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MATLAB Programming Solution For Critical And Normal Depth In Trapezoidal Channels

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Abstract

Critical depth and Normal depth are an essential parameter in the analysis of varied flow in open channels. The governing equations for critical depth are implicit and no analytical solutions exist. In trapezoidal channel, the governing equations are highly nonlinear in the normal and critical flow depths and thus solution of the implicit equations involves numerical methods. So many solutions already exist in the form of empirical relations and tabulated form. Equations are lengthy and chances of getting error are more. As in advance computing age MATLAB programming got importance for these types of computational problems. In this study result obtained by programming is compared with different explicit solutions and method of genetic algorithm. It is found that result obtained from recent study is up to millimeter of accuracy from results obtained from genetic algorithm and explicit solution suggested by Ali and Essa.

Introduction

Critical depth and Normal depth calculation is an important task for hydraulic engineers in the design of conveyance open channels for irrigation, drainage, and water supply projects. The flow depth corresponding to the minimum specific energy for a given discharge in an open channel is known as the critical depth. The critical depth concept is used as the basis for flow characterization, i.e., sub- or supercritical. The flow at critical condition of an open channel is unstable. At critical condition, a small change in specific energy will cause abrupt fluctuation in water depth of the channel. This is because the specific energy curve is almost vertical at critical state. Therefore, if the design depth of the channel is near or equal to critical depth of the channel, the shape of the channel must be altered to avoid a large fluctuation in water depth. Open-channel design is often based on the assumption of uniform flow and normal depth [5]. Because of the implicit nature of governing equations, direct solution of normal depth is also not possible as critical depth and one has to resort to tedious iterative techniques [6]. But any iterative technique is easy to use with the help of programming interface. In this current study the calculation of critical and normal depth has been done with the help of MATLAB programming. Solution established by MATLAB programming is compared with some explicit solution for critical and normal depth [1-3] and the critical depth of trapezoidal section is compared with genetic algorithm method [4].

Methodology

Newton's method use derivative calculus to find the roots of a function or relation by first taking an approximation and then improving the accuracy of that approximation until the root is found. Newton's Method is used to find the root of an equation provided that the function f[x] is equal to zero. $f: [a, b] \rightarrow R$ is a differentiable function defined on the interval [a, b] with values in the real numbers R. Better approximation, x_{n+1} can be calculated from current approximation x_n and the definition of the derivative at a given point that it is the slope of a tangent at that point. Here, f ' denotes the derivative of the function f. Then by simple algebra we can derive

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)} \qquad (1)$$

The process was started with some arbitrary initial value variable.

Basic Equations and Functions

Specific energy (E) in an open channel is calculated from Eq. (2)

$$E=y\cos^2\theta+\frac{\alpha Q^2}{2gA^2} \qquad (2)$$

Where, θ= Bed slope; α=kinetic energy correction factor; y=flow depth; A=flow area; Q=discharge; and g=gravitational acceleration. By differentiating we get the Eq. (3).

$$\frac{dE}{dy}=\cos^2\theta-\frac{\alpha Q^2\frac{dA}{dy}}{gA^3} \qquad (3)$$

For the minimum specific energy first derivative with respect to variable y should be zero. If the slope of the channel is small (less than 10%) than cosθ = 1 and the governing equation for the critical depth as in Eq. (4).

$$\frac{\alpha Q^2 T}{gA^3}=1 \qquad (4)$$

Where T = Top Width of the channel (dA=Tdy). For trapezoidal section both top width and area is the function of y and it cannot be separated for the solution of y. Manual calculation is time taking and cumbersome to calculate the critical depth for the trapezoidal section [1]. Especially where the practical design is concern MATLAB coding will serve the purpose by using Newton method of derivative for finding the root of function. For calculating critical depth trapezoidal section, function and its derivative is defined as Eq. (5) & Eq. (6)

$$F(y)=\frac{A^{3/2}}{\sqrt{T}}-\frac{\alpha Q}{\sqrt{g}} \qquad (5)$$

Where; A= $F_1(y)$ & T = $F_2(y)$

$$F'(y)=\frac{3\sqrt{AT}}{2}-z\left(\frac{A}{T}\right)^{3/2} \qquad (6)$$

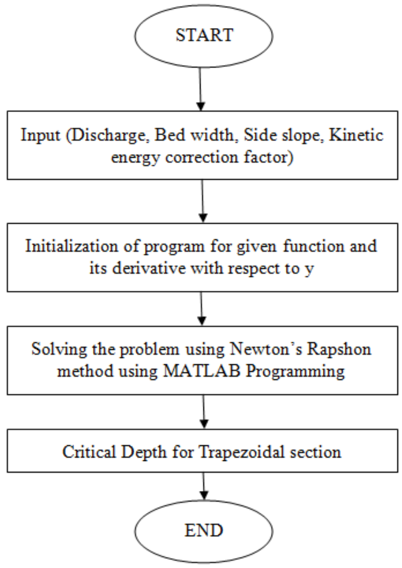
For calculating normal depth trapezoidal section, function and its derivative is defined as Eq. (7) & Eq. (8)

$$F(y)=\frac{\sqrt{s}AR^{2/3}}{\eta}-Q \qquad (7)$$

Where; P= wetted perimeter; R= Hydraulic Radius (A/P); η= Manning's Coefficient; s= Bed Slope; z= Side Slope (H: V) and A= $F_1(y)$; P = $F_2(y)$; T= $F_3(y)$ & R = $F_4(y)$.

$$F'(y)=\frac{\sqrt{s}}{\eta}\left[R^{2/3}T+\frac{T}{P}-\frac{2yR}{P}\right] \qquad (8)$$

Flow Chart for solving the problem



MATLAB Programming Code:

1. For Critical Depth

```
Input Parameters for Trapezoidal cross section
Q= input('discharge in cumec=');
b= input('bottom width of channel in m=');
z= input('side slope H to V=');
a= input('kinetic energy correction factor=');
yinitial=0.01; %Initial Guess to start the iterations
% Critical Depths Calculation
   yc(1)=yinitial;
   ic=1;
   dyc(1)=1e-2;
   while (abs(dyc(ic))>1e-4)
       Ac(ic)=b*yc(ic)+z*(yc(ic))^2;
      Tc(ic)=b+2*z*yc(ic);
      Pc(ic)=b+2*(z^2+1)^(0.50)*yc(ic);
      Rc(ic)=Ac(ic)/(Pc(ic));
      Dc(ic)=Ac(ic)/(b+2*z*yc(ic));
      fc(ic)=Ac(ic)^(3/2)*Tc(ic)^(-1/2)-
Q/sqrt(9.81/a); %
      ffc(ic)=Ac(ic)^(3/2)*(-1/2)*Tc(ic)^(-
3/2)*2*z+Tc(ic)^(-1/2)*(3/2)*Ac(ic)^(1/2)*Tc(ic); %
f' Derivative
      yc(ic+1)=yc(ic)-fc(ic)/ffc(ic);
      dyc(ic+1)=-fc(ic)/ffc(ic);
      ic=ic+1;
   end
   criticaldepth=yc(ic);
```

2. For Normal depth

```
% Input Parameters for Trapezoidal cross section
Q= input('discharge in cumec=');
b= input('bottom width of channel in m=');
z= input('side slope H to V=');
n= input('manning coefficient=');
S= input('Longitudinal slope=');
```

```
yinitial=0.01; %Initial Guess to start the iterations
% Normal Depth Calculation
  yn(1)=yinitial;
  in=1;
  dyn(1)=1e-2;
  while (abs(dyn(in))>1e-4)
    An(in)=b*yn(in)+z*(yn(in))^2;
    Tn(in)=b+2*z*yn(in);
    Pn(in)=b+2*(z^2+1)^(0.50)*yn(in);
    Rn(in)=An(in)/(Pn(in));
    Dn(in)=An(in)/(b+2*z*yn(in));
    fn(in)=sqrt(S)*An(in)*Rn(in)^(2/3)*n^(-1)-Q;
    ffn(in)=(sqrt(S)*n^(-
1))*((Rn(in)^(2/3)*Tn(in))+(Tn(in)/Pn(in))-
((2*yn(in)*Rn(in))/Pn(in)));
    yn(in+1)=yn(in)-fn(in)/ffn(in);
    dyn(in+1)=-fn(in)/ffn(in);
    in=in+1;
  end
  Normaldepth=yn(in);
```

Results

An open channel having a flow of 17 m^3/s and bottom width of the channel is 6 m. Manning's coefficient is taken as 0.0145 with side slope 2:1 (Horizontal: Vertical). Longitudinal slope of the channel is taken as 0.002 and kinetic energy correction factor is taken as 1. Critical depth and Normal depth for above mentioned problem obtained from some researchers by explicit solution and genetic algorithm is given in Table below.

Table 1 Critical Depth and Normal Depth

Suggested Solutions	Critical Depth (m)
Ali R. Vatankhaha [3]	0.8487
Ali R. Vatankhaha,Said M. Easa [2]	0.8465
Wang et al. [1]	0.8557
A Kanani et al.(GA Method) [4]	0.8462
Proposed Methodology (MATLAB Programming)	**0.8468**
	Normal Depth (m)
Ali R. Vatankhaha [3]	0.665
Proposed Methodology (MATLAB Programming)	**0.6647**

Order of accuracy found from Ali & Easa [2], Genetic Algorithm (GA) from Kanani et al. [4] and MATLAB programming is up to millimeter. Critical depth obtained from Ali & Easa [2] explicit solution is 0.8465 m and Kanani et al. [4] found the critical depth from the GA method is 0.8462 m. From the proposed MATLAB programming it is seen that critical depth is 0.8468 m. Normal depth from Ali [3] 0.665 m for the same above mentioned problem is same as 0.6647 m.

References

1. Wang ZZ, Yuan S, Wu CL. "An inquiry on a formula for calculating critical depth of open channel with trapezoidal cross section". J Hydraul Eng 1999;43(4):14–7

2. Vatankhah Ali R, Easa Said M. Explicit solutions for critical and normal depths in channels with different shapes. Flow Meas Instrum 2011;22(1):43–9.

3. Ali R. Vatankhah, "Explicit solutions for critical and normal depths in trapezoidal and parabolic open channels". (Article in press), http://dx.doi.org/10.1016/j.asej.2012.05.002

4. A. Kanani; M. Bakhtiari; S. M. Borghei; and D.-S. Jeng, **"**Evolutionary Algorithms for the Determination of Critical Depths in Conduits", *Journal of Irrigation and Drainage Engineering*, Vol. 134, No. 6, December 1, 2008. ©ASCE, ISSN 0733-9437/2008/6-847–852
5. David C. Froehlich, "Width and depth-constrained best Trapezoidal section", *Journal of Irrigation and Drainage Engineering*, Vol. 120, No. 4, July/August, 1994. @ASCE, ISSN *0733-9437/94/0004-*0828
6. K. Babaeyan-Koopaei ,"Dimensionless curves for normal-depth calculations in Canal sections" *Journal of Irrigation and Drainage Engineering*, Vol. 127, No. 6, November/December, 2001. @ASCE, ISSN 0733-9437/01/0006-0386–0389