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Micromechanics Analysis of Fiber Reinforced Composite

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***Abstract*—This dissertation concerned with the micromechanical analysis of the unidirectionally fiber reinforced composites. Two idealized packing have been developed to carry out the analysis in finite element software package ABAQUS. Translational symmetric transformation has been applied to make the simplest model for the analysis which can save the time for the analysis. Appropriate periodic boundary conditions have been derived for both the packing by using uniform microscopic field. Unit cell obtained from this section can be subjected to arbitrary combination of the macroscopic strains and the response of the unit cell is observed to take out the effective material properties using the simple elasticity approach. Mathematical models are presented to obtain the values of the effective material properties so that they can compared with the numerical model. Further parametric study has been carried out to check the dependency of fiber and matrix on the overall effective material properties. More significant analysis has been done on effective material properties to make it useful for the selection of the appropriate material for the specific application**

Keywords-Effective material properties, finite element method, ABAQUS, unit cell

I. Introduction

Composites are defined as a multiphase material which consists of different materials in order to obtain desired properties that the individual constituent by themselves cannot attain. Composite materials can be tailored for various properties by appropriately choosing their components, their proportions, their distributions, their morphologies, their degrees of crystalline, their crystallographic textures, as well as the structure and composition of the interface between components (Deborah 2009). Due to this strong tailor ability, they are capable for different applications like automobiles, aerospace, construction, electronics, energy, biomedical and other industries. Composites are popular for their high strength to weight ratio and stiffness to weight ratio. Composites are broadly classified as fibrous, laminated, particulate, and hybrid composites. Present study aims on developing analysis procedure for unidirectional fiber reinforced composites. Usually fiber reinforced composites consist of two phases: one is matrix and the other is fiber.

Micromechanics

Micromechanics is an increasing trend in order to understand the behaviour of modern material with sophisticated microstructures, e. g. fibre or particulate reinforced composites, textile composites, etc. (Li 2007). Micromechanics have become an important means of understanding the mechanical behaviour of the composite material. Certain assumptions are taken into account like idealized packing of the fibre in the matrix. First step for this kind of analysis is to introduce representative unit cell and then apply uniform strain boundary condition to analyze the material properties of the composite.

A physical property of the composites depends upon the microstructure, which is design during its manufacturing. Volume fraction of the reinforcing material must be known before its processing. However, final design of composite is limited to some extent, which results in complex and micromechanical interaction. This develops dilemma in modelling the relation between microstructure and the characteristics of the material. In many studies assumption has been taken that the dispersion of the reinforcing element is regular within the matrix material. Due to this assumption it significantly simplifies the calculations and gives acceptable results. It also helps to simplify the finite element model to some extent but in case of crack propagation and plastic zones this assumption leads to seriously incorrect results (Pyrz 2008). So, it is required to mention the dispersion characteristics of microstructure if one analyzes the micro cracks and plastic zones.

If the material is statistically homogeneous, which means that the local material properties are constant when averaged over a representative volume element, then it is possible to replace the real disordered material by a homogeneous one in which the local material properties are the averages over the representative volume element in the real material. Modern technology has found extensive use for unidirectionally fibre-reinforced composite materials. To make effective use of these materials, knowledge of their properties and performances when subjected to loads is essential. Many aspects of their behavior are directly associated with the microscopic structure of these materials. The desire to understand these materials drives the research in this field into the micromechanics of this type of materials (Hashin 1983).

Unidirectionally fiber reinforced composites has been considered in this dissertation, by assuming idealized fiber-matrix arrangement in square packing and hexagonal packing. In previous study square packing is analyzed in (Adams, Crane and Donar 1984), (Li 1999) and for hexagonal packing in (Avril and Carman 1992), (Li 1999) and (Zou and Li 2000). The superiority of the hexagonal packing to the square packing is that it preserves this characteristic while the effective properties obtained from square packing show significant transverse anisotropy. The transverse isotropy achieved through a hexagonal packing, however, is at a price, i.e. the unit cell from it is substantially more sophisticated than that from a square packing (Li 2000). Apart from the square and hexagonal many author have tried different symmetries like cylindrical unit cell, used in literature of (Rosen and Hashin 1964) and (McCartney 1992). This model showed good results for the material properties.

II. LITERATURE REVIEW

In (Xia, et al. 2006), the macrostructure was considered as a periodic array of a repeated unit cell (RUC). RUC was constructed assuming a uniform distribution and the same geometry for the reinforcing phase. The uniqueness of solution by applying unified displacement difference periodic boundary conditions on the repeating unit cell models (RUCs) was proved. Illustrative examples were presented and advantages of applying this type of boundary conditions were discussed. Uniqueness was proved by analyzed the RUCs in displacement based FEM analysis. By applying enough sets of global strains in the unified periodic boundary condition, entire stiffness or a flexibility matrix for a periodic composite structure was predicted. Further it was suggested that the proposed unified boundary conditions can also be applied to non-linear micro mechanical analysis of composites under any combination of multi axial load.

The mechanical behavior of the composites was derived from the use of the micromechanics modeling method which provide the whole behavior by using the known properties of the constituents. Nature of the composites was predicted using the repeating volume element or unit cell model in (Aboudi 1991).A mathematical presentation of periodic composites, called asymptotic homogenization theory, can be found, e.g. (Moorthy, et al. 2001)in among others.

(Aboudi 1991) Has developed a unified micro-mechanical theory based on the study of interacting periodic cells, and it was used to predict the overall behavior of composite materials both for the elastic and inelastic constituents. In his work and many other references, homogeneous displacement boundary conditions equivalent to the ‘‘plane-remains-plane’’ conditions were applied to the RVE or unit cell models. In fact, the ‘‘plane-remains-plane’’ is only valid for the symmetric RUC subjected to normal tractions. Many researchers, e.g., (Needleman and Tvergaard 1993), have indicated that the “plane-remains-plane” boundary conditions are over-constrained boundary conditions.

(Li 2008) Has devoted his study to the generation of such an account, where boundary conditions were derived entirely based on the symmetries which present in the microstructure. The implication of the boundary condition was discussed. Also, it was demonstrated that unit cell of same appearance but subject to boundary condition derived based on the different symmetry consideration may behave rather differently. It also depicted to inform the user of unit cell that to introduce a unit cell one needs not only the mechanically correct boundary condition but also a clear sense of microstructure under consideration.

(Paley and Aboudi 1992) Has analyzed fibrous composites with periodic structure, the repeating volume element consists of four interacting sub-cells. It also offers generalization of method to an arbitrary number of sub-cells for the modeling of multiphase periodic composites. Effective constitutive laws that govern overall behavior of the elastic-visco-plastic composite material were established. Comparison between the response of boron/aluminum composite obtained and finite element solution were given.

Finite element method is very useful in analyzing the RUCs. It determines the mechanical and damage mechanisms of composites. Many authors have done the finite element analysis on different types of composites like unidirectional laminates (Allen and Boyd 1993), cross ply laminates (Bigelow 1993), woven and braided textile composites (Dasgupta, Agrawal and Bhandarkar 1996). High computer performance in combination with easy-to-use commercial model-creation software (Pro/Engineer, AutoCAD, etc.) and FEM software has contributed to this development. Thus it has become relatively easy to apply FEM to solid RUCs with all levels of complexity.

III. METHODOLOGY

Representative unit cells (RUCs)

Figure 1 shows continuum and point Q is surrounded by infinitesimal material elements. When the micro element is magnified, it may have a complex structure consisting of voids, cracks, inclusion, grains and other defects.

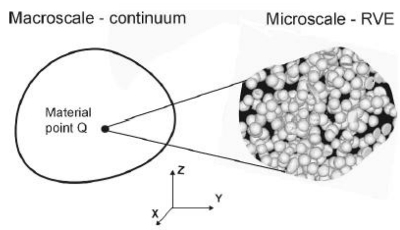


Figure 1: Representative Volume element (Pyrz 2008)

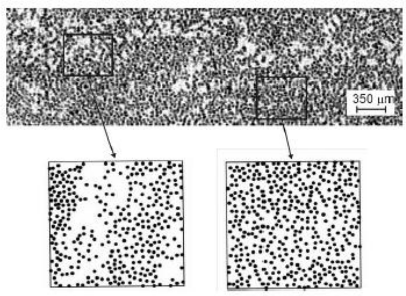


Figure 2: Cross-section of unidirectional glass/epoxy composites from X-ray microtomographic scanning (Pyrz 2008)

Figure 2 shows random distribution of the fibre in matrix by using X-ray microtomographic scanning. As shown in the figure the rich area of fibre may behave differently than matrix rich area. Therefore, the matrix rich areas constitute the smallest entity of the microstructure and are judged to have pronounced effects on the overall response of the continuum. The representative volume element is expected to have the same effective properties that the whole material (Pyrz 2008).

Figure 3 shows the cross-section of unidirectional fiber reinforced composites with different selection of periodic elements. By taking an assumption of idealized arrangement of fiber in matrix, many different unit cells can be chosen to simplify the geometric consideration.

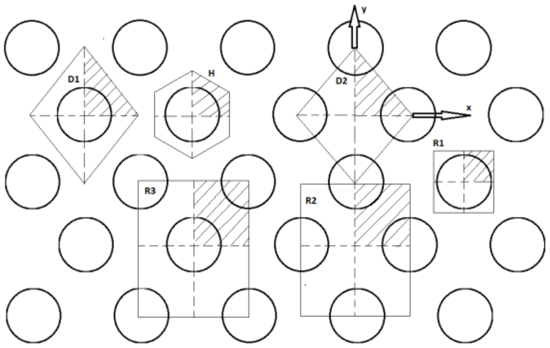


Figure 3: Square and Hexagonal fiber-matrix layout and various periodic elements

As shown in fig.3 many different shapes of the repeating element can be considered. When this periodic element (D1, D2 and H) is translated in two selected direction 60° apart and X- direction, the whole region can be covered. Similarly, the rectangular shapes R can be chosen and translation in X-direction and y-direction, the whole region can be covered. Also by employing symmetries about x=0 and y=0 respectively, various geometries can be obtained as shown in the figure 2.3 by shaded region. Triangular elements can be obtained from D1 and D2, trapezoidal unit cell from H and smaller rectangular element from R1, R2 and R3 (Li 1999). As mentioned earlier this dissertation concerns with idealized packing of the fibers in matrix, which simplifies the geometry and also reduce the cost of modelling in finite element software. Also this dissertation focuses on the 2D problems rather than 3D problem, which reduce the simulation time. So, layouts were selected for the analysis i.e. square layout and hexagonal layout. Layouts are shown in the figure 4 and 5.

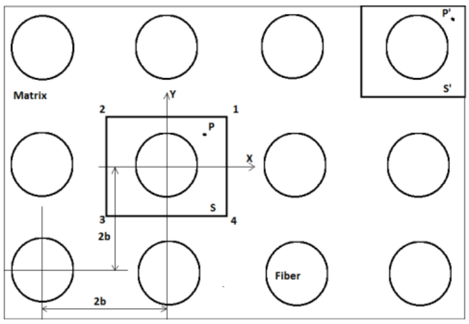


Figure 4: Square packing

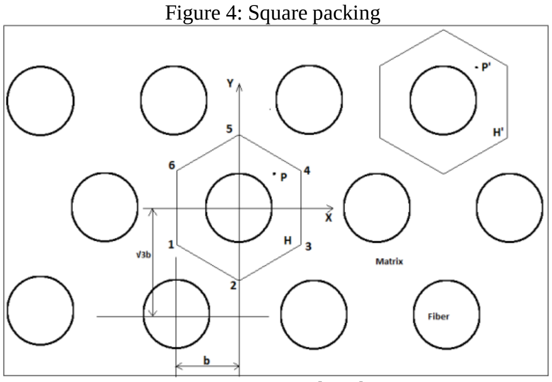


Figure 5: Hexagonal packing

Voronoi tessellation method for unit cells

Assumptions are considered for the unidirectionally fiber reinforced composites are that the fibers are infinitesimally long and every cross-section of the composite perpendicular to the fiber are identical. The micromechanical analysis of such a material can then be simplified to a two-dimensional problem in the plane of a cross-section of the composite. In this plane, a mathematical approach, the Voronoi tessellation, can be adopted to tessellate the domain of interest in the plane with the centers of the fibres being the centers of Voronoi cells as mentioned in (Li 2000). These cells are called Voronoi cells.

In this method cells are separated by segments and passing through mid-point of the cell and those neighboring cells. Such a method can be employed the unidirectional fiber reinforced composites consists of random arrangement of the fibers. However, the performance of random distributed fiber in matrix is still unavailable. Square and hexagonal packing can be obtained from this method in which all Voronoi cells are identical and can be reproduced more by using translational symmetry transformation. Thus, the obtained cells are the natural choice for the unit cell method involves square and hexagonal packing.

Geometrical consideration

A. Square packing

As shown in the figure 2.4 the obtained unit cell S from Voronoi tessellation gives region covered by two pairs of sides.

$x = \pm b$ and $y = \pm b$

So, if we consider an unit cell, then b=1

$x = \pm 1$ and $y = \pm 1$

Where, b is the half spacing between two fibers. Area of the unit cell will be

$A = 4\boldsymbol{b^2}$

(1)

Let us consider the radius of the fiber than fiber volume fraction will be,

$V_f = \boldsymbol{\pi a^2/4b^2}$

(2)

To evaluate the compactness of the unit cell, the maximum achievable fiber volume fraction is,

$V_f = \pi/4 = 78.54$ %, when a = b

Below table 2.1 shows the corresponding value of the fiber radius using the equation 2.

Fiber volume fraction (%)	Fiber radius a (mm)
40	0.35
50	0.39
60	0.43
70	0.47

Table 1: Corresponding value of fiber radius

B. Hexagonal packing

As shown in figure 2.5, hexagonal unit cell can be obtained from the use of the Voronoi tessellation which consists of three pair of sides

$x = \pm b$ and $\pm x + \sqrt{3}\, y = \pm 2b$

Where, b is the half spacing between two fibers. Then the area of the unit cell will be

$A = 2\sqrt{3}\, \boldsymbol{b^2}$
(3)
Similarly, as shown in the relation between fiber volume fraction and fiber radius can be obtained as,
$V_f = \pi\boldsymbol{a^2} / 2\sqrt{3}\boldsymbol{b^2}$
(4)
The maximum achievable fiber volume fraction is,
$V_f = \pi/2\sqrt{3} = 90.69\%$, when a=b

In this dissertation, the analysis has been done for the fiber volume fraction of 40%, 50%, 60% and 70%, which covers most of the available material criteria. Below table 2.2 shows the corresponding value of the fiber radius using the equation 4.

Fiber volume fraction (%)	Fiber radius a (mm)
40	0.33
50	0.37
60	0.40
70	0.43

Table 2: Corresponding values of fiber radius

The maximum fiber volume fraction in both the packing is different. As specially in hexagonal packing any fiber remains at equal distance from the next fiber which does not happen in the square packing. In square packing the distance between 0°, 90° direction of the fiber are same but for 45° direction it does not remain same. This is because of the large transverse isotropy in hexagon packing and transverse anisotropy in square packing. (Li 2000). This obtained value of radius was used for the modeling of the unit cells in ABAQUS finite element software package.

IV. Periodic Boundary Conditions

A. Displacement field under uniform macroscopic strain

Use of translational symmetry has been made to derive the boundary condition for the square unit cell and relation between displacement and strain was found out...As, this dissertation concerned with the 2D problem, the strains applied to the unit cell are ε_{xx}, ε_{yy} and γ_{xy} and displacements are *u* and *v*. Then, kinematic equations can be given by:

$$\varepsilon_{xx} = \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}} \;,\quad \varepsilon_{yy} = \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{y}} \;,\quad \gamma_{xy} = \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{y}} + \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{x}}$$
(5)
Equation 5 can be written as,
$u = \int \boldsymbol{\varepsilon_{xx} dx} + f_1(y) = \varepsilon_{xx}.x + f_1(y)$
(6)
$v = \int \boldsymbol{\varepsilon_{yy} dx} + f_1(y) = \varepsilon_{yy}.y + f_1(x)$
(7)
Substituting the equation 6 and 7 in equation 5 of shear strain,
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial f1(y)}{\partial y} + \frac{\partial f2(x)}{\partial x}$$
Where,
$f_1 = ay + b$, and
(8)
$f_2 = cx + d$
(9)
Substituting equation 8 and 9 in equation 6 and 7respectively,
$u = \varepsilon_{xx}.x + ay + b$
(10)
$v = \varepsilon_{yy}.y + cx + d$
(11)
Further substitution if displacement in the kinematic equation gives,
$\gamma_{xy} = a + d$
At x = y = 0 and u = v = 0 then, (rigid body constraint)
b = 0, d = 0
At x=1, y= 0 and v = 0 then,
c = 0 and $a = \gamma_{xy}$
Substituting all the obtained values in equation 10 and 11 will give,
$u = \varepsilon_{xx}.x + \gamma_{xy}\, y,$
(12)
$v = \varepsilon_{yy}.y$
(13)
Where, x and y are co-ordinate of arbitrary point P in x-y axis.
Similarly, the relation for the image point P' can be given by,
$u' = \varepsilon_{xx}.x' + \gamma_{xy}\, y',$
(14)
$v' = \varepsilon_{yy}.y'$
(15)
Subtracting equation 14 and 15 with equation 12 and 13,
$u' - u = \varepsilon_{xx}.(x' - x) + \gamma_{xy}.(y'-y)$
(16)
$v' - v = \varepsilon_{yy}.(y' - y)$
(17)

Above equations will be used to derive the displacement boundary conditions for the required unit cells derive from the Voronoi tessellation.

B. Displacement boundary condition for the square unit cell

Use of this type of packing is fully justified because of the assumption of regular arrangement of the fiber in matrix. The results of this type of packing will be compared with the hexagonal packing. Also this type of packing was considered by many authors because of its simplicity inn modeling and easy derivation to find the boundary condition for the square unit cell. Square unit cell is shown in the figure 2.4. In this dissertation x-y plane is considered for the 2D problem. By considering area S', arbitrary point P(x, y) on S can be mapped on S' as P' (x', y'). But square unit cell has a unit dimension in both x and y direction. So, the distance between these two arbitrary points will be unity i.e. $\Delta x = \Delta y = 1$.
Thus,
$u' - u = \varepsilon_{xx}.\Delta x + \gamma_{xy}.\Delta y$
(18)
$v' - v = \varepsilon_{yy}.\Delta y$

For the micromechanical analysis of the square unit cell, it is necessary to apply boundary condition around the edge of the unit cell. Translational symmetry transformation was used to derive these boundary conditions. The selection of an arbitrary point P and P' is important for the analysis. If point P is on one side than P' must be on opposite side of the unit cell.
Then, the equation boundary condition for two pair of sides are given by,
For E4 and E2 ($\Delta y = 0$, $\Delta x = 1$),

$u_4 - u_2 - \varepsilon_{xx} = 0$ and
(19)
$v_4 - v_2 = 0$
For E3 and E1 ($\Delta x=0, \Delta y = 1$),
$u_1 - u_3 - \gamma_{xy} = 0$ and
$v_1 - v_3 - \varepsilon_{yy}= 0$

It is also needed to consider corners because corners are not fully independent. Corners are shared by two sides. Many publications did the mistake to formulate the boundary condition for the corners. As shown in the figure 8 corners V1, V2, V3, V4 are selected precisely to formulate the boundary conditions for them. Symmetry transformation has been used for the formulation. Corners were selected on the basis of their sharing to the next unit cell. Pair of corners are selected which was like V1 and V2, V4 and V3, V2 and V3. Boundary condition for them is as follows:
For V1 and V2 ($\Delta y = 0, \Delta x = 1$)
$u_1 - u_2 - \varepsilon_{xx} = 0$ and
(20)
$v_1 - v_2 = 0$
For V4 and V3 ($\Delta y = 0, \Delta x = 1$)
$u_4 - u_3 - \varepsilon_{xx} = 0$ and
$v_4 - v_3 = 0$
For V2 and V3 ($\Delta x = 0, \Delta y = 1$)
$u_2 - u_3 - \gamma_{xy} = 0$ and
$v_2 - v_3 - \varepsilon_{yy} = 0$

Where, subscript indicates displacement of the corresponding corner in the square unit cell. Input file for ABAQUS is created in which these boundary conditions were introduced to simulate micromechanical analysis. With the use of this boundary condition micromechanical analysis has been performed. Many publications had taken an assumption of the sides keep straight after the deformation which restricts the use of unit cell. No such a restriction has been applied in this dissertation. As mentioned by the (Li 2000) only displacement boundary conditions are not enough to determine the solution yet, traction boundary conditions are required for the completeness of the presentation of the problem. But in this dissertation traction boundary conditions are eliminated from the analysis.

C. Displacement boundary condition for Hexagonal packing

As stated by the (Li 2000) hexagonal packing delivers the transversely isotropic characteristics which a real composite possesses in a statistical sense by having fibres distributed in matrix completely at random over the cross-section perpendicular to the fibres. In this dissertation symmetry has been employed translation in x-y direction, rotation and reflection about x-y axis about a special point. As shown in the figure 2.5 chosen arbitrary point P (x, y) in H is transformed to P' (x', y') in H'. So, by using equation 18, boundary condition for the hexagonal packing can be derived. But before deriving the boundary condition one needs to assign each side and corner with specific variable and also the distance between the each corner and edge are needed to derive the boundary condition for the hexagonal packing.

To determine the distance between the corresponding edge and the corner trigonometric mathematics has been used. Let the horizontal distance between E1E4 and V1V3 will be *a*, horizontal distance between the pair of E2E5, E3E6, V1V5, V2V4, and V2V6 will be *b*, vertical distance between pair of E2E5, E3E6, V1V5, V2V4, and V2V6 will be *c*. Distance are formulated as follows:

$$a = 2 * \cos 30 * 0.5 \ = 0.8660$$

$$b = \cos 30 * 0.5 \ = 0.4330$$

$$c = 1 - \sin 30 * 0.5 \ = 0.75$$

Then, substitute the values in equation 18 will provide the boundary condition for the hexagonal packing.

For E1 and E4 ($\Delta x = a$, $\Delta y = 0$)

$u_4 - u_1 - a.\, \varepsilon_{xx} = 0$ and
(21)

$v_4 - v_1 = 0$

For E2 and E5 ($\Delta x = b$, $\Delta y = c$)

$u_4 - u_1 - b.\, \varepsilon_{xx} - c.\, \gamma_{xy} = 0$ and

$v_4 - v_1 - c.\, \varepsilon_{yy} = 0$

For E3 and E5 ($\Delta x = b$, $\Delta y = c$)

$u_4 - u_1 - b.\, \varepsilon_{xx} - c.\, \gamma_{xy} = 0$ and

$v_4 - v_1 - c.\, \varepsilon_{yy} = 0$

For V1 and V3 ($\Delta x = a$, $\Delta y = 0$)

$u_4 - u_1 - a.\, \varepsilon_{xx} = 0$ and

$v_4 - v_1 = 0$

For V1 and V5 ($\Delta x = b$, $\Delta y = c$)

$u_4 - u_1 - b.\, \varepsilon_{xx} - c.\, \gamma_{xy} = 0$ and

$v_4 - v_1 - c.\, \varepsilon_{yy} = 0$

For V2 and V4 ($\Delta x = b$, $\Delta y = c$)

$u_4 - u_1 - b.\, \varepsilon_{xx} - c.\, \gamma_{xy} = 0$ and

$v_4 - v_1 - c.\, \varepsilon_{yy} = 0$

For V2 and V6 ($\Delta x = b$, $\Delta y = c$)

$u_4 - u_1 - b.\, \varepsilon_{xx} - c.\, \gamma_{xy} = 0$ and

$v_4 - v_1 - c.\, \varepsilon_{yy} = 0$

Equations 21 are all required displacement boundary conditions for the hexagonal unit cell. Subscript indicates the displacement of the corresponding corner or edge. According to the (Li 2000) use can be made to reduce the size of the unit cell to quarter of it, However , this is at the price of having to apply different boundary condition for different loading condition. Many publications are published which used the quarter model of the hexagon.

V. MODELLING AND ANALYSIS

A. Type of analysis

In previous sections the boundary conditions and mesh are derived for the square and hexagonal unit cell. Macroscopic strains are introduced for the derivation of the boundary conditions. Nodes of the edges and corners are extracted from the meshed model and then applied each node an individual degree of freedom. Concentrated force was applied as a loading condition for the unit cell. Longitudinal, transverse and shear load was applied to derive the effective material properties of the fiber reinforced composites. Macroscopic stress and strain can be obtained from the post processing section of the ABAQUS. Data file provides the needed results from the corresponding unit cell model. The effective material properties can be calculated using elasticity approach.

Element used for the square unit cell was CPS4R (4 node quadratic element) for both matrix and fiber. Displacement is selected for the output data. After submitting the job file the input file executes and creates another file called data file, in which the all the input and output data are stored.

B. Effective material properties

The two basic approaches to the micromechanics of composites material are mechanics of material and elasticity. The mechanics of material embodies the vastly simplifying assumptions regarding hypothesized behavior of mechanical system. The properties of the composites can be defined in terms of its constituent properties and also in terms of relative volume fraction.

Many mathematical models have been found to determine the effective material properties of the composites. Several authors have devoted their study to develop the mathematical model to determine longitudinal young's modulus, Transverse young's modulus, Shear modulus, and poison's ratio. This mathematical model can be referred in (Chamis 1984), (Peters 1997), (Kuno 1996), (Tsai and Thomas 1980), (Vinson and Sierakowski 1987).

C. Material selection

Isotropic materials were selected for the micromechanical analysis of the unit cell. Both matrix and fiber are isotropic, and homogeneous in nature. The input file is independent for the material selection. One can change the Young's modulus and Poisson's ratio of the matrix and fiber according to their need for the analysis. By just editing the input file the analysis can be done for various materials. Effective material of the combination of two different parts can be derived using ABAQUS and available mathematical model by the different authors. For the analysis one particular material is chosen whose material properties are shown in the below table 5.1 which was also used by the (Li 2000). So that the result obtained for the numerical model can be validated.

Material properties	Fiber	Matrix
Young's Modulus(GPa)	10	1
Poisson's Ratio	0.2	0.3

Table 3 material property of glass epoxy

The relation between the stresses and force can be obtained as follows.

$$\sigma_x = \frac{Fx}{A}\ ,\sigma_y = \frac{Fy}{A}\ ,\ \tau_{xy} = \frac{Fxy}{A}\ .$$

Where, F_x, F_y, F_{xy} are the longitudinal, transverse and shear force which can be obtained from the ABAQUS input file attached in Appendices A and B. the 2D model the volume considered is the area of the unit cell model i.e. 1x1 mm^2. So, by using these stresses one can easily find the effective elastic material properties of the unit cell considering microscopic strain and the applied concentrated force of 100N. These effective material properties can be derived as follows.

$$E_x = \frac{\sigma_x}{\varepsilon_x} = \frac{Fx}{A} * \frac{1}{\varepsilon_x} = E_y,$$

$$\nu_{xy} = \nu_{yx} = \frac{-\varepsilon_y}{\varepsilon_x},$$

$$G_{xy} = G_{yx} = \frac{\tau_{xy}}{\gamma_{xy}}.$$

Where, E_x, E_y, ν_{xy} and G_{xy} are the effective material properties of the material combined by two different phases. ε_x, ε_y and γ_{xy} are the strains related to the node where the force was applied. From this equation one can able to find out the required material properties of the unit cell consist of two different materials.

D. ANALYTICAL MODEL

Mathematical models are based on the translational symmetry in y-z direction. Mathematical model for Young's Modulus, Shear modulus and Poisson's ratio are described below.

(1) Mathematical model for Young's modulus

1. Rule of mixture:-

$$E_y = E_z = \frac{E_m E_{fy}}{V_f E_m + (1-V_f)E_{fy}}$$

2. Puck:-

$$E_y = E_z = \frac{E_{m*}(1+0.85*V_f^2)}{V_f*\frac{E_{m*}}{E_f}+(1-V_f)^{1.25}} \qquad \text{Where, } E_{m*} = \frac{E_m}{1-V_m^2}$$

3. Chamis:-

$$E_y = E_z = \frac{E_m}{1-V_f^{0.5}(1-\frac{E_m}{E_{fy}})}$$

4. Halpin-Tsai:-

$$E_y = E_z = \frac{E_m(1+\beta\alpha V_f)}{1-\alpha V_f} \qquad \text{Where, } \alpha = \frac{\frac{E_f}{E_m}-1}{\frac{E_f}{E_m}+\boldsymbol{\beta}}$$

β = 1, for hexagonal packing

β = 2, for square packing

(2). Mathematical model for Poisson's ratio

1. (Hull and Clyne 1996)

$$\nu_{yz} = \nu_{zy} = 1 - \nu^* - \frac{\boldsymbol{E_y}}{\boldsymbol{k}},$$

$$\text{Where, } \nu^* = [\boldsymbol{V_f \nu_f} + \boldsymbol{V}_m \nu_m]\frac{E_y}{E_x}$$

$$k = \left[\frac{V_f}{k_f} + \frac{V_m}{k_m}\right],$$

$$\text{Where, } k_f = \frac{E_f}{3(1-2\nu_f)} \text{ and, } k_m = \frac{E_m}{3(1-2\nu_m)}$$

2. Chamis:-

$$\nu_{yz} = \nu_{zy} = \frac{E_y}{2G_{yz}} - 1 \qquad \text{or}$$

$$\nu_{yz} = \nu_{zy} = [V_f \nu_f + V_m \nu_m]\left[\frac{1+\nu_m-\nu_*\frac{E_m}{E_x}}{1-\nu_m^2-\nu_m\nu_*\frac{E_m}{E_x}}\right],$$

$$\text{Where, } \nu_* = [V_f \nu_f + V_m \nu_m]\frac{E_y}{E_x}$$

(3). Mathematical model for Shear Modulus

1. Elasticity approach:-

$$G_{yz} = \frac{E_y}{2(1-\nu_{yz})}$$

2. Tsai- Hahn:-

$$G_{yz} = \left[\frac{1}{V_f+\beta_G V_m}\left(\frac{V_f}{G_f} + \beta_G V_m \frac{1}{G_m}\right)\right]$$

Where, $\beta_G = 0.62$

3. Chamis:-

$$G_{yz} = \frac{G_m}{1-V_f^{0.5}\left(1-\frac{G_m}{G_{fyz}}\right)}$$

$$\text{Where, } G_{fyz} = \frac{E_{fy}}{2(1+\nu_f)}$$

	40%	50%	60%	70%
Young modulus	2.3213	2.7502	3.3018	4.0484
shear modulus	1.5690	1.3219	1.0747	0.8276
poison's ratio	0.3671	0.3531	0.3392	0.3252

Table 4 Material properties of different volume fraction by analytical model

C. SIMULATION

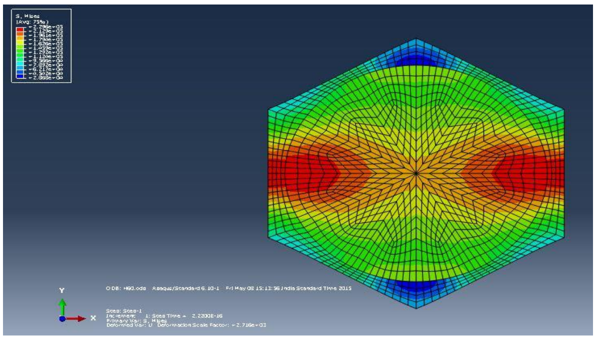


Figure 6.Stress value for 60% hexagonal packing

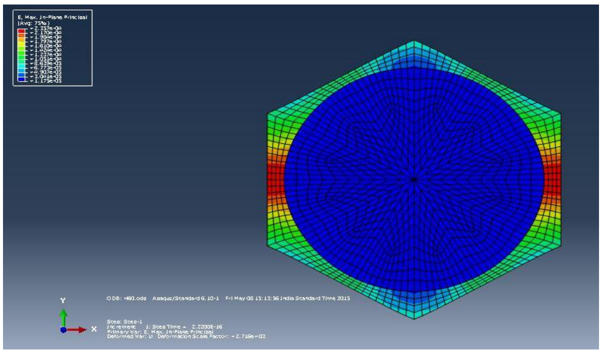


Figure 7.Strain value for 60% hexagonal packing

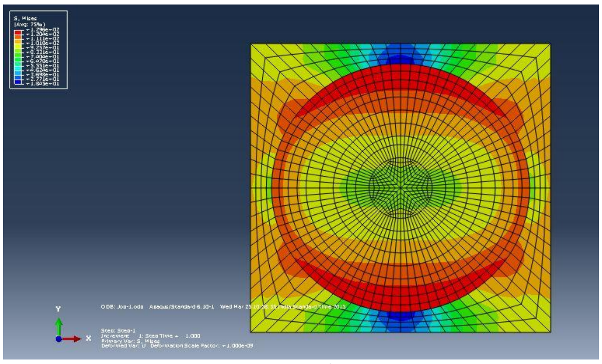


Figure 8.Stress value for60% square packing

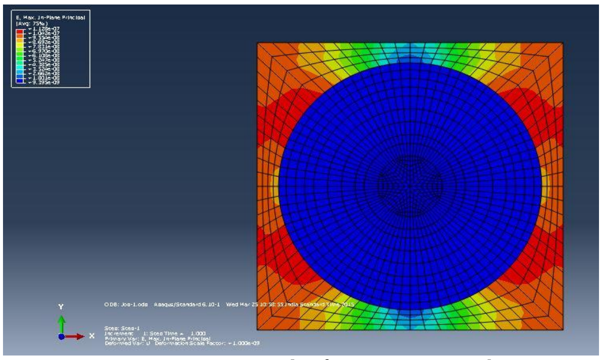
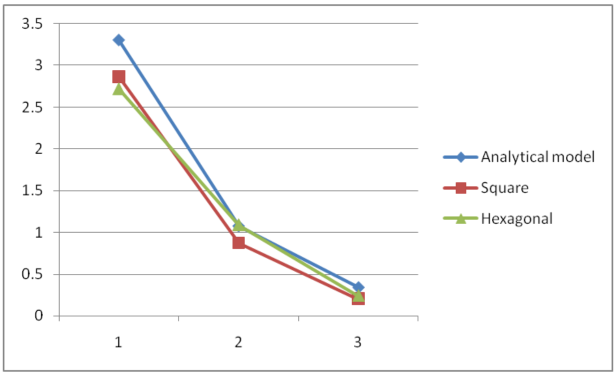


Figure 9.Strain value for 60% square packing

VI. RESULTS

property	Analytical model	Numerical model (ABAQUS)	
		Square	Hexagonal
Young modulus	3.3018	2.87	2.72
shear modulus	1.0747	0.87	1.09
poison's ratio	0.3392	0.20	0.24

Table 5.comparison of mathematical and numerical model



Graph 1 comparison of analytical and numerical model

VII. CONCLUSION

In this dissertation methodology is introduced to formulate the effective material properties of the model consists of two different materials. 2D Square and hexagonal repeating unit cell have been established for the comparison of the obtained results. Mathematical model has been introduced to validate the numerical model created by ABAQUS finite element package. Different fiber volume fractions have been considered to present the graphical presentation or behavior of the each material property and dependency of each material property on the effective properties was carried out. Finally, some observations on micromechanics behavior of the effective material properties have been presented.

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