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Minimization of Average Error Using Parks-McClellan & Genetic Algorithm in Farrow Structure Based FIR Filter Design

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Abstract

A linear phase FIR filter is designed with Farrow Structure and minimization of filter coefficients is done using Parks–McClellan (PM) algorithm and Genetic Algorithms (GA). In the transfer function of the Farrow Structure, different subfilters of even order are weighted by different powers of the fractional delay value. Both the fractional delay value and its powers are taken to be smaller than 0.5 and these are used as diminishing weighting functions. The approximation error for each subfilter is then increased in proportion to the powers of the FD value. These diminishing weighting functions are then used in the filter design so as to obtain the optimum values iteratively. The algorithms are coded in MATLAB software and the design goal is successfully achieved using the GA and compared with that obtained using the PM method and it is found that GA outperforms PM method in various design cases considered.

1. Introduction

Digital filters in hardware form can now routinely perform tasks that were almost exclusively performed by analog systems in the past whereas software digital filters can be implemented using low-level or user-friendly high-level programming languages. Nowadays digital filters can be used to perform many filtering tasks which in the not so distant past were performed almost exclusively by analog filters and are replacing the traditional role of analog filters in many applications. Besides the inherent advantages such as high accuracy and reliability, small physical size, and reduced sensitivity to component tolerances or drift, digital implementations allow one to achieve certain characteristics not possible with analog implementations such as exact linear phase and multirate operation. Digital filtering can be applied to very low frequency signals, such as those occurring in biomedical and seismic applications very efficiently. In addition, the characteristics of digital filters can be changed or adapted by simply changing the content of a finite number of registers, thus multiple filtering tasks can be performed by one programmable digital filter without the need to replicate the hardware. With the ever increasing number of applications involving digital filters, the variety of requirements that have to be met by digital filters has increased. As a result, design techniques that are capable of satisfying the required design requirements are becoming an important necessity.

2. Finite Impulse Response Filters

A Finite impulse response filter has a large number of useful properties which generally make it preferable to an infinite impulse response (IIR) filter.FIR filters are digital filters with finite impulse response. They are also known as non recursive filters as they do not require any feedback. This means that any approximate errors are not computed by summed iterations. The same relative error may occur in each calculation. This also makes implementation simpler. They are inherently stable. This is because of the fact that, there is no requirement of feedback. In the transfer function all the poles are located at the origin and thus are located within the unit circle which is the required condition for stability of the filter in a discrete and linear-time invariant system. They can be easily designed to have a precise linear phase by making the coefficient sequence symmetric, phase change or linear proportional to frequency, corresponding to equal delay at all the frequencies. This property of FIR filter is commonly desired for phase sensitive applications, for example crossover filters, data communications and mastering where there is a need for a linear phase characteristic within the passband of the filter.FIR filters have only zeros and no poles in their transfer function and therefore it does not oscillate and have a constant delay. Therefore the

FIR filter has finite length. The phase shift of the filter will be product of the time delay and frequency. TheFD filter must be time varying and in practice,the situation is even made harder by the fact that the sampling rate ratios are not only incommensurate but also time varying.FIR filters can have exactly linear phase. They are inherently stable. The design methods are generally linear. They can be realized efficiently in hardware. The filter start up transients have finite duration The main drawback of FIR filters is that considerably more computation power in a general purpose processor is required as compared to an IIR filter with similar sharpness and selectivity, especially when lower frequency (relative to the sample rate) cutoffs are needed. However many digital signal processors provides specialized hardware features to make Finite impulse response filters approximately as efficient as IIR for many application. Moreover FIR filters commonly require a much higher filter order than IIR filters to achieve a good level of performance.

2.1 Farrow Filter Structure

Variable Fractional delay FIR Filter is designed using the Farrow Structure, a most common method for implementing the time-varying Finite Impulse Response Fractional Delay filters. Implementation complexity is lower for Farrow based filtering methods. Filter design with Farrow structure gives the flexibility that there is only one way to manipulate with the filters characteristics, mainly by changing the fractional delay value. To redesign the filter there is no need to redesign the subfilters as the filter coefficients remain fixed. To calculate the output of a FIR FD filter, one need to estimate the values of the applied input signal between the existing discrete-time samples. Special interpolation filters can be used to calculate new sample values at arbitrary points. The polynomial-based filters are of special interest because a special structure, the Farrow structure, permits simple handling of coefficients. In particular, the tunability of the Farrow structure makes it well-suited for practical hardware implementations. In the digital signal processing ,to design a digital filter implies that selecting the filter coefficients such that the system has specific characteristics. The required characteristics are stated in filter specifications. Most of the time filter specifications refer to the frequency response of the filter. The benefit of using the Farrow structure over a Direct-Form FIR filter structure resides in its tunability. In many practical applications, the delay is time-varying. For every new delay parameters one would need a new set of filter coefficients in the Direct-Form filter implementation but with a Farrow structure implementation, the polynomial coefficients remain constant. In this design each of the impulse response coefficients are modeled as Mth order polynomials of the delay variable which implemented the variable filter as a linear combination of M+ 1 filters.

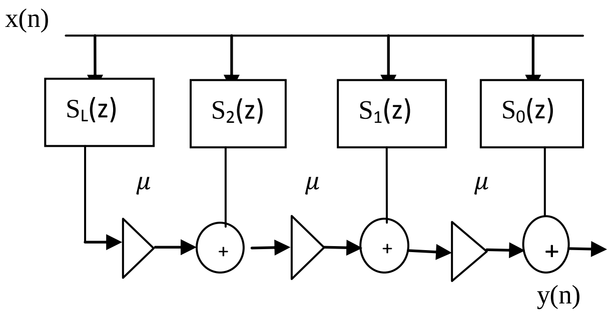

Figure 1. Farrow Structure with fixed subfilters $S_k(z)$ and a variable FD of μ

The Farrow Structure shown in Figure1. is composed of fixed linear phase finite length impulse response (FIR) subfilters $S_k(z)$,k=0,1…….,L,of order N_k as well as the variable multipliers μ.The transfer function is given by

$$H(z,\mu)=\sum_{k=0}^{L} S_k(z)\mu^k, \quad |\mu|\leq 0.5 \qquad (1)$$

The design parameters are hence the number of subfilters, L+1,the order of the subfilters ,N_k, and the coefficients of the subfilters $S_k(z)$.The overall structure can approximate FD filters with an adjustable μ over a frequency range of ω € [0,ω_c]. In the transfer function of the Farrow Structure, different subfilters are weighted by different powers of the FD value. As both the FD value and its powers are smaller than 0.5, these are used as diminishing weighting functions. The approximation error for each subfilter is then increased in proportion to the powers of the FD value. A new distribution to the orders of the Farrow subfilters is given. These diminishing weighting functions are then used in the filter design so as to obtain the optimum values iteratively. Sub filters of even order are considered.

2.3 Parks-McClellan Method

Parks-McClellan method which is also known as the Equiripple, Optimal, or Minimax method with the Remez exchange algorithm is used to find an optimal equiripple set of coefficients to design an optimal linear phase filter. This is a standard method for the FIR filter design which minimizes the filter length for a particular set of design constraints. This method is used to design linear phase, symmetric or antisymmetric filters of any standard type. Better filters result from minimization of maximum error in both, the stopband and the passband of the filter which leads to equiripple filters. Such filters are an optimum approximate and can be achieved using algorithmic techniques. In this algorithm to design FIR filters, some of its parameters such as the filter length (*M*), passband and stopband normalized frequencies (*w*p, *w*s), maximum of the absolute ripple in the passband and stopband (*δ*p, *δ*s) are fixed and the remaining parameters are to be optimized. Parameters *M, δp*, and *δ*s are fixed while the remaining parameters are optimized. The Parks–McClellan (PM) algorithm is the most popular approach for optimum FIR filters design due to its flexibility and computational efficiency. In the PM algorithm, an approximate error function is defined by

$$E(\omega)=G(\omega)[H_d(e^{j\omega}) - H(e^{j\omega})] \quad (2)$$

Where $H_d(e^{j\omega})$ and $H(e^{j\omega})$ are the frequency responses of the desired and the approximate filters respectively $G(\omega)$ is the weighting function. It is used to provide weighting of the average error differently in different frequency bands. The filter is optimized in the sense that the maximum weighted error is minimized. However this algorithm does not allow explicit selection of the maximum of the absolute ripple in the passband and stopband, one can only specify their ratio. Furthermore, the PM gives floating point coefficients which require quantization. The user needs to specify the desired frequency response, a weighting function for average errors from the frequency response, and a digital filter order N. The algorithm is used to obtain the set of N+ 1 coefficients that minimize the maximum deviation from the ideal filter response. This finds the filter that is as close to the desired response. This method is easy in practice since it uses a program that takes the desired filter and N which is the order of the filter, and returns the optimum filter coefficients. The resulting filters minimize the maximum average error between the desired frequency response and the actual frequency response by spreading the average error uniformly over each band. The filters that have equiripple behavior in the passband and the stopband of the filter response are called an equiripple filters. The computational efforts in the filter design are linearly proportional to the length of the filter.In Matlab software this method is available as remez().We can use the (remezord) command to estimate the order of the optimal Park-McClellan FIR filter.Generalized optimization techniques are then used to minimize (or maximize) a given function, known as the objective function, or cost function. A linear Optimization problem is the one whose objective function is a linear function of the input. Optimization algorithms generally take a starting guess point and change the variables subjected to the constraints in such a way that it decrease (or increase) the objective function. Some sort of termination condition is then required.

3. Optimization Strategy

The first step of any optimization routine is having a basic knowledge of the problem and its limitations. In the case of filter optimization the designer must be familiar with the electrical performance of each filter design. This method of approach requires the performance of each new design to be calculated and compared against the ideal optimum specification. Generalized optimization techniques minimize (or maximize) a given function, known as the objective function, or cost function. A linear Optimization problem is the one whose objective function is a linear function of the input. Optimization algorithms generally take a starting guess point and change the variables subjected to the constraints in such a way that it decrease (or increase) the objective function. Some sort of termination condition is then required. First, a starting point x0 is chosen and f(x) is evaluated at that point. Then, a new value of x is chosen to reduce the objective function. This process is repeated until some termination condition is met. The minima or maxima of functions occur at points where the derivative of that function is equal to zero.In filter design problems, the cost function is generally an error function. Thus the cost function must derive the magnitude response of the given input x, and subtract it from a desired response. This generates an error signal that can be minimized. The two most common types of objective functions used in filter design are least squares and minimax .The least-squares optimization problem is concerned with minimizing the sum of the squared error vector, while minimax optimization problems are concerned with minimizing the maximum value in the error vector. A weighting function is generally a constant function that applies a weight to each value in the error vector.

3.1 Genetic Algorithm Optimization

In our design, we utilize genetic algorithm to optimize the design of the digital filter. Genetic Algorithms are searching and optimization techniques inspired by two biological principles, namely the process of “nature selection” and the mechanics of “natural genetics”. The genetic algorithm is an artificial genetic system based on the process of natural selection and genetic operators. It is also a heuristic algorithm which tries to find the optimal results by decreasing the value of objective function (error function) continuously. In a genetic algorithm, a population of initial solutions (called individuals, creatures, or phenotypes) in an optimization problem is evolved for better solutions.

Each candidate solution in the initial population has a set of properties which are its chromosomes or genotype are mutated and altered. Traditionally these solutions are represented in binary form as binary strings of 0s and 1s, but other encodings are also possible. The algorithm starts with an initial population representing random candidate solutions. Each individual in the population is awarded a score based on its performance. The individuals with the best scores are most likely to be selected to produce a new generation. The selected individuals are used to produce a new population based on two main genetic operators, crossover and mutation. In crossover, two individuals are used to produce two new individuals by genes exchange between the two selected individuals. Random mutation is also applied to add some diversity to the population. The produced children are also scored, with the best performers are likely to be parents in the next generation. The process is repeated until attaining a termination criterion.The Genetic Algorithm optimization procedure in our design is summarized as follows .Each loop consists of the following steps.

Step1 In the first step many individual solutions are randomly generated in order to form an initial population. The size of population depends up on the nature of the problem, but contains several hundreds or thousands of possible solutions. Traditionally, the population is generated randomly, allowing the entire range of possible solutions in the search space. Occasionally, the solutions can be "seeded" in areas where optimal solutions are likely to be found.

Step2 In the second step each successive generation, some proportion of the existing population is selected in order to produce a new generation. In generation of Individual solutions are then selected by a fitness-based method, where fitter solutions which are measured by a fitness function are typically more likely to be selected .Other methods rate only a random sample of the population, as the former process may be time-consuming. The fitness function is defined over the genetic representation and measures the quality of the represented solution. The fitness function is always depends upon the problem.

Step3 In the third step we have to generate a second generation population of solutions from those which are selected by genetic operators which are crossover also known as recombination, and mutation.

Step4: This generation process is repeated until a termination criterion has been reached. Common terminating conditions which are when a solution is found that satisfies the minimum conditions and the fixed numbers of generations are reached. Also the allocated budget which includes the computation time and money must be reached for the termination. The highest ranking solution's fitness is reaching a termination condition or it has reached a plateau such that successive iterations no longer produce better results. Manual inspection is done in the next step of the termination step. Finally the combinations of the above steps are required for the last termination condition.

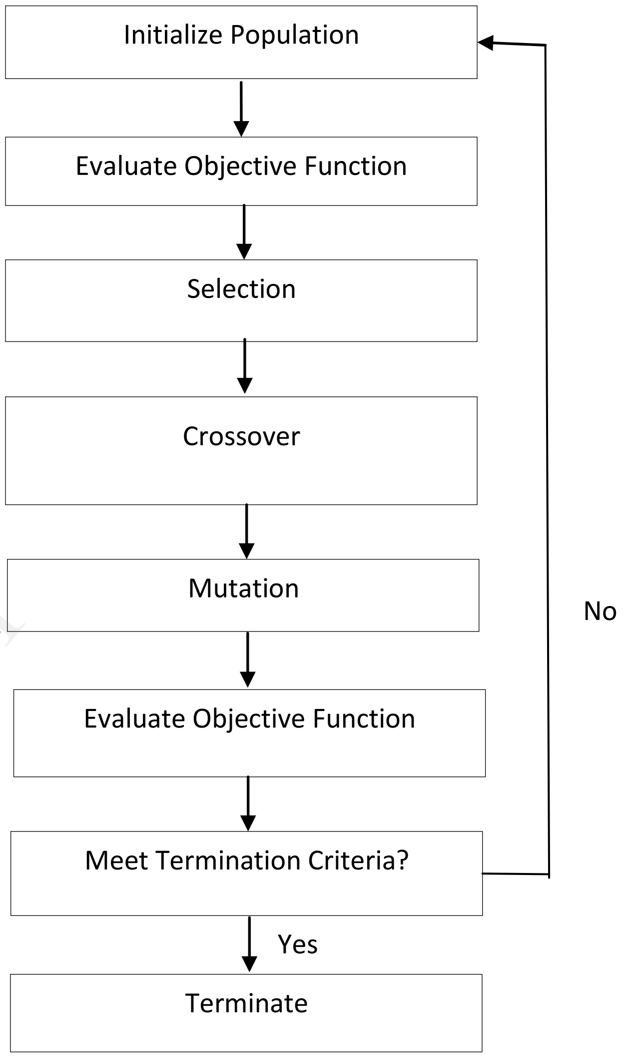

Figure 2. Block diagram of Genetic Algorithm function used in the design

4. Results

The design of digital filters basically means finding the values of filter coefficients so that given filter specification are achieved. A minimum-order low pass filter with a 10 MHz pass band cutoff frequency and 25 MHz stop band cutoff frequency, with a sampling frequency of 50 MHz, at least 30dB attenuation in the stopband, and less than 3 dB of ripple in the pass band is designed according to the design specifications in the Figure 3. The order of the designed lowpass filter using the Park-McClellan method is calculated to be 12.

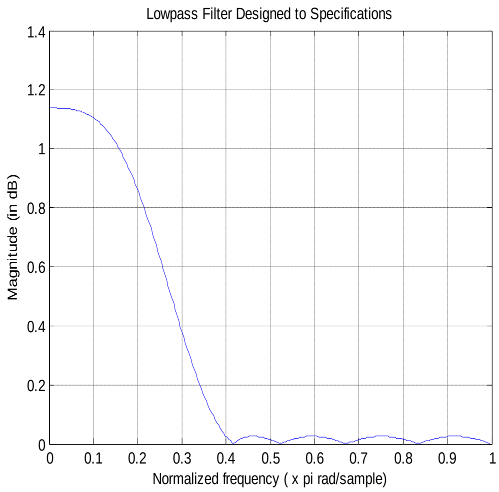

Figure 3. LPF through PM of order 12

Table 1.compares the values of the three filter coefficients obtained by Ideal Filtering, through Fractional Delay and PM algorithm and after GA optimization.Filter design with Farrow structure gives the flexibility that there is only one way to manipulate with the filters characteristics, mainly by changing the fractional delay value. The value of Fractional Delay (μ) is 0.30 in this case. Fig.4. shows the magnitude response of the designed filter structure using the PM algorithm. Maximum magnitude response is obtained when an input signal completely passes through the passband of the filter. Here the normalized frequency lies between 0 to 1.The cut off frequency is 0.4 to 0.6. It is clear from the graph that after 0.4 frequency, the signal starts attenuating resulting in 30 db reduction in the magnitude response of the filter.

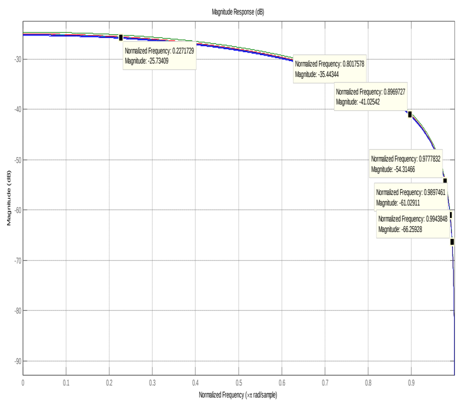

Figure 4. Farrow filter structure with the PM coefficients

Table 1. Calculated Filter Coefficients

Filter Coefficients	Ideal filtering	Through fractional delay and PM Algorithm.	After optimization through GA Algorithm
a0	0.3982	0.0248	-0.6250
a1	0.0025	0.0259	0.4391
a2	0.0040	-0.0487	-0.8750
a3	0.0024	-0.1146	1.0000
a4	0.0010	0.0092	-0.4063
a5	0.0036	0.3002	1.0000
a6	0.0007	0.4609	-0.6250
a7	0.0085	0.3002	0.9375
a8	0.0022	0.0092	-0.9844
a9	0.0498	-0.1146	1.0000
a10	0.0022	-0.0487	-1.0000
a11	-0.002	0.0259	1.0000
a12	0.0012	0.0248	-0.5625

Fig.5. shows the magnitude response of the designed filter structure after optimization using the GA algorithm. Maximum magnitude response is obtained when an input signal completely passes through the passband of the filter. The passband response is the filter's effect on frequency components that are passed through unchanged. Frequencies within a filter's stopband are highly attenuated. Transition bandwidth describes how quickly a filter transitions from a passband to a stopband, or vice versa. The more rapid this transition, the higher the transition bandwidth and the more difficult the filter is to achieve. Here the normalized frequency lies between 0 to 1.The cut off

frequency is 0.4 to 0.6. It is clear from the graph that after 0.4 frequency, the signal starts attenuating resulting in less reduction in the magnitude response of the filter. The transition band has reduced more rapidly after performing optimization with Genetic Algorithm.

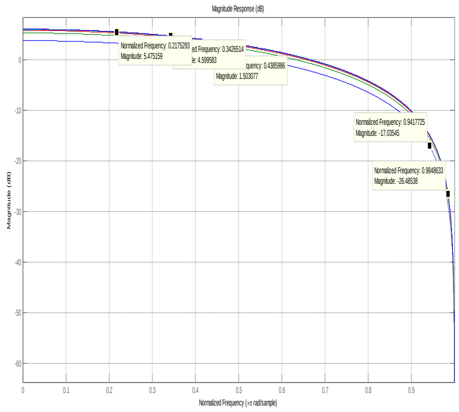

Figure 5. Magnitude response after Optimization with GA

Fig.6. describes the Phase delay in the filter designed using the PM algorithm. Delay is a fractional value and can be varied with time. Fractional delay means a delay that is a non integer multiple of the sample interval. Ideal fractional delay system uses an ideal low pass filter where the delay merely shifts the impulse response in the time domain. Firstly minimum phase delay of 0.5 occurs and then the phase delay rises abruptly to 1.This rise in phase delay occurs because fractional delay factor (μ) in μ^K depends on K. Where K are the total number of subfilters used in the Farrow structure filter design. Every time the value of K increases, it causes an increase in the phase delay.

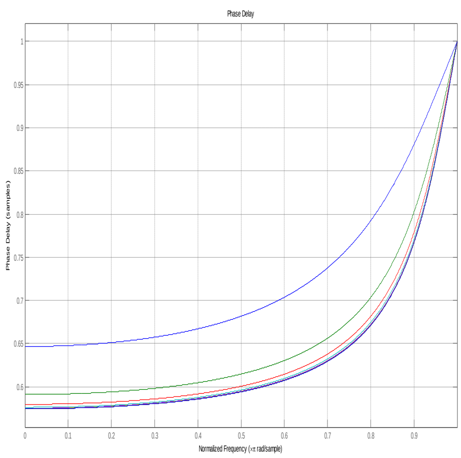

Figure 6. Phase delay before Optimization

Fig.7. shows the phase response of the filter designed with order 12 after optimizing with GA algorithm. Ideal filters have a linear phase response characteristics within the passband of the filter. Such filters are not physically realizable but serve as a mathematical idealization of practical filters. Delay is a fractional value and can be varied with time. Fractional delay means a delay that is a non integer multiple of the sample interval. Ideal fractional delay system uses an ideal low pass filter where the delay merely shifts the impulse response in the time domain. The number of sub filters used is 8 which are even and the value of fractional delay (μ) is 0.30. For any value of delay, the ideal filter should have both a flat magnitude response and a flat phase response. A linear phase response with zero phase delay is obtained after optimization at 0.5 frequency which is a basic property of the FIR filters.FIR filters allow the design of linear phase filters, which eliminate the possibility of signal phase distortion. With the help of GA, the number of operations in design process is reduced and coefficient calculation is easily realized. Besides, the desired responses are obtained, the ripples in the pass band and in the stopband regions are attenuated successfully. This shows approximately linear phase delay response with this proposed method.

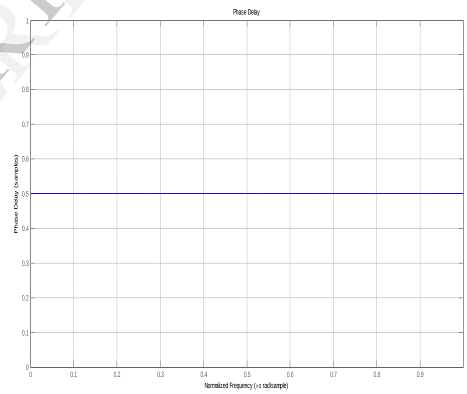

Figure 7. Phase delay after Optimization

4.1 Comparison of Average Error Calculati- -on using PM and GA Algorithms

The resulting filters minimize the maximum error between the desired frequency response and the actual frequency response by spreading the approximate average error uniformly over each band. An approximate error function is given by

$$E(\omega)= [H_d(e^{j\omega}) - H(e^{j\omega})] \qquad (3)$$

Where $H_d(e^{j\omega})$ and $H(e^{j\omega})$ are the frequency responses of the desired and the approximate filters respectively. The design objective is to find the approximate filter coefficients that result in the optimum filter. The average error calculated through PM algorithm filter coefficient values is given by

% average error after PM = (0.8543 – 0.4342)/ 0.8543 = 49.17%

The percent average error calculated after using GA optimal filter coefficients is given by

% average error after GA = (0.4342 – 0.3802) /0.4342 = 12.3%

The percent average error calculated using GA has been minimized to a great extent and gives lower absolute error as compared to that of using PM algorithm which has higher ripple in the passband. Park-McClellan algorithm does not allow explicit selection of the maximum of the absolute ripple in the passband and stopband, one can only specify their ratio. Furthermore, the PM gives floating point coefficients which require quantization. Better filters result from minimization of maximum error in both the passband and stopband of the filter which leads to equiripple filters. Such filters are an optimum approximate and it shows that the GA outperforms PM and is achieving the design requirement.

4.2 Comparison of Implementation and Cost of Filter Design with PM and GA Algorithms

In the Figure 8 ,with the Park-McClellan method, the number of operations in design process has increased .As the number of Multipliers and Adders used in the filter designing with PM algorithm are 32and 21 respectively, the arithmetic and computational complexity of the design process has increased. So the cost of Filter design is large with the greater number of multipliers required in the design obtained with PM algorithm.In the figure 9. With the help of GA, the number of operations in design process is reduced and coefficient calculation is easily realized. Besides, the desired responses are obtained, the ripples in the pass band and in the stopband regions are attenuated successfully. Also the number of Multipliers and Adders used in the filter designing with GA algorithm are 24 and 21 respectively, the arithmetic and computational complexity of the design process has decreased considerably with the greater reduction in the number of multipliers as compared to that of Park-McClellan Method. The number of additions scales proportionally to the number of fixed multiplications in the Farrow structure filter design. For this reason our complexity comparisons will only consider the number of fixed multiplications. So by using Genetic Algorithm the cost of Filter design and computational complexity has reduced to a great extent.

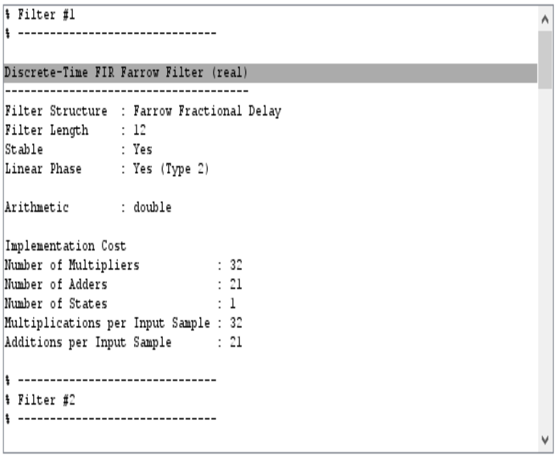

Figure 8. Filter information with Parks-McClellan Method

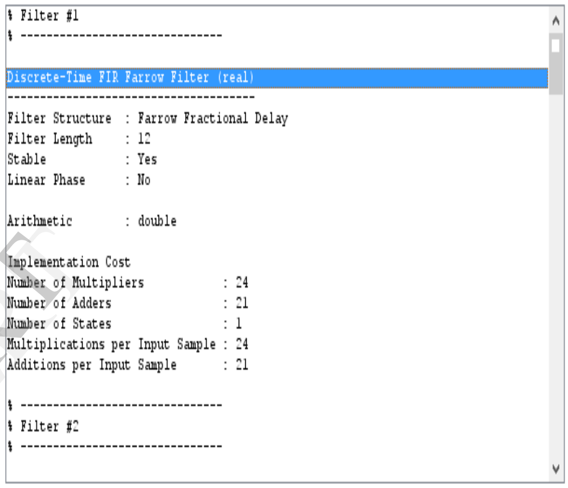

Figure 9. Filter Information with Genetic Algorithm

5. Applications

However many digital signal processors provides specialized hardware features to make Finite impulse response filters approximately as efficient as IIR for many applications. Moreover FIR filters commonly requires a much higher filter order than Infinite impulse Response filters to achieve a good level of performance.Various other examples include synchronization of digital modems.digital simulation of the doppler effect in virtual reality systems or elimination of wow in old gramophone disc recordings.These are used in numerous applications such as communications,audio and music technology.Another class of application is modelling of musical instruments which involve discretization of differential equations describing a physical system describing the accoustical vibrations. Propagation delays are caused by a finite speed of vibrations in strings,tubes and other musical resonators which must be simulated accurately otherwise the instrument will give sound out of tune.Such instances are important applications of Fractional Delay filters where uniform sampling is used and interpolation between samples is required. Also widely employed in speech coding and synthesis,antenna and transducer arrays and time delay estimation.Digital Fractional delay(FD) filters provide a useful building block that can be used for fine tuning the sampling instants ie.implement the required bandwidth interpolation.

6. Conclusion

In this present work, FIR filter is designed using GA and Parks McClellan in MATLAB. The magnitude response, phase variation and phase delay have improved to great extent. The response is studied by keeping values of fixed order, crossover probability and mutation probability. Out of the two techniques GA offers a quick, simple and automatic method of designing low pass FIR filters that are very close to optimum in terms of magnitude response, frequency response and in terms of phase variation. FIR has a very high, large phase deviation and lack of control of critical frequencies ω_p and ω_s. To overcome this problem, Parks McClellan is used. But as the order of the filter increases, this method is not suitable. Therefore, to solve all these problems, GA is used. With the help of GA, the number of operations in design process is reduced and coefficient calculation is easily realized. Besides, the desired responses are obtained, the ripples in the pass band and in the stopband regions are attenuated successfully. Seeing from the frequency response graph of the FIR filter, the actual frequency response is much close to the desired frequency response when the order of the filter is higher.FIR digital filter has a precise linear phase much than IIR digital filter because IIR digital filter has a nonlinear phase. Therefore, FIR digital filter should be choosed to use when calculation and accuracy are the much requirements.

7. Future Scope

The work has been restricted to low pass filters, it could be extended to High Pass, Band Pass and Band Stop filters. In the future, work of error minimization of optimized filter coefficients can be implemented with IIR Filter design.

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