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Mixed Convection in a Differentially Heated Square Cavity with Moving Lids

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Abstract— **Heat transfer in a two−dimensional steady mixed convection problem for a two−sided lid−driven differentially heated square cavity is investigated numerically. The top and bottom moving walls are maintained at different constant temperatures while left and right walls are thermally insulated. The transport equations are solved numerically with finite element approach using characteristic based split (CBS) algorithm.**
Comparisons with previously published work on the basis of special cases are performed and found to be in excellent agreement. Based upon the numerical predictions, the effect of Richardson number is presented. Governing parameters were 0.01 ≤ Ri ≤ 10 and Prandtl number, Pr = 0.71. Flow pattern and temperature contour lines for various cases are presented.

Keywords— Characteristic Based Split (CBS) Algorithm;Mixed Convection; Computational Fluid Dynamics

I. Introduction

Mixed convection lid−driven flow problems are encountered in many engineering applications including cooling of electronic devices, furnaces, lubrication technologies, chemical processing equipment, drying technologies, etc. In engineering applications there are many transport processes which are governed by the combined action of the natural convection and forced convection. The modern technologies require thorough understanding of the pertinent processes involving such mixed convection problems. Often the practical processes are governed by the combined action of natural and forced convection and, depending on the situation; one may predominate over the other. The orientation of the cavity with respect to the gravitational field, for instance, provides various scenarios depending whether the density gradient is parallel or perpendicular to gravity vector Numerous investigations have been conducted on lid−driven cavity flow and heat transfer, considering various combinations of the imposed temperature gradients and cavity configurations. This is because the driven cavity configuration is encountered in many practical engineering and industrial applications. Such configurations can be idealized by the simple rectangular geometry with regular boundary conditionsyielding a well−posed problem. The resulting flow however, is rather complex even when the flow is purely shear driven for the isothermal case without any temperature gradient. When atemperature gradient is imposed such that the shear driven and buoyancy effects are of comparable magnitude then theresulting flow falls under the mixed convection regime and the interaction and coupling of these effects makes the analysis more complex.

Fluid flow and heat transfer in rectangular or square cavities have been studied extensively. Natural and mixed convection problems in a differentially heated square cavity were extensively studied by R. Iwatsu et. al[1−3]. Similar studies in deep lid driven cavities were done by A.K. Prasad et. al.[4]. Combination of buoyancy forces due to temperature gradient and forced convection due to shear results in a mixed convection heat transfer, which is a complex phenomenon due to interaction of these forces. Mixed convection in two−sided lid−driven differentially heated square cavity was studied byHakan F. Oztop et. al.[5]. They showed how the interaction of natural and forced convection affects the flow and heat transfer in a square cavity. Al−Amiri [6] made a numerical study to perform laminar flow and heat transfer in a lid−driven square cavity heated from a driving wall filled with a porous medium. He wrote the governing equation in the streamline−vorticity form and these equations were solved via the finite−volume method. Moallemi and Jang[7] numerically studied mixed convection in a bottom heated driven square cavity and investigated the effect of Prandtl number on the flow and heat transfer process.

Almost all of the studies in the existing literature addressing mixed convection in lid−driven cavities considered configurations where the sidewalls are aligned with the direction of gravity and are subjected to constant heat flux or isothermal conditions[9]. In some applications it is necessary to position the cavity such that the heated walls are aligned perpendicular to the direction of gravity. Understanding themixed convection heat transfer process in such cavities is very important for design purposes. Furthermore, analysis of the unexplored mixed convective flow behavior in such cavities is of academic interest as well. These are the motivation and main focus of this study.

II. PROBLEM DESCRIPTION

Consider a double lid−driven differentially heated square cavity filled with an incompressible fluid. The horizontal lids are at different constant temperatures. The bottom lid is assumed to be at a higher temperature with respect to top wall. The vertical walls are assumed to be adiabatic and impermeable to mass transfer.

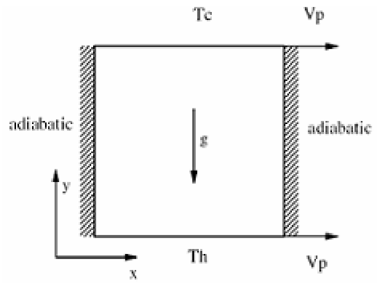

Fig.1: Both walls are moving towards right

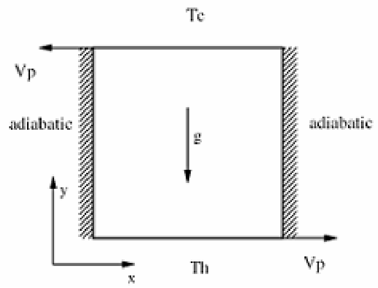

Fig.2: Bottom wall (hot) is moving towards right while top wall (cold) is moving towards left.

Two different cases were considered as shown in Fig.1 and Fig.2. In case I both walls are moving towards right. In case II, the bottom wall (hot) is moving towards right while top wall (cold) is moving towards left. In all the cases the moving walls have the same speed.

A. Mathematical Modeling

The flow is assumed to be two−dimensional, steady, laminar and incompressible. The properties of the fluid at a reference temperature are assumed to be constant, except for the buoyancy term of the momentum equation (Boussinesq approximation). Radiation heat transfer among sides is assumed to be negligible compared to other modes of heat transfer. From the assumptions mentioned above continuity, momentum and energy equations can be written as follows[13−14]:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

x−momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial x}+\upsilon_f\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

y−momentum equation:

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial y}+\upsilon_f\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$
$$+\beta_f g\left(T-T_c\right)$$

Energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_f\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

The following dimensionless parameters are used to convert the above equations into non-dimensional form.

$$x^*=\frac{x}{H},\ y^*=\frac{y}{H},\ u^*=\frac{u}{V_p},\ v^*=\frac{v}{V_p}$$

$$p^*=\frac{p}{\rho V_p^2},T^*=\frac{T-T_c}{\Delta T}$$

$$Gr=\frac{g\beta H^3\Delta T}{\nu_f^2},\mathrm{Re}=\frac{V_pH}{\upsilon_f},\mathrm{Pr}=\frac{\upsilon_f}{\alpha_f}$$

The dimensionless governing equations can now be written as:

Continuity equation:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0$$

x−momentum equation:

$$\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p^*}{\partial x^*}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

y−momentum equation:

$$\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p^*}{\partial y^*}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right)+RiT^*$$

Energy equation:

$$\frac{\partial T^*}{\partial t^*}+u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T}{\partial y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\partial^2 T^*}{\partial y^{*2}}\right)$$

The local variation of the Nusselt number can be expressed as

$$Nu=\frac{\partial T^*}{\partial y^*}$$

The average Nusselt number along the left wall is calculated as

$$\overline{Nu}=\int_0^1 Nu\,dx$$

The above set of equations was solved numerically using characteristic based split scheme [10–12].

B. Validation of the Code

The present code is validated for natural convection heat transfer by comparing the results of buoyancy driven laminar heat transfer in a square cavity with differentially heated side walls. The left wall was kept cold while the right wall was hot. The top and bottom walls are insulated. Table1: presents a comparison of the present data with those of De Vahl Davis[8] work. Numerical predictions, using the developed algorithm, have been obtained for Rayleigh numbers between 103 and 106.

TABLE I. COMPARES THE RESULTS WITH THOSE BY DE VAHL DAVIS.

Grashof number (Gr)	De Vahl Davis	Present solution
10^3	1.118	1.104
10^4	2.243	2.217
10^5	4.519	4.440
10^6	8.779	8.493

The computed results are in very good agreement with the benchmark solution. The maximum percentage deviation in results observed is only 3.25%, and is at Gr = 10^6.

C. Grid Independence Study

The grid independence test is performed for grids, 30 X 30,
40 X 40, 50 X 50 and 60 X 60. Unstructured triangular grid has been used for all the computations. The variation of the u−velocity in the horizontal mid−plane and temperature in the vertical mid was compared. It was observed that the curves overlap with each other for 50 X 50 and 60 X 60. So a grid number of 50 X 50 is chosen for further computation.

III. RESULTS AND DISCUSSIONS

The range of Richardson number, Ri, for this investigation is varied between $0.01 \leq Ri \leq 10$. To vary Richardson number, Ri, Grashof number is fixed at Gr = 104 while changing Reynolds number through the plate velocity.

Case I: Here both the lids are moving towards right. Both streamlines and isotherms were found to be symmetric about the mid y−plane (Fig.3−10). As the Reynolds number was increased the centre of streamlines shifted towards the direction of plate movement and the isotherms moved closer to the walls. As a result the temperature gradient near the wall increases. So the Nusselt number increases with Reynolds number as shown in Fig.11.

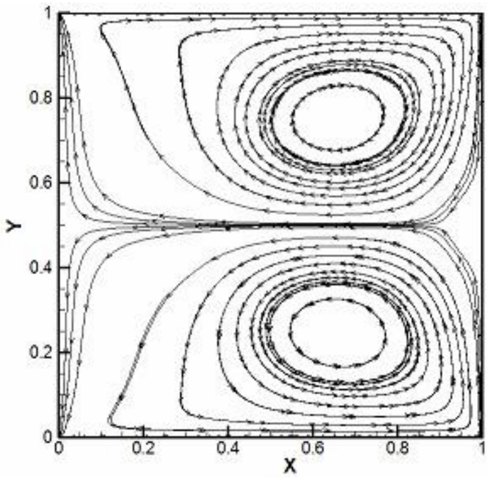

Fig.3: Streamlines for Case I with Ri = 0.01

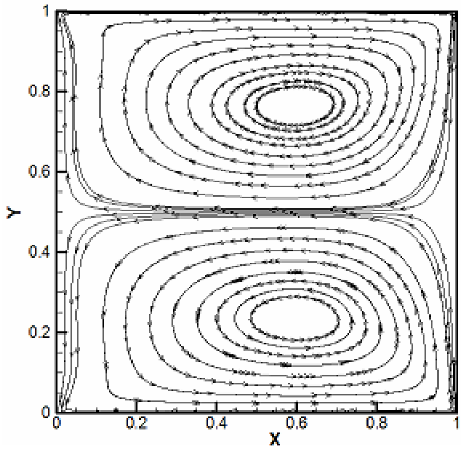

Fig.4: Streamlines for Case I with Ri = 0.1

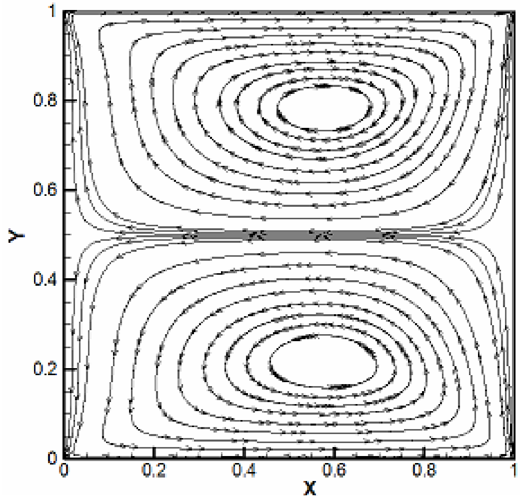

Fig.5: Streamlines for Case I with Ri = 1

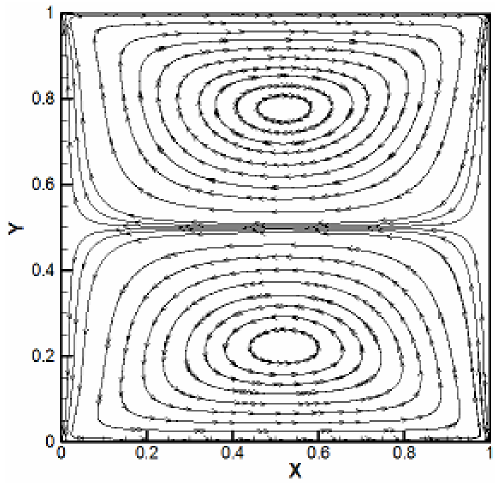

Fig.6: Streamlines for Case I with Ri = 10

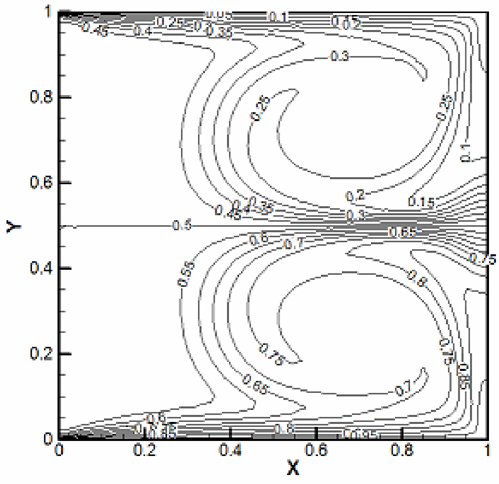

Fig.7: Isotherms for Case I with Ri = 0.01

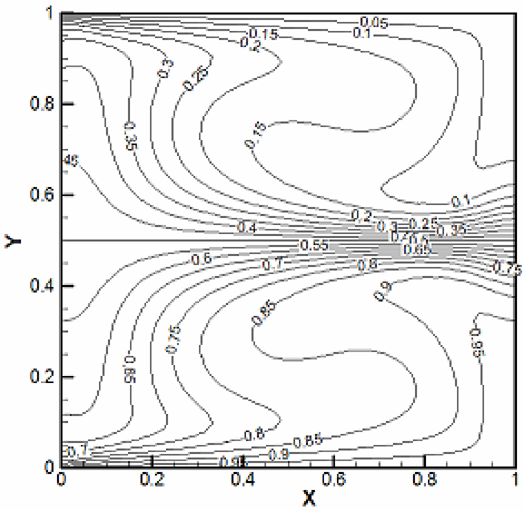

Fig.8: Isotherms for Case I with Ri = 0.1

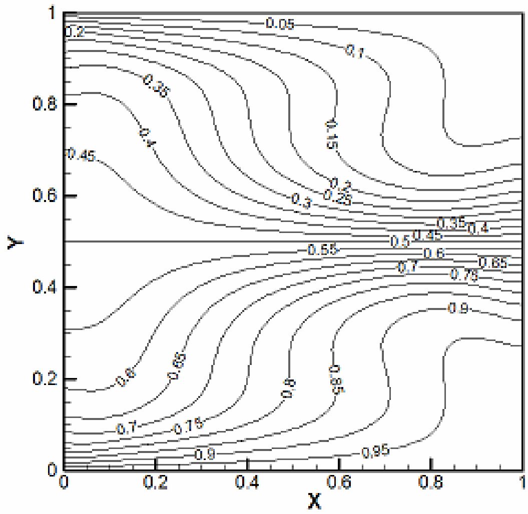

Fig.9: Isotherms for Case I with Ri = 1

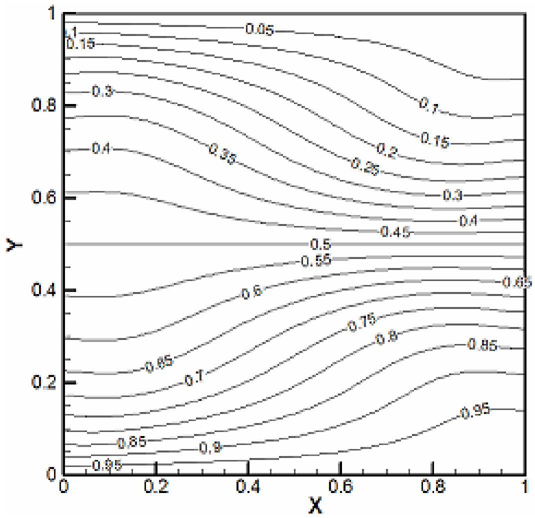

Fig.10: Isotherms for Case I with Ri = 10

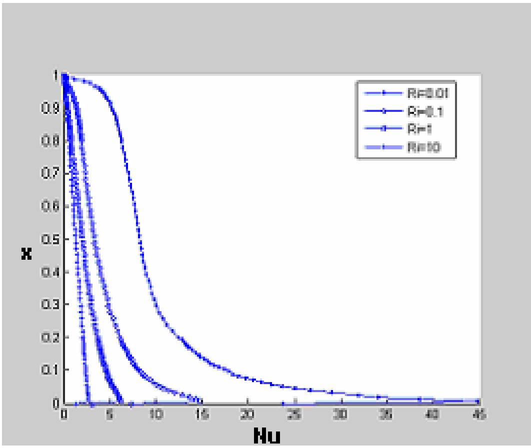

Fig.11: Variation of Nusselt number for case I

Case II: Here the top lid is moving towards left and bottom wall is moving towards right. As the Reynolds number increases the Nusselt number is also increases as a result of the increased influence of forced convection. When Reynolds number increases the isotherms move closer to the walls (Fig.17–20) resulting in the increase of temperature gradient near the wall. So the Nusselt number increases with Reynolds number as shown in Fig.12. Also we can see that as Ri increases the streamlines lines become more and more circular (Fig.13–16). This is because the natural convection process becomes dominant over forced convection as Ri increases.

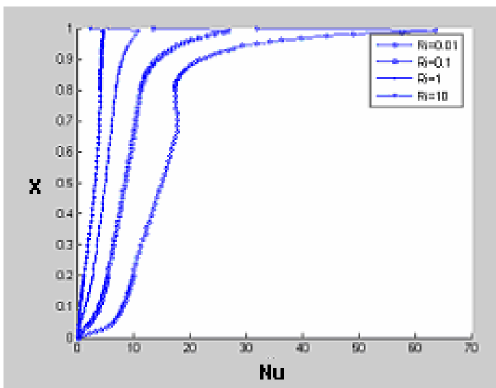

Fig.12: Variation of Nusselt number for case II

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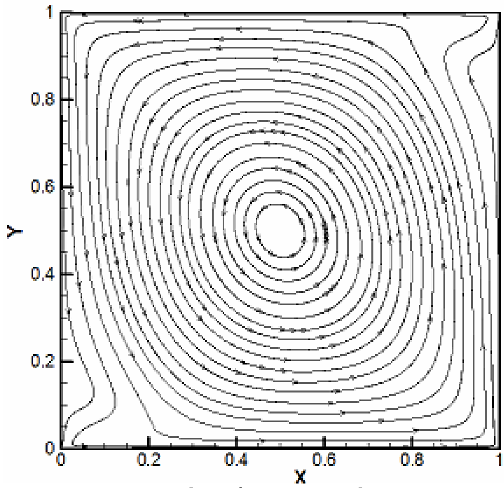

Fig.13: Streamlines for Case II with Ri = 0.01

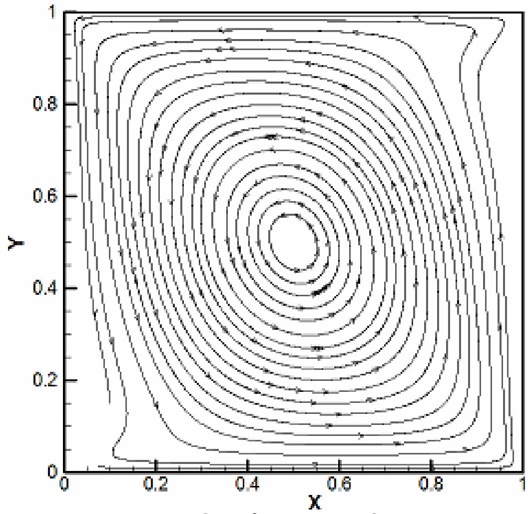

Fig.14: Streamlines for Case II with Ri = 0.1

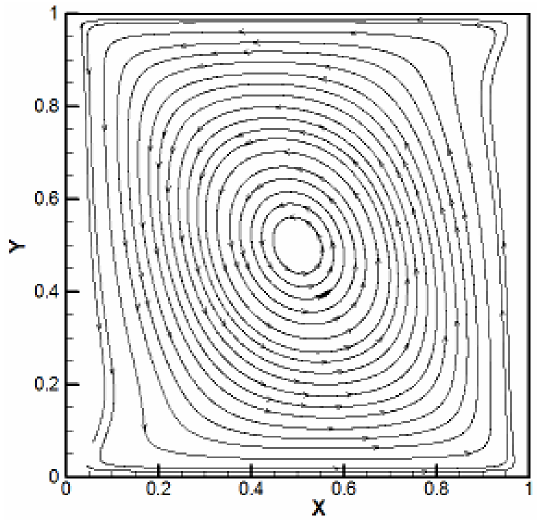

Fig.15: Streamlines for Case II with Ri = 1

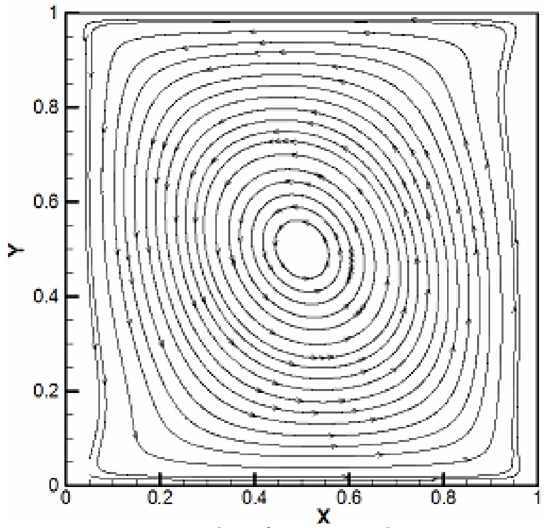

Fig.16: Streamlines for Case II with Ri = 10

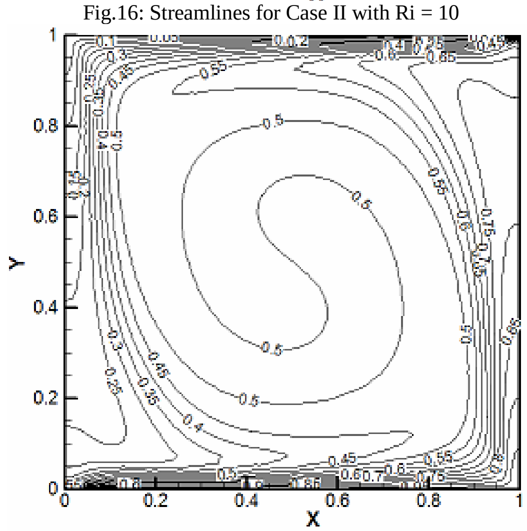

Fig.17: Isotherms for Case II with Ri = 0.01

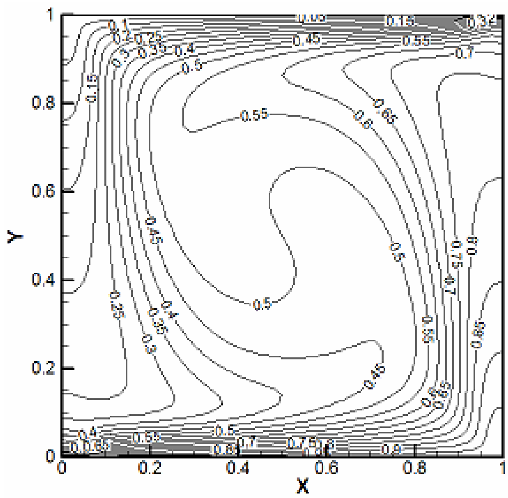

Fig.18: Isotherms for Case II with Ri = 0.1

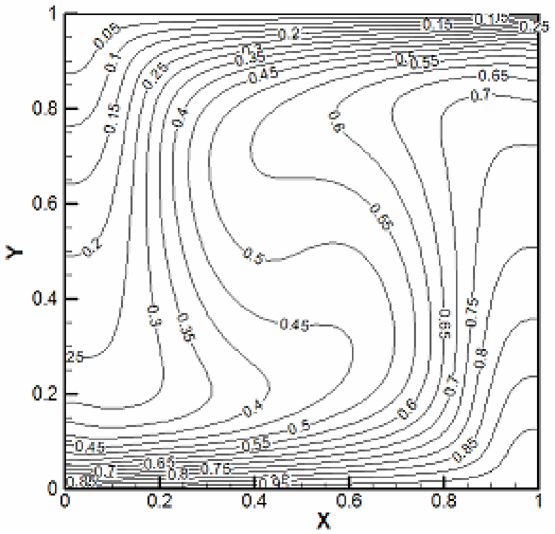

Fig.19: Isotherms for Case II with Ri = 1

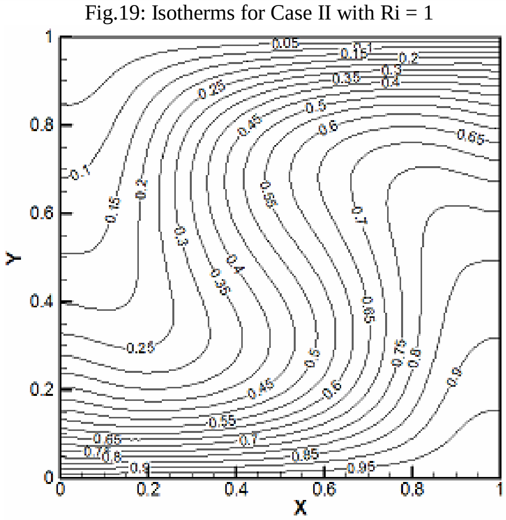

Fig.20: Isotherms for Case II with Ri = 10

IV. CONCLUSION

In this study, mixed convection heat transfer enhancements of double lid−driven differentially heated square cavity is investigated for various values of Richardson number and different cases of lid movement. The buoyancy force dominates the flow for Ri >>1; the inertia force becomes of the same order of magnitude of buoyancy force for Ri _1, and the inertia force dominates the flow for Ri < 1. For Ri in the range of 0.1−10.0, both inertia and buoyancy forces contribute to the flow structure, and the fluid motion can be considered as mixed convection.

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