Modal Analysis of Thin Frp Skew Symmetric Angle-Ply Laminate with Circular Cut-Out

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ABSTRACT

The present work deals with the free vibration analysis of a thin Fiber Reinforced Plastic (FRP) skew laminated composite plate with a circular cut-out at the geometric centre. The laminate considered for the present analysis is a four layered symmetric [45^0/-45^0/-45^0/45^0] angle-ply and Anti-symmetric [45^0/-45^0/45^0/-45^0] angle-ply laminate. The problem is modelled in ANSYS software based on the Classical Lamination Theory (CLT) which is suitable for the analysis of thin laminates. The first five natural frequencies of the skew plate are evaluated by changing the skew angle (α), thickness ratio (s), the diameter to length ratio (d/l) and the boundary conditions (B.C).

Keywords: FRP, FEM, CLT, Free vibration, Circular cut-out

"1. Introduction"

A review of literature is presented to establish better understanding of the background issues involved related to this report. Fiber reinforced composite materials consists fibers of significant strength and stiffness embedded in a matrix with distinct boundaries between them. Both fibers and matrix maintain their physical and chemical identities, yet their combination performs a function which cannot be done by each constituent acting singly. Fibers of fiber reinforced plastics (FRP) may be short or continuous. It appears obvious that FRP having continuous fibers is indeed more efficient.

Natural frequencies and buckling stresses of angle-ply laminated composite plates are analyzed by Matsunaga [2] taking into account the effects of shear deformation, thickness change and rotatory inertia. Cheung et.al. [3] developed a three-dimensional linear, small deformation solution for the free vibration of thick, layered rectangular plates with various boundary conditions using a finite layer method. Yoshiki et.al. [4] analyzed the natural frequencies for the free vibration of stiffened FRP laminate plates and the applicability of the model is discussed by comparing with the numerical results obtained from finite element analysis. Anderson et.al. [5] presented a technique “Eigen Value Economizer” which permits very fine subdivisions of elements with eigen value calculations of limited size and evaluated a numerical integration in deriving element properties, permitting thickness variation within an element. Slyper et.al. [6] developed a explicit transverse bending stiffness and mass matrices for a triangular finite element having a linear thickness variation. This element is more suitable for general application than the triangular elements previously described for determining the static deflections and vibrational characteristics of plates in transverse bending. Raju et.al. [7] studied on large amplitude vibrations of Mindlin plates using Lagrangian, isoparametric, quadrilateral elements with selective integration. Negm et.al. [8] presented an application of a new improved rectangular finite element to the problems of free and forced harmonic oscillations of thin plates in bending. A higher order theory of plate deformation, laminated plates by Lo et.al. [9] modeled the behavior of laminated plates by comparing with elasticity solutions. Reddy et.al. [10] developed a higher-order shear deformation theory by taking into account parabolic distribution of the transverse shear strains through the thickness of laminated composite plates, and obtained exact closed-form solutions of symmetric cross-ply laminates and the results are compared with 3-D elasticity solutions and first-order shear deformation theory solutions. Reissner et.al. [11] discussed that for analysis of composite plates, a satisfactory transverse shear deformation theory for laminated anisotropic plates is needed. Liou et.al. [12] developed a three-dimensional eight-node hybrid stress finite element method for the analysis of fundamental frequencies for the cross-ply laminates and also compared with numerical values obtained from the elasticity solution. Mallikarjuna et.al. [13] presented a C^0 finite element formulation of the higher-order theory to determine the natural frequencies of isotropic, orthotropic and layered anisotropic composite and sandwich plates. Spilker et.al. [14] developed an invariant eight-node hybrid-stress element for thin and thick multi-layer laminated plates using the finite element formulation for higher order theories to evaluate the free vibration frequencies of symmetric and anti-symmetric laminated plate. It is generally known that the classical lamination theory works efficiently for the static as well as dynamics problems of thin laminated structures and simplifies the problem to the major extent. The present investigation intends to apply the finite element technique, based on classical lamination theory, for the free vibration analysis of symmetric and anti-symmetric thin laminates. The lowest five natural frequencies are studied.

"2. Finite Element Modelling"

Geometry and Finite Element Model

The geometry and finite element model of the problem are shown in Figure 1. The sides of the plate are taken as 1m. The laminate considered for the present analysis is a symmetric and anti-symmetric angle-ply laminate with fiber angle of 45^0. The length-to-thickness ratio is maintained as 50. The skew angle is varied from 0^0 to 50^0 and the diameter of the circular cut-out is selected from the d/l ratio which is taken as 0.2.

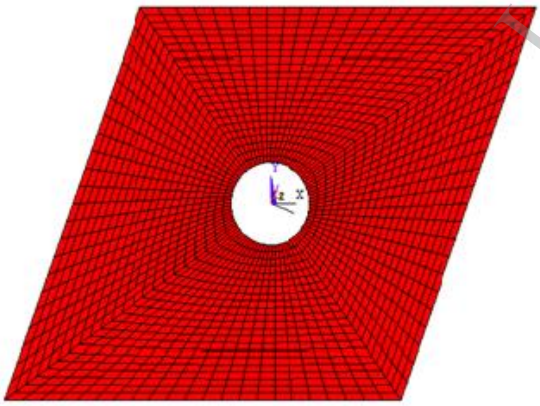

Figure 1: Laminated Composite Skew plate.

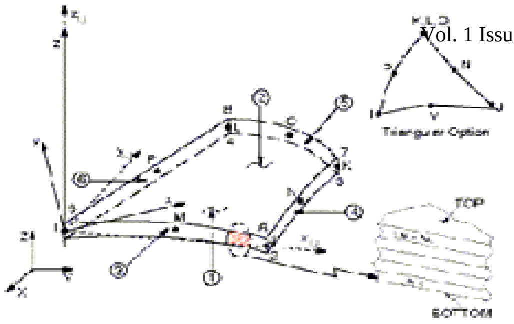

Figure 2: Layered Shell Element (SHELL 99)

Laminated composite general shell element SHELL99 of ANSYS [15] is used for meshing the geometry of the problem. This element is suited for modelling moderately thick to thin laminated composite shells. As shown in Figure 2, the element consists of number of layers of perfectly bonded orthotropic materials. The element is quadratic and has six degrees of freedom per node namely, translations in x, y and z directions respectively, and rotations about x, y and z axes respectively. SHELL99 allows up to 250 layers. This element gives results of high accuracy and discretization involves fewer elements. As shown in Figure 2, the lamination sequence is $[45^0/-45^0/-45^0/45^0]$ for symmetric and $[45^0/-45^0/45^0/-45^0]$ for Anti-symmetric angle-ply laminates.

"3. Material properties"

The following material properties are used. Carbon Epoxy Fiber Reinforced Plastic Composite:

E_1 =147 GPa, E_2 =10.3 GPa, E_3 =10.3 GPa,

G_{12} =7 GPa, G_{23} = 3.7 GPa, G_{13} =7 GPa,

$\nu_{12} = 0.27$, $\nu_{23} = 0.54$, $\nu_{13} = 0.27$,

$\rho = 1.6 \times 10^3$ kg/m^3

Modal Analysis of Thin FRP Symmetric $[45^0/-45^0/-45^0/45^0]$ and Anti-Symmetric $[45^0/-45^0/45^0/-45^0]$ Angle-Ply Laminate

"4. Results"

Figure 3 & 4 show the variation of natural frequency with respect to skew angle. It has been observed that the natural frequency is increasing with increase in the skew angle for 4 modes and for the fifth mode in the anti-symmetric case there is a slight decrease and increasing in the frequency. This is due to the increase in resultant stiffness and decrease in mass of the skew plate with increase in skew angle. The rate of increase is different for different modes which are due to the specific deformed mode of the plate. Figure 5 & 6 show the variation of the natural frequency with respect to thickness ratio. Fiber orientation greatly effects the natural frequency with respect to thickness ratio varying from 20-100. With increase in thickness mass reduces but with the change in fiber angle will affect the stiffness of the skew plate. As a result natural frequency decreases and will be different for five modes. Figure 7 & 8 shows the variation of the natural frequency with respect to d/l ratio. The size of circular cut-out is also increasing the natural frequency for the considered range of d/l ratio (0.2-0.6). This is due to the reduction in mass of the plate. Figure 9 & 10 shows the variation of the natural frequency with respect to boundary conditions and observed that the natural frequency is very high in case of all edges clamped for both symmetric and anti-symmetric laminated plates and low in both symmetric and anti-symmetric for the horizontal sides of the skew plate are free and the vertical sides are simply-supported.

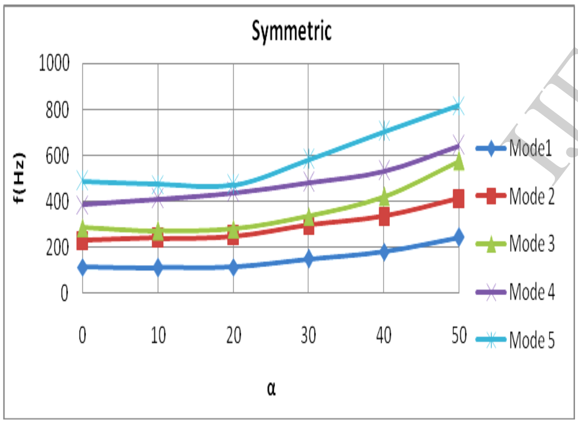

Figure 3: Variation of natural frequencies v/s skew angle (S=50, d/l=0.2, B.C=S-S-S-S)

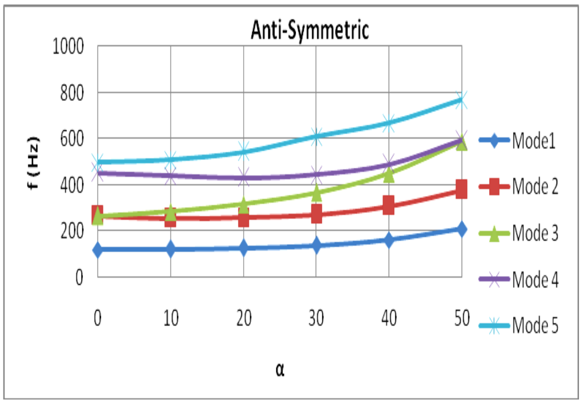

Figure 4: Variation of natural frequencies v/s skew angle (S=50, d/l=0.2, B.C=S-S-S-S)

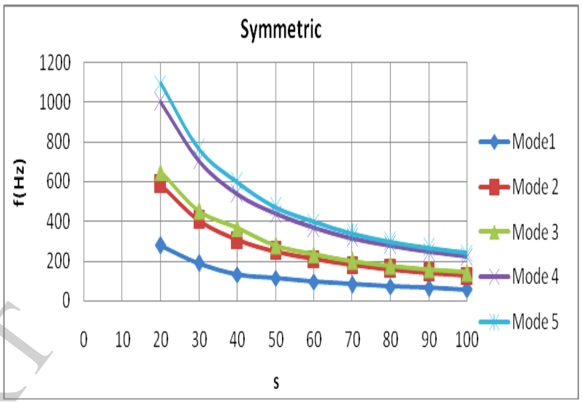

Figure 5: Variation of natural frequencies v/s thickness ratio ($\alpha=20^0$, d/l=0.2, B.C=S-S-S-S)

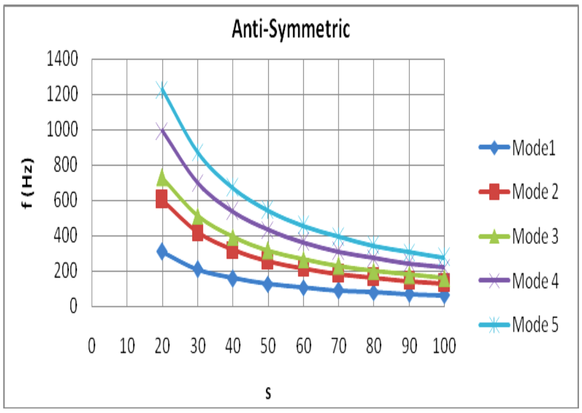

Figure 6: Variation of natural frequencies v/s thickness ratio ($\alpha=20^0$, d/l=0.2, B.C=S-S-S-S)

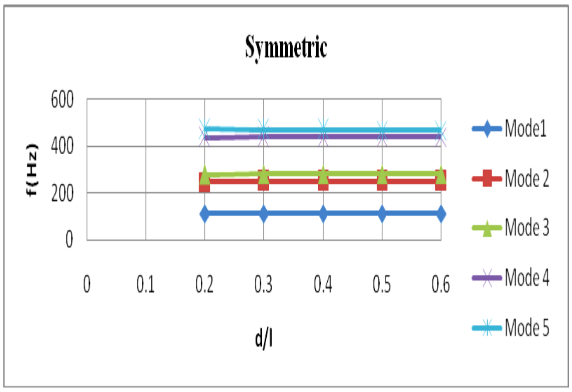

Figure 7: Variation of natural frequencies v/s d/l ($S=50$, $\alpha=20^0$, B.C=S-S-S-S)

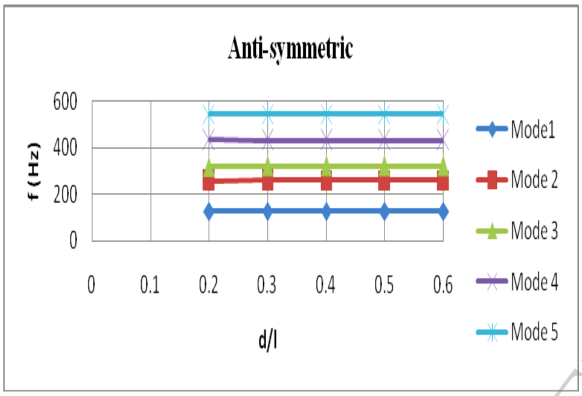

Figure 8: Variation of natural frequencies v/s d/l ($S=50$, $\alpha=20^0$, B.C=S-S-S-S)

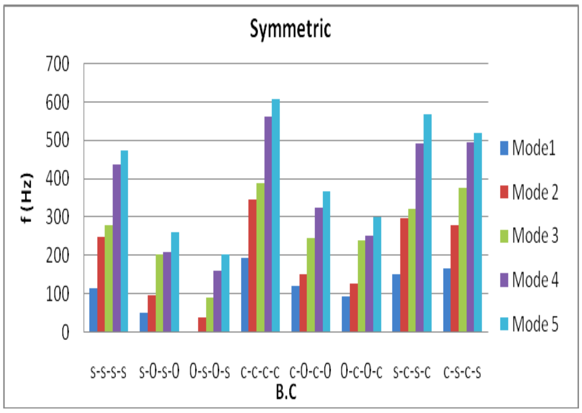

Figure 9: Variation of natural frequencies v/s boundary condition ($S=50$, $d/l=0.2$, $\alpha=20^0$)

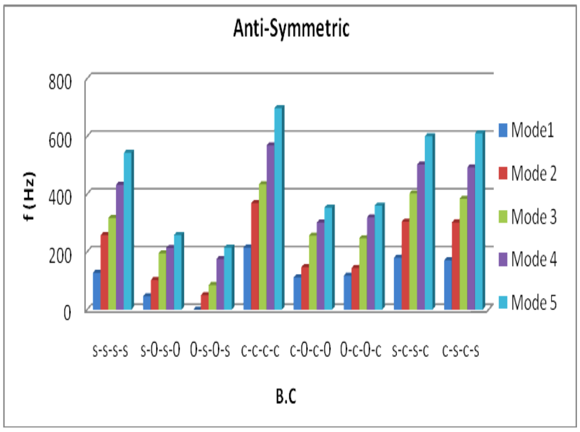

Figure 10: Variation of natural frequencies v/s boundary condition ($S=50$, $d/l=0.2$, $\alpha=20^0$)

"5. Conclusions"

Free vibration analysis of a thin Fiber Reinforced Plastic (FRP) skew laminated composite plate with a circular cut-out has been studied in the present work. The laminate considered for the present analysis is a four-layered symmetric and anti-symmetric angle-ply laminate. The effect of skew angle (α), thickness ratio (s), diameter to length ratio (d/l) and the boundary conditions (B.C) on natural frequencies in each mode is studied and the effect of each parameter on the natural frequencies is presented.

➢ Natural frequency increases with increase in the skew angle from (0º-50º) and is different for different mode shapes. Increase in skew angle increases the internal stiffness of the skew plate and the mass of the plate decreases.

➢ Natural frequency decreases with increase in thickness to length ratio (s) from (20-100) of skew plate. As thickness increases both stiffness and mass of the skew plate contribute to decrease natural frequency at a greater rate.

➢ Natural frequency increases with increase in diameter to length of the plate ratio (d/l) from (0.2-0.6). As d/l ratio increases the internal stiffness of the skew plate increases and the mass of the plate decreases. As a result the natural frequency increases and variation of increase is different for different modes.

➢ The effect of boundary conditions (B.C) on natural frequencies in each mode is studied. The natural frequency is more in case of any edge clamped when compared to simply-supported.

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