usually be found in most university control labs. It is generally linked to real control problems such as horizontally stabilizing an airplane during landing and in turbulent airflow. There are two degrees of freedom in this system. One is the ball rolling up and down the beam, and the other is beam rotating from its one end. Ball and beam system is a very interesting system by its nonlinear dynamics and its under-actuated phenomenon.

These types of systems have wide range of industrial applications including passenger's platform balancing for comfort in luxury cars, control of exothermic chemical process where addition of heat accelerate the process, control of rocket and aircraft vertical take-off and liquid carrying tankers on the roads where liquid behaves like a ball on beam. Together with the interesting application, its dynamics gives a wide range of flexibility to implement classical, modern and advanced control techniques. It is ideal laboratory equipment to test classical, modern and advanced control system theory. The aim of the system is to control the position of the ball to a desired reference point, and reject disturbances such as a push from a finger. The control signal can be derived by feeding back the position information of the ball. The control voltage signal goes to the DC motor via a power amplifier, and then the torque generated from the motor drives the beam to rotate to the desired angle. Thus, the ball can be located at the desired position. It is important to point out that the open loop of the system is unstable and nonlinear. The problem of 'instability' can be overcome by closing the open loop with a feedback controller.

The modern state-space method can be employed to stabilize the system. The nonlinear property is not significant when the beam only deflects a small angle from the horizontal position. In this case, it is possible to linearise the system. However, the non-linearity become significant when the angle of the beam from the horizontal is larger than +50 degrees, or smaller than -50 degrees. In this case, a simple linear approximation is not accurate. Thus a more advanced control technique such as nonlinear control should work better.

Fig. 1 Ball and beam module

II. MATHEMATICAL MODELING

A. BALL AND BEAM SYSTEM:

The ball and beam module consists of a steel rod in parallel with a nickel-chromium wire wound resistor forming the track on which the metal ball is free to roll. The position of the ball is obtained by measuring the voltage at the steel rod. When the ball rolls along the track, it acts as a wiper similar to a potentiometer resulting in the position of the ball. When coupled to the servo plant, the DC motor will drive the beam such that the motor angle controls the tilt angle of the beam. The ball then travels along the length of the beam. The aim is to design a control system to track the ball to a commanded position.

For this problem, it is assumed that the ball rolls without slipping and the friction between the beam and ball is negligible. The constants and variables for this example are defined as:

L: Beam Length (42.5cm) ηg: Gearbox efficiency (0.85)
X:Ball position ηm: Motor efficiency (0.69)

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R:Radius of ball(1.27cm) Km: Back emf constant (0.00767V/(rad/sec))
m: Mass of the ball (0.064 kg) Kt: Motor torque constant (0.00767 Nm)
θ: Servo load gear angle Kg : Gearbox ratio (70)
r: Lever arm offset J: Balls moment of inertia (3.87*10^-7 kgm^2)
Rm : armature resistance(2.6Ω) Ftx: Translational force on ball
Beq :Viscous damping Coefficient Frx: Rotational force on ball
Tm : motor torque g:Gravitational force(9.8 m/s^2)
Zm : Motor impedence Jeq: Equivalent high gear moment of inertia

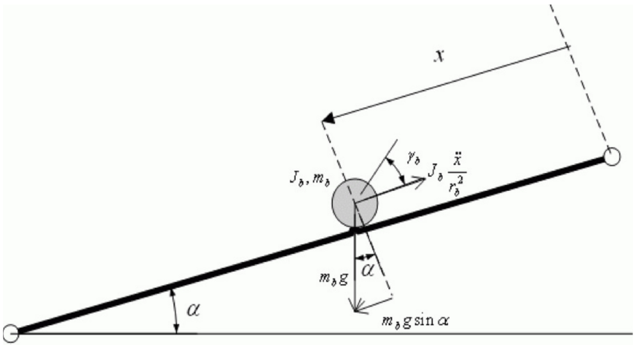

Fig.2 Ball and beam free body diagram

Let us begin by examining the forces acting on the ball. We have the translational force due to gravity, and we have a rotational force due to the torque produced by the rotational acceleration of the ball. The two forces are:
Gravitational force in the x-direction:

$$F_{tx}=mg \sin \alpha \quad (1)$$

The torque produced by the rotational motion of ball is equal to the radius of the ball multiplied by the rotational force (opposing the direction of travel). Using Newton's 2ndequation of motion, the torque is equal to the moment of inertia of ball, multiplied by its angular acceleration, which then can be written as its moment of inertia multiplied by the double derivative of its translational motion (x) divided by its radius yielding the following expression:

$$T_r = F_{rx}R= J a=J \frac{\ddot{x}}{R} \quad (2)$$

We now take the moment of inertia of ball and re-arrange equation (2) to solve for the rotational force:

$$J = \frac{2}{5}mR^2 \quad (3)$$

$$F_{rx}=\frac{2}{5}m\ddot{x} \quad (4)$$

Given all the forces acting on the rolling ball, we can again apply Newton's 2nd law of motion and equate the mass of ball multiplied by its acceleration to the sum of forces acting on the ball.

$$m\alpha= m\ddot{x}=\Sigma F= F_{tx}- F_{rx}= mg \sin\alpha - \frac{2}{5}m\ddot{x} \quad (5)$$

$$\therefore \ddot{x} = \frac{5}{7}g\ \alpha \quad (6)$$

$$\therefore \frac{X(s)}{\alpha(s)}=\frac{5g}{7S^2} \quad (7)$$

From fig.2 and the arc length traveled by the lever arm as Arc. If we measure both angles in radians, we get the following expression:

$$Arc=\theta r = \alpha L \rightarrow \therefore \theta = \frac{L}{r}\alpha \quad (8)$$

B. DC SERVO MOTOR:

To complete the system derivations, we need to know the transfer function from motor voltage Vm to output angle θ_L.
A common actuator in control systems is the DC motor. It directly provides rotary motion and coupled with wheels or drums and cables, can provide translational motion. The electric equivalent circuit of the armature and the free-body diagram of the rotor are shown in the following figure,

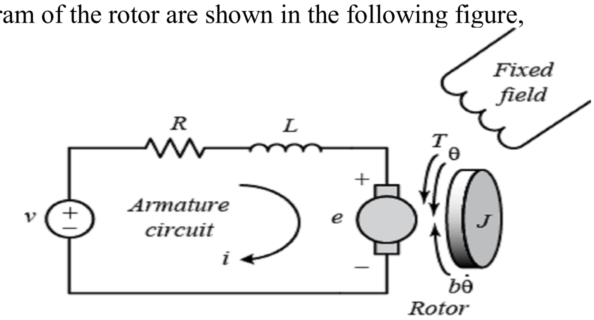

Fig.3 Equivalent circuit and the free-body diagram of the rotor of DC servo motor

In this example, it is assumed that the input of the system is the voltage source (Vm) applied to the armature of motor, while the output is the position of the shaft (θ). The rotor and shaft are assumed to be rigid. It is further assumed that, a viscous friction model, that is the friction torque is proportional to shaft angular velocity.

In general, the torque generated by a DC motor is proportional to the armature current and the strength of the magnetic field. In this example, we assume that the magnetic field is constant and, therefore, the motor torque is proportional to only the armature current i by a constant factor Kt as shown in the equation (8). This is referred to as an armature-controlled motor.

Motor torque (Tm)= Kt I (9)

The back emf, e, is proportional to the angular velocity of the shaft by a constant factor Km,

Back emf (e)= Km (10)

$$J_{eq}\ddot{\theta}_L + B_{eq}\dot{\theta}_L=(K_t K_g \eta_g \eta_m)\ i \quad (11)$$

Applying KVL to motor armature circuit,

$$Z_m i = V_m - K_m K_g \dot{\theta}_L \quad (12)$$

Taking Laplace transform of above two equations, and
By rearranging the terms finally we get transfer function

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of system,

$$\frac{\theta_L(S)}{V_m\,(S)} = \frac{K_t K_g \eta_g \eta_m}{[R_m J_{eq}]S^2 + [B_{eq} R_m + K_t K_g{}^2 K_m \eta_g \eta_m]S} \qquad (13)$$

III. CONTROLLER DESIGN AND MATLAB SIMULATION RESULTS

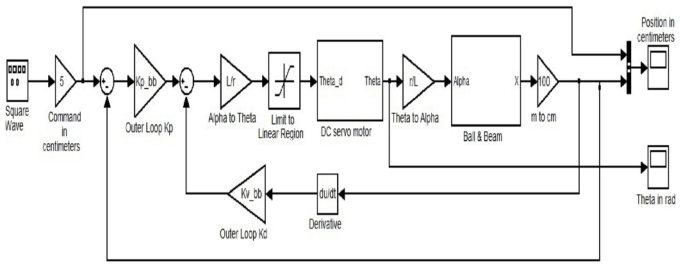

Fig.4 Ball and beam with PD controller

The outer loop (ball and beam) controls the ball position using the feedback law

$$\alpha = K_{p\_bb}(X_d - X) - K_{v\_bb} S\, X \qquad (14)$$

For outer loop

$$G(s) = \frac{X(s)}{\alpha(s)} = \frac{5g}{7S^2} \qquad (15)$$

And feedback path

$$H(s) = K_{p\_bb}(X_d - X) - K_{v\_bb} S\, X \qquad (16)$$

using Close loop transfer function of system,

$$\frac{1}{1+G(s)H(s)}$$

and Comparing it with standard second order system

$$\frac{\omega_n^2}{S2 + 2\xi\,\omega_n\, S + \omega_n^2}$$

Controller gains for outer loop (ball and beam),

$$K_{v\_bb} = \frac{2\xi\,\omega_{n\_bb}}{7} \qquad (17)$$

and

$$K_{P\_bb} = \frac{\omega_{n\_bb}{}^2}{7} \qquad (18)$$

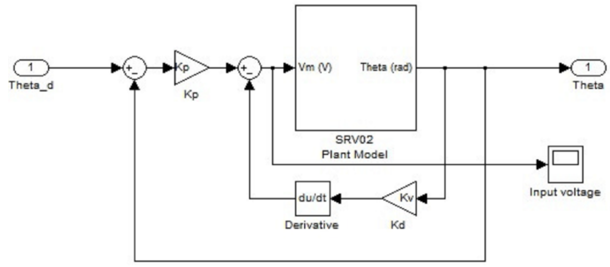

Fig.5 DC servo motor with PD controller

$$G(s) = \frac{\theta_L(S)}{V_m\,(S)} = \frac{K_t K_g \eta_g \eta_m}{[R_m J_{eq}]S^2 + [B_{eq} R_m + K_t K_g{}^2 K_m \eta_g \eta_m]S} \qquad (19)$$

The inner loop (DC servo motor) feedback law ,

$$H(s) = V_m\,(s) = K_p(\theta_d - \theta_L) - K_v\, S\, \theta_L \qquad (20)$$

Using Close loop transfer function of system

$$\frac{1}{1+G(s)H(s)}$$

and Comparing it with standard second order system

$$\frac{\omega_n^2}{S2 + 2\xi\,\omega_n\, S + \omega_n^2}$$

We get Controller gains for inner loop (DC servo motor),

$$K_p = \omega_n{}^2 \frac{R_m J_{eq}}{K_t K_g \eta_g \eta_m} \qquad (21)$$

And

$$K_v = \frac{2\xi\omega n\, R_m J_{eq} - (B_{eq} R_m + K_t K_g{}^2 K_m \eta_g \eta_m)}{K_t K_g \eta_g \eta_m} \qquad (22)$$

Now ball position response with these controllers when they are used together in system is as shown in figure below,

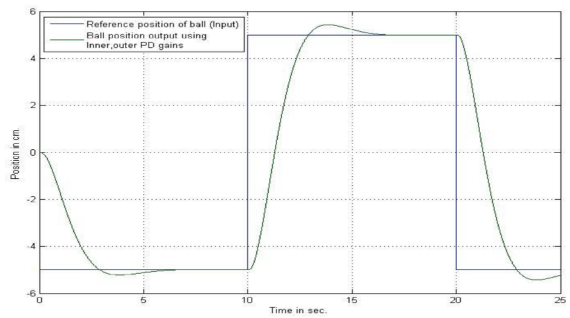

Fig.6 Position response of ball and beam system with double PD controller

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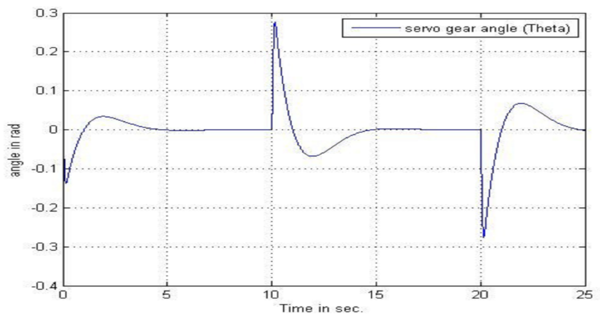

Fig.7 Servo gear angle response of ball and beam system with modified PD controller

IV. PARTICLE SWARM OPTIMIZATION FOR MODIFIED PD CONTROLLER

Several optimization techniques using swarming principle have been adopted to solve a variety of engineering problems in past decade. Particle Swarm Optimization (PSO) is such swarming techniques which can be used for PD controller parameter optimization for ball and beam system. PSO is applied and its results are compared to the results of modified PD controller to find out which is better. Time domain analysis is tabulated for our model controlled with optimal value of modified PD controller obtained using PSO algorithm.

A modified PD controller using the PSO algorithm is developed to improve the response of system. It was also called the PSO-PD controller. The PSO algorithm is mainly utilized to determine optimal controller parameters Kp, Kv, Kp_bb and Kv_bb.

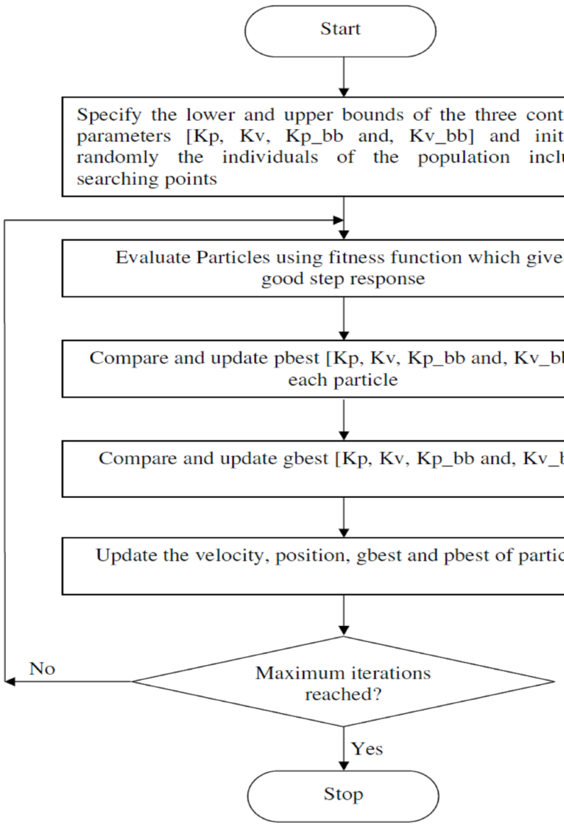

Fig.8 The flow chart for modified PD- PSO controller

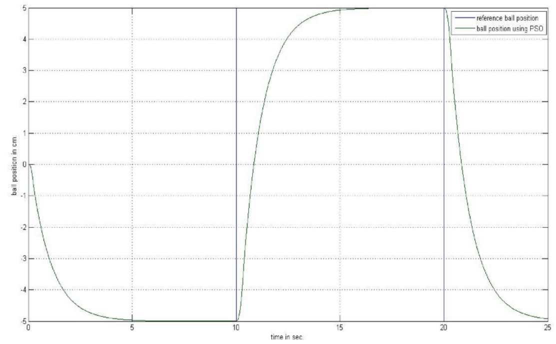

Fig.9 Position response of ball with modified PD-PSO controller

V. COMPARISON OF SYSTEM RESPONSES OF MODIFIED PD CONTROLLER AND MODIFIED PD-PSO CONTROLLER

By comparison of system response with modified PD controller and with modified PDPSO controller, it shows that modified PD-PSO method can improve the performance of the system in better way.

Comparative results for ball position using modified PD and controllers are given below in Table1,

Table1: Comparison of system responses of modified PD controller and modified PD-PSO controller

Method	Kp, Kv, Kp_bb and, Kv_bb	Overshoot (%)	Settling Time (s)	Rise Time (s)
With modified PD	K_p=11.1206 V/rad K_v=0.1110 V/rad/sec K_{p_bb}=0.00176rad/cm K_{v_bb}=0.0022rad/cm/sec	9.67	6.875	2.81
With modified PD-PSO controller	K_p=22.0657 V/rad K_v=0.6194 V/rad/sec K_{p_bb}=0.3853 rad/cm K_{v_bb}=0.3800rad/cm/sec	0	6.349	6.349

CONCLUSION

A new design method to determine PD controller parameters using the PSO method is presented. Simulation results obtained show that the proposed controller can perform an efficient search for the optimal PD controller. By comparison with modified PD controller, it shows that, this method can improve performance of the system. The PSO-PD controller is the best which presented satisfactory performances and possesses good robustness (no overshoot, minimal settling time).

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