Mixed Convection Flow Through A Porous Medium Bounded By Two Vertical Walls With Slip Boundary Conditions

Akhilesh K. Mishra,
Department of Mathematics, Gombe State University, Gombe, Nigeria

Abstract

This paper investigates the steady fully developed mixed convection flow between two vertical walls filled with porous materials having slip boundary for the velocity and temperature. The analytic solutions for the velocity and temperature profiles has been obtained for different cases depending upon the values of the Darcy number, Rayleigh number and the ratio of the effective viscosity of the porous domain to the viscosity of the fluid. The effects of the various parameters entering into the problem, on the velocity and the temperature are depicted graphically and on the skin friction is depicted in tabular form, and discussed in detail.

1. Introduction

The study of flow and heat transfer through a porous medium has become of main interest in science and technology because of several engineering applications, particularly when the fluid flow is caused by shearing motion of a plate. The mechanism of mixed convection in the porous media has important applications in the utilization of geothermal energy. The recent books by Nield and Bejan [1] and Ingham and pop [2] have extensively documented the works devoted in this area.

Radiation Effects on Unsteady Flow through a Porous Medium Channel with Velocity and Temperature Slip Boundary Conditions has discussed by Chauhan and Kumar [3]. Mazumdar [4] studied the dispersion of solute in natural convection flow through a vertical channel with a linear axial temperature variation. Chen [5] has presented the effects of non-Darcian flow phenomena on mixed convection in porous medium adjacent isothermal horizontal plates. Chen *et. al*. [6] have investigated the fully developed mixed convection in a vertical porous channel with a uniform heat flux imposed at the plates by using the Brinkman-Forchheimer extended Darcy model. Sekhar, Reddy and Prasad [7] investigated chemically reacting on MHD oscillatory slip flow in a planer channel with varying temperature and concentration. Effects of slip conditions on forced convection and entropy generation in a circular channel occupied by a highly porous medium: Darcy extended Brinkman-Forchheimer model analysed by Chauhan and Kumar [8].

Jain and Sharma [9] and Jain and Gupta [10] have studied three dimensional coutte flows with slip boundary conditions and suction velocity varies sinusoidaly. Sharma [11] investigate the effect of periodic heat and mass transfer on the unsteady free convection flow past a vertical flat plate in slip flow regime when suction velocity oscillates in time. Chaudhary and Jha [12] studied the effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime. Khaled [13] investigated the effect of slip condition on Stokes and Couette flows due to an oscillating wall. A more insight into the subject of the slip flow regime is given by Mahmoud, [14]. A series of investigations have been made on slip flow regime viz. Derek *et.al* [15].

In this paper, steady fully developed free and forced convection flow through porous bounded by two walls is considered. The boundary conditions for velocity and temperature are slip boundary conditions at both the walls. In which there is a uniform axial temperature variation along the walls and analytic solutions have been obtained for velocity and temperature profile for different cases and the effects of the pertinent parameters on the flow and temperature fields are examined and discussed.

2. Mathematical Analysis

Consider the fully developed laminar tree convection flow between two vertical walls filled with a fluid saturated porous medium under a constant pressure gradient. The walls are separated by a distance 2L apart and having an axial temperature variation. The x'-axis is taken along the vertical direction while y'-axis is perpendicular to it. For fully developed laminar flow, the velocity has only the vertical component and is a function of y' only.

As a result of these assumptions, the equation of motion in x' direction and energy equation are obtained as follows

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 3, March - 2013

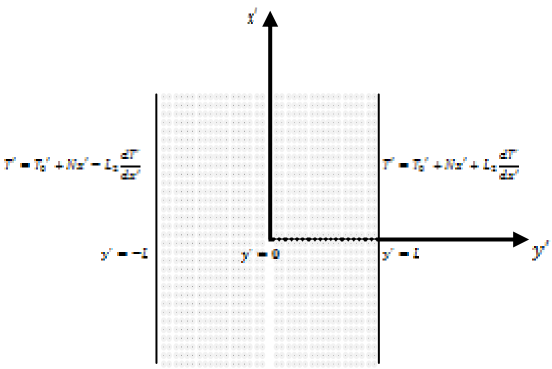

Figure 1. Physical configuration of the model

$$0=-\frac{\partial p'}{\partial x'}-\rho g+\mu_{eff}\frac{\partial^2 u'}{\partial y'^2}-\frac{\mu_f}{k}u' \quad (1)$$

$$u'\frac{\partial T'}{\partial x'}=\alpha\frac{\partial^2 T'}{\partial y'^2} \quad (2)$$

The boundary conditions for the velocity and temperature fields are

$$u'=L_1\frac{du'}{dy'},\qquad T'=T_0'+Nx'+L_2\frac{dT'}{dy'}\qquad \text{at } y'=L$$

$$u'=L_1\frac{du'}{dy'},\qquad T'=T_0+Nx'-L_2\frac{dT'}{dy'^2}\qquad \text{at } y'=-L$$

(3)

Under the usual Bossinesqs approximation the equation of state is assumed to be

$$\rho=\rho_0\left[1-\beta\left(T'-T_0'\right)\right]$$

(4)

After assuming the uniform the uniform axial temperature variation along the channel walls, the temperature of the fluid can be written as

$$T'-T_0'=Nx'+\theta'(y') \quad (5)$$

Using (1) and (5) and introducing dimensionless quantities

$$y=\frac{y'}{L},\quad u=\frac{u'L}{\alpha P_x},\quad \theta=\frac{\theta'}{NLP_x}$$

$$P_x=\frac{\rho L^3}{\alpha\mu_t}\left[-\left(\frac{1}{\rho_0}\frac{\partial p_1'}{\partial x'}+g\right)\partial x'+g\beta Nx'\right] \quad (6)$$

The equation (1) and (2) in dimensionless form obtained as follows

$$Rv\frac{d^2u}{dy^2}-\frac{u}{Da}+Ra=-1 \quad (7)$$

$$\frac{d^2\theta}{dy^2}-u=0 \quad (8)$$

In the above equations, Rv, Da and Ra are defined as

$$Rv=\frac{\mu_{eff}}{\mu_f},\quad Da=\frac{k}{L^2},\quad Ra=\frac{\rho gNBL^4}{\alpha\mu_f}$$

$$h_1=\frac{L_1}{L},\qquad h_2=\frac{L_2}{L} \quad (9)$$

Where Rv= ratios of viscosities
Ra= Rayleigh Number
Da= Darcy Nimber
h_1= Velocity slip parameter
h_2= Temperature slip parameter.

The boundary conditions for the model are as follows:

$$u=h_1\frac{du}{dy},\quad \theta=h_2\frac{d\theta}{dy},\qquad \text{at } y=1$$

$$u=-h_1\frac{du}{dy},\quad \theta=-h_2\frac{d\theta}{dy},\qquad \text{at } y=-1 \quad (10)$$

Now by using (7) and (8) resulted into a fourth order differential equation in u as

$$Rv\frac{d^4u}{dy^4}-\frac{1}{Da}\frac{d^2u}{dy^2}+Ra\frac{du}{dy}=0 \quad (11)$$

The auxiliary roots $m_1,\ m_2,\ m_3$ and m_4 of the above equations are as follows:

$$m_1,\ m_2=\sqrt{\frac{1\pm A}{2DaRv}}$$

(12)

$$m_3,\ m_4=-\sqrt{\frac{1\pm A}{2DaRv}}$$

Where,

$$A = \sqrt{1 - 4RaRvDa^2} \tag{13}$$

The Auxiliary equation roots given by the equations (12) and shows that the solution for u and depends on the values of $Da^{2,}$ Ra and Rv. Thus three different cases arise and the solutions have been obtained as under.

<u>Case I</u>: When $0 < A < 1.$

The solution for u and θ by solving (7) and using (8) and (10) is obtained as

$$u(y) = C_1 \cosh(m_1 y) + C_2 \cosh(m_2 y) \tag{14}$$

$$\theta(y) = -\frac{1}{Ra} + A_1 C_1 \cosh(m_1 y) + A_2 C_2 \cosh(m_2 y) \tag{15}$$

$$\text{where } A_1 = \frac{1}{RaDa} - \frac{Rv}{Ra} m_1^2$$

$$A_2 = \frac{1}{RaDa} - \frac{Rv}{Ra} m_2^2$$

$$A_3 = m_1 h_1 \cosh(m_2)\sinh(m_1) - m_2 h_2 \sinh(m_2)\cosh(m_1)$$

$$A_4 = h_2 m_2 \sinh(m_2)\cosh(m_2) - h_2 m_1 \sinh(m_1)\cosh(m_2)$$

$$A_5 = (m_1^2 - m_2^2 \cosh(m_1)\cosh(m_2) + m_1 m_2 h_1 h_2 (m_1^2 - m_2^2)\sinh(m_1)\sinh(m_2)$$

$$A_6 = (m_1 h_2 \sinh(m_1)\cosh(m_2) + m_2 h_1 \sinh(m_2)\cosh(m_1)$$

$$A_7 = m_1 h_2 \sinh(m_1)\cosh(m_2) + m_2 h_2 \sinh(m_2)\cosh(m_1)$$

$$A_8 = -\frac{1}{Ra}(A_3 + A_4),$$

$$A_9 = RvA_5 + m_1^2 A_6 - m_2^2 A_y$$

$$A_{10} = \cosh(m_1) + m_1 h_1 \sinh(m_1)$$

$$A_{11} = A_8 + A_9,$$

$$A_{12} - \{\cosh(m_2) + h_1 m_2 \sinh(m_2)\}$$

$$C_2 = \frac{A_{10}}{A_{11}}, \quad C_1 = \frac{A_{12}}{A_{11}}$$

<u>Case II</u>: When $A = 1.$

The velocity and temperature profiles for this case by using the boundary conditions are as follows:

$$u(y) = B_1 \cosh(Ey) + B_2 \tag{16}$$

$$\theta(y) = \frac{-1}{Ra} + B_1 E_1 \cosh(Ey) + \frac{B_2}{RaDa.} \tag{17}$$

$$\text{where } E = \frac{1}{\sqrt{DaRv,}}, \quad E_1 = \frac{1}{RaDa} - \frac{Rv}{Ra} E^2$$

$$E_2 = (RaDaE_1 - 1)\cosh(E), \quad E_3 = (\mathrm{RaDaE}_1 h_2 - Eh_1)\sinh(E)$$

$$E_4 = Eh_1 \sinh(E) + \cosh(E),$$

$$B_2 = \frac{-DaE_4}{(E_2 + E_3)}, \quad B_1 = \frac{Da}{(E_2 + E_3)}$$

<u>Case III</u>: When A>1.

The velocity and temperature profiles for this case by using the boundary conditions are as follows:

$$u(y) = C_3 \cosh(m_1 y) + C_4 \cos(m_2 y) \tag{18}$$

$$\theta(y) = \frac{-1}{Ra} + D_1 C_3 \cosh(m_1 y) + D_2 C_4 \cosh(m_2 y) \tag{19}$$

$$\text{Where } D_1 = \frac{1}{RaDa} - m_1^2 \frac{Rv}{Ra}$$

$$D_2 = \frac{1}{RaDa} + m_2^2 \frac{Rv}{Ra}$$

$$D_3 = D_2 \cosh(m_2) - h_2 D_2 m_2 \sinh(m_2)$$

$$D_4 = \cosh(m_1) + m_1 h_1 \sinh(m_1)$$

$$D_5 = \cosh(m_2) - h_1 m_2 \sinh(m_2)$$

$$D_6 = D_1 \cosh(m_1) + h_2 D_1 m_1 \sinh(m_1)$$

$$D_7 = Ra(D_3 D_4 - D_5 D_6)$$

$$C_3 = -\frac{D_5}{D_7}, \quad C_4 = \frac{D_4}{D_7}$$

The expressions for the skin friction in non-dimensional form for the different cases are obtained by using the relation

$$\tau = \left.\frac{du}{dy}\right|_{y=1}$$

They are as follows:

Case I-

$$\tau = C_1 m_1 \sinh(m_1) + C_2 m_2 \sinh(m_2)$$

ISSN: 2278-0181
Vol. 2 Issue 3, March - 2013

Case II

$$\tau = B_1 E_1 \sinh(E_1)$$

Case III

$$\tau = C_3 m_1 \sinh(m_1) + C_4 m_2 \sin(m_2)$$

3. Result and Discussion

In order to point out the effects of different parameters on velocity (u), following discussions are set out. Numerical calculations are carried out for different values of the Darcy number (Da), ratio of viscosities, (Rv), Rayleigh number (Ra), velocity and temperature slip parameters $(h_1,\ h_2)$. The results have been shown graphically for the three cases $A>\ =\ <1$.

The Figure 1 illustrates the physical configuration of the model. Both the walls are separated by distance 2L and other parameters are explained there.

The velocity profiles for different values of Ra(10, 15) and Rv (1.0, 1.5, 2.0) are shown in figure 2, when Da=10^{-1} , for case I and figures 3 and 4 for case II and case III respectively, when Da=10^{-1} ,$10^{-2}$and 10^{-3} and h_1 and h_2 is fixed. As expected the flow is symmetrical about y=0. These figures demonstrate that the flow is parabolic type up to certain values of Rayleigh number. As usual, the velocity increases with increase of Darcy number (Da) due to fact that Darcy number is directly proportional to permeability (K) of the medium, but the reverse situation occurs in the case of ratio of viscosities (Rv). It is also evident that velocity decreases with the increase of Rayleigh number.

In figures 5 and 6 the impact of $(h_1,\ h_2)$ on the velocity profiles have been shown. Figures 5 and 6 illustrates the velocity profiles for the case I and case III respectively when Da=$10^{-1,}$ Ra=, h_1 =0.4, 0.8, 1.2 and h_2 = 0.2, 0.4, 0.6. It noticed that the velocity increases with the increase of slip parameter h_1, which, represents that the increase in the slip parameter has the tendency to reduce the friction forces which increases the fluid velocity. While reverse phenomena occurs in the case of temperature slip parameter h_2.

4. Conclusion

The present study investigates the fully developed mixed convection flow of an incompressible viscous fluid between two vertical walls filled with porous medium saturated by the same fluid. The Brinkman Darcy model is used to analyse the porous domain. The velocity profiles increase with the Darcy number while decreases with ratios of viscosities. The velocity slip parameter promotes the velocity profiles while temperature slip parameter has reverse impact on velocity profiles.

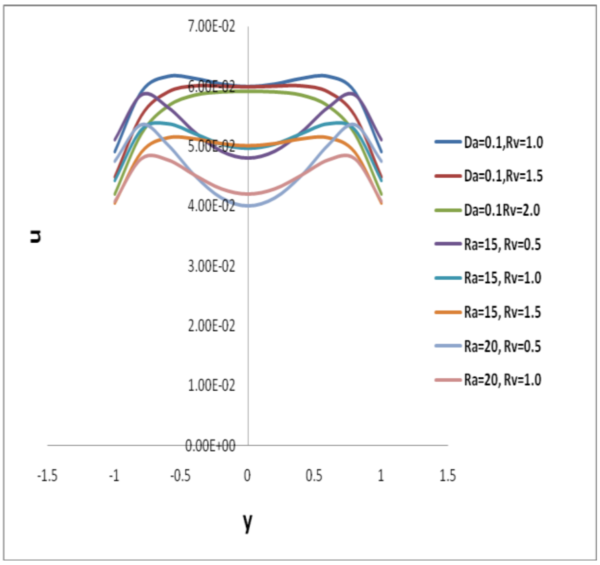

Figure 2: Velocity Profiles for Case I for different values of Ra and Rv, when Da=10^{-1}.

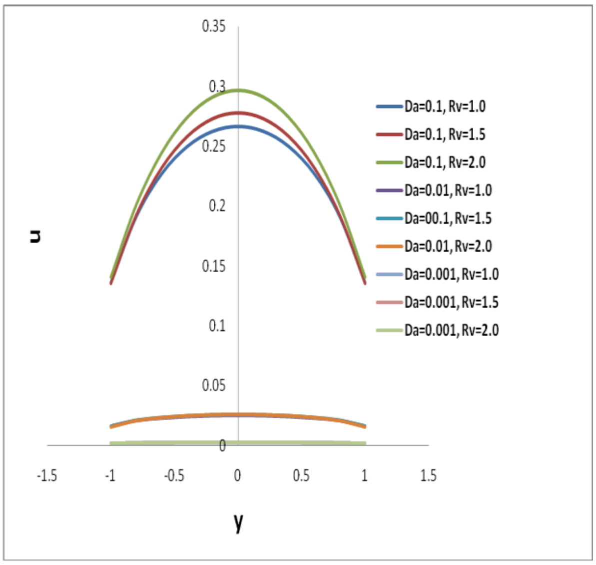

Figure 4: Velocity Profiles for Case III for different values of Rv, when Da=10^{-1}, 10^{-2} and 10^{-3}

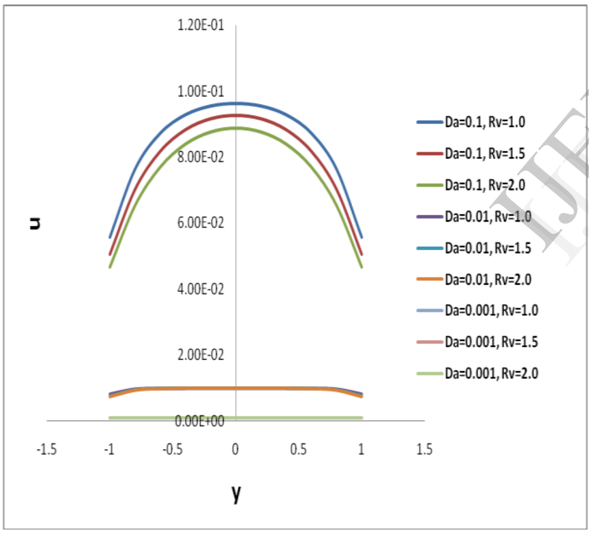

Figure 3: Velocity Profiles for Case II for different values of Rv, when Da=10^{-1}, 10^{-2} and 10^{-3}

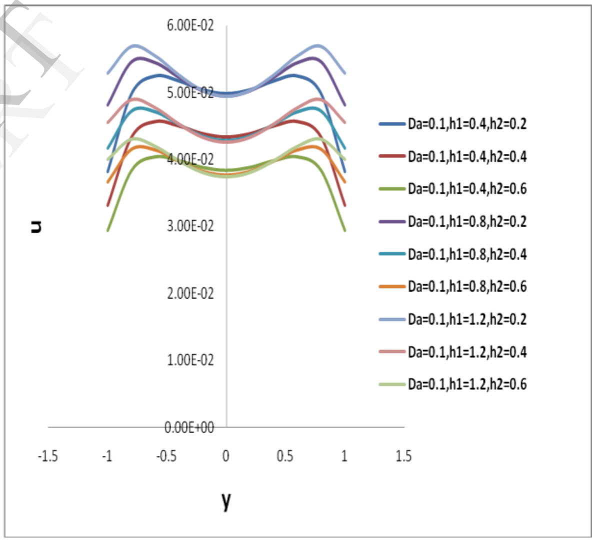

Figure5: Velocity Profiles for Case I for different values of h_1 and h_2, when Da=10^{-1}

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 3, March - 2013

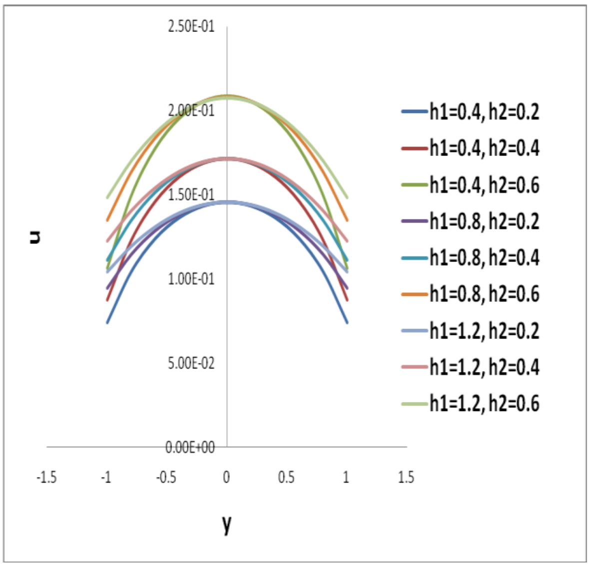

Figure 6: Velocity Profiles for Case III for different values of h_1 and h_2, when Da=10^{-1}

5. References

[1] Nield DA and Bejan A (1999), Convection in porous media (2nd ed.) Springer, New York.

[2] Ingham DB and Pop I (1998), Transport phenomena in porous media, Pergamon press, Oxford.

[3] Chauhan DS and Kumar V, (2012) Radiation Effects on Unsteady Flow through a Porous Medium Channel with Velocity and Temperature Slip Boundary Conditions has discussed, Applied Mathematical Sciences, 36(6), 1759 – 1769.

[4] Mazumdar BS (1981), Taylor's diffusion for a natural convection flow through a vertical channel. Int. J. Engineering Sciences 19, 771-779.

[5] Chen CH (1997), Analysis of non-Darcian flow phenomena on mixed convection in porous medium adjacent isothermal horizontal plates, Int. J. Heat Mass transfer 40, 2993-2997.

[6] Chen YC, Chung JN, Wu CS, Lue YF (2000) Non Darcy mixed convection in vertical channel filled with porous medium, Int. J. Heat Mass transfer 43, 2421-2429.

[7] Sekhar KR, Reddy GVR and Prasad BD, (2012) Chemically reacting on MHD oscillatory slip flow in a planer channel with varying temperature and concentration, Advances in Applied Science Research, , 3 (5):2652-2659.

Jain NC and Gupta P., (2006), Three dimensional free convection coutte flow with transpiration cooling. J. Zhejiang Univ. SCIENCE A, 7(3), p.p 1-8.

[8] Chauhan DS and Kumar V,(2009) Darcy extended Brinkman-Forchheimer model.Turkish J. Eng. Ev. Sci, (33),91-104.

[9] Jain NC and Sharma B, (2009), On three dimensional free convection coutte flow with transpiration cooling and tempera-ture jump boundary condition. Int. J. App. Mech. Eng., 14(3), 715-732.

[10] Jain NC and Gupta P., (2006), Three dimensional free convection coutte flow with transpiration cooling. J. Zhejiang Univ. SCIENCE A, 7(3), p.p 1-8.

[11] Sharma, P K., (**2005**) Effect of periodic heat and mass transfer on the unsteady free convection flow past a vertical flat plate in slip flow regime when suction velocity oscillates in time *Latin American Applied Research*, 35, 313-319,.

[12] Chaudhary, RC and Jha, AK, (**2008**), Effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime, *Applied Mathematics and Mechanics*, 29 (9), 1179-1194.

[13] Khaled, A.R.A. and Vafai, K. (2004) The effect of the slip condition on Stokes and Couette flows due to an oscillating wall: exact solutions, Int. J. Non-Linear Mech., 39(5), 795-809.

[14] Mahmoud, M.A.A. (2010). Chemical reaction and variable viscosity effects on flow and mass transfer of a non-Newtonian visco-elastic fluid past a stretching surface embedded in a porous medium. Journal Meccanica, 10(1007):9292

[15] Darek C, Tretheway DC and Mainhart CD, (2002), A series of investigations on slip flow regime, *Physics of the fluids*. **14**, L9.