Modal and Harmonic Analysis of Master Leaf Spring

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***Abstract*— Vehicles with spring leaf suspension work in different road conditions. An important condition is that of irregular roads. Irregular roads can generate low frequencies vibrations on a car system. In the performance of a car, the natural behavior of the spring leaf is bending. The bending mode shapes caused by low frequencies release a large amount of energy. If the excitation created by the road coincides with one of the natural frequencies of the suspension system, resonance appears. In this work, modal, harmonic, and analytical analyses of a master leaf spring are carried out in order to get the bending mode shapes for low vibration frequencies. The master leaf spring characteristics are described in [4]. The model was done in Solidworks and analyses in ANSYS Workbench.**

Keywords*— Master leaf spring, Modal analysis, Harmonic analysis, Finite element analisys, Euler-Bernoulli equation, Mode shapes, ANSYS Workbench.

I. INTRODUCTION

A spring is an elastic body with the property to absorb and liberate energy by deforming itself. In most of the cases, light and heavy vehicles have a semi-elliptical leaf spring as an important component of the suspension system.

A leaf spring consists of a number of flat plates with different lengths, called leaves. All the leaves are stacked together. The longest leaf is called master leaf and the rest are called graduated leaves. Graduated leaves are attached to the master leaf by rebound clips to share the rebound load.

A master leaf has two eyes at both ends. Through these eyes, leaf springs are attached to the body of a vehicle. One of the eyes is connected to the frame with a simple pin joint so that the eye can rotate freely around the pin but no translation is allowed. The other eye is connected to the frame through a shackle. The shackle is a flexible link that connects the master leaf and the frame. This last eye has the flexibility to slide along the X-axis when a load is applied and it can rotate around the pin.

When a vehicle is running on an irregular path, sometimes it is projected from the surface. When this happens, the spring absorbs the energy of the propulsion by deforming itself. When the vehicle is again on the road, the spring releases the energy. This deformation is the change of the distance between both eyes. If both eyes were totally fixed, the spring leaf would not be able to adjust to the changes of distance. In order to allow this deformation, a shackle is provided at one end, to give a flexible conection.

A vibration is a movement that can make a complete system unstable. In many structures, vibrations are undesirable since they can cause waste of energy, stress concentrations, considerable damage, etcetera.

The natural frequency of a system is the frequency that has a tendency to make vibrate a system. All systems have one or more natural frequencies. If these frequencies are excited, an important increase in vibration will be caused. Natural frequencies are independent from applied forces in a system.

Modal analysis is the study of natural frequencies and mode shapes of systems and structures under free vibration. The most used method for modal analysis is the Finite Element Analysis (FEA). FEA is a computational method that allows to analyze objects with arbitrary forms, getting acceptable results.

Harmonic analysis can find the stable-state response of linear structures to loads that vary sinusoidal (harmonically) with time. Its purpose is to determine the structure's response at different frequencies and obtain a graph of some response quantity (displacement, velocity or acceleration) versus frequency.

When the frequency of an external force that excites a system coincides with the natural frequency of the system, the phenomenon of resonance appears. Resonance causes an excessive increase in vibration's amplitude and consequently faults in systems. Resonance is a dangerous condition that should be avoided. This is the main reason to carry out a modal and harmonic analysis.

II. LITERATURE REVIEW

B. MAHESH BABU, D.MURALIDHAR YADAV AND N.RAMANAIAH [1] performed a dynamic analysis of a master leaf spring by using FEA with help of ANSYS Workbench. Natural frequencies produced by excitation frequencies at different velocities are determined. The applied boundary conditions: one eye fixed and the other connected to a shackle that allowed master leaf to straighten.

A. L. AISHWARYA, A. ESWARA KUMAR, AND V. BALAKRISHNA MURTHY [2] analyzed free vibration of a composite material spring leaf (Carbon/Epoxy) to determine the effects of variation of different parameters. The applied boundary conditions were one eye fixed and the other was allowed to slide longitudinally. It is concluded that modification of friction coefficient has no significant effects on natural frequencies whereas the increase of the distance between both eyes improve them.

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PRESHIT B. WAGHMARE AND RAOSAHEB B. PATIL [3] performed static and dynamic analysis of steel leaf spring of an Armada vehicle by using ANSYS in order to compare natural frequencies with three different materials: steel, manganese silicon, and carbon epoxy. The applied boundary conditions were: one eye was allowed to rotate on Z-axis and the other was allowed to slide on X-axis and rotate on Z-axis.

ASHISH P. BORHADE AND J. T. PATTIWAR [4] performed dynamic analysis of a steel leaf spring of an Armada vehicle. Experimental analyses by using a FFT analyzer and FEA by using ANSYS were carried out. The applied boundary conditions were: one eye has zero degree of freedom (DOF) and the other had one DOF (X-axis displacement).

III. PROBLEM DEFINITION

The objective of this work is to do modal, harmonic, and analytical analyses of the master leaf spring analyzed in [4]. Boundary conditions described in [3] were applied in order to determine the low natural frequencies and mode shapes of vibration.

IV. METHODOLOGY

Dimensions and material properties (Silicon Magnesium Steel) of master leaf spring were taken from [4]:

Dimensions:
Span= 875 mm (center to center)
Camber= 130 mm.
Width at end = 62 mm.
Width at center = 62 mm.
Thickness = 5mm.
Inner diameter = 35 mm.
Outer diameter = 45 mm.

Material properties:
Young's modulus=2.1*e5 N/mm^2
Density=7.860*e-6 kg/mm^3
Poisson's ratio= 0.3
Tensile strength=1962 N/mm^2

The master leaf spring model was built in Solidworks by following the instructions given in [4].

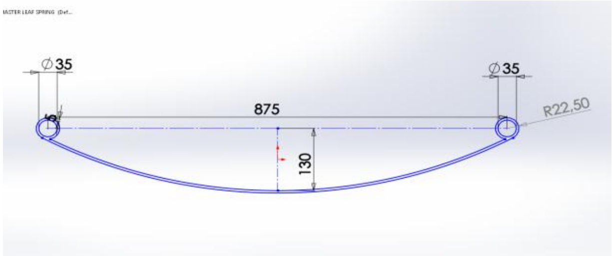


Fig. 1. Master leaf spring model in Solidworks

The model was imported to ANSYS Workbench. Once the material properties were added, boundary conditions described in [4] were applied:

Eye 1: Fixed (0 DOF)
Eye 2: only allowed to slide in X-axis (1 DOF)

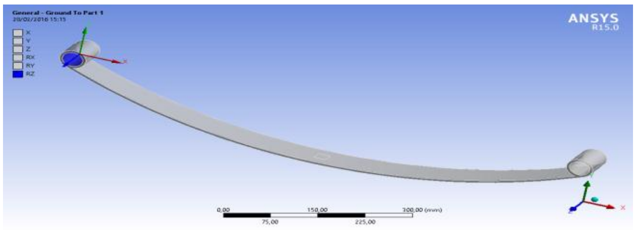
Natural frequencies obtained after FEA are shown in the table below.

Mode	Natural frequencies (Hz) FEA
1	29.643
2	87.151
3	174.53
4	269.67
5	288.85
6	435.19

Table.1. Natural Frequencies obtained by applying boundary conditions described in [4]

Based on [3] new boundary conditions were proposed. These conditions consider the rotation around the Z-axis in both eyes:

Eye 1: connected to the frame with a pin, thus rotation was only allowed around Z-axis. ROT Z.
Eye 2: connected to the frame through a shackle. It was allowed to slide along X-axis and rotation around Z-axis. RX and ROT Z.



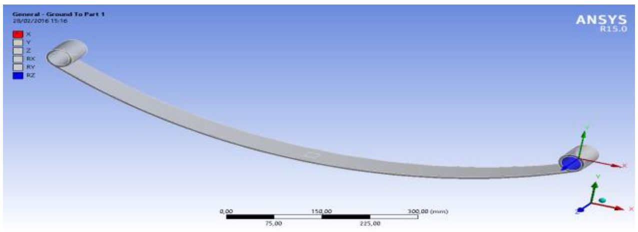


Fig.2 and 3. Boundary conditions proposed in this work based in [3].

Natural frequencies obtained by applying the new boundary conditions in ANSYS Workbench are shown in table.2:

Mode	Natural frequencies (Hz) FEA
1	12.034
2	49.324
3	111.91
4	195.14
5	269.66
6	290.45

Table.2. Natural frequencies obtained by applying new boundary conditions

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Mode shapes were as follows:

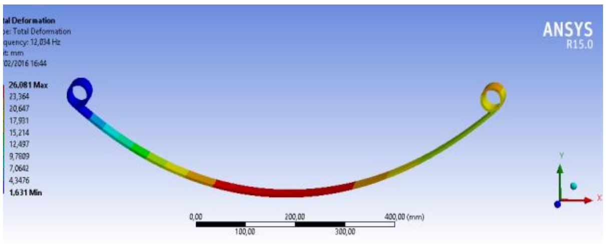

Fig.4 Mode shape for the first natural frequency.

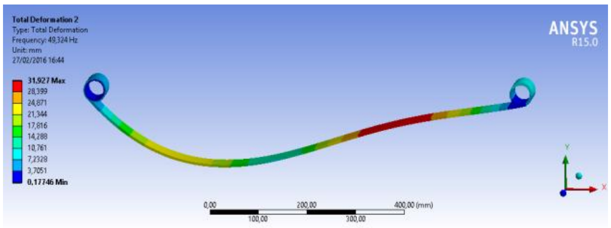

Fig.5 Mode shape for the second natural frequency.

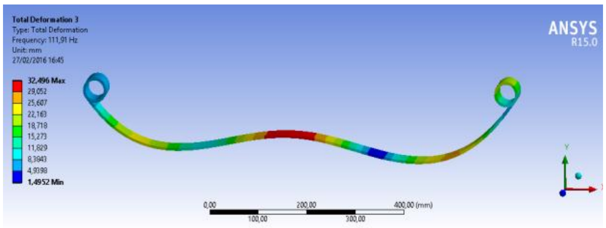

Fig.6 Mode shape for the third natural frequency.

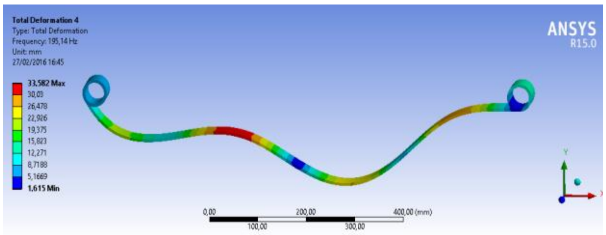

Fig.7 Mode shape for the fourth natural frequency.

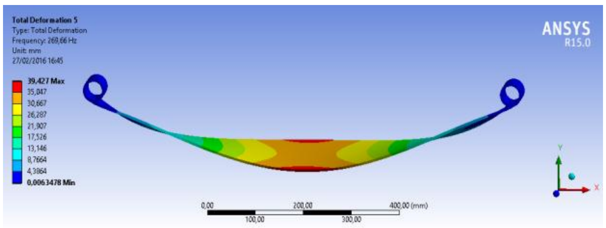

Fig.8 Mode shape for the fifth natural frequency.

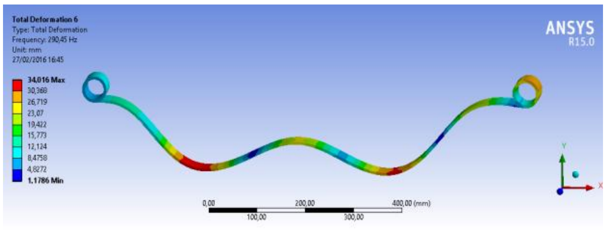

Fig.9 Mode shape for the sixth natural frequency.

With the dimensions and material properties described before, it is possible to calculate analytically the natural frequencies of the first mode shapes of vibrations. Euler-Bernoulli beam theory can be applied on leaf springs [5]:

$$EI\,\frac{\partial^4 \omega(x)}{\partial x^4} + \rho S\,\frac{\partial^2 (x,t)}{\partial t^2} \qquad (1)$$

Where E, I, ρ, and S are the Young's modulus, second moment of area, density, and cross-sectional area, respectively.

The boundary conditions and the derivation of the partial differential equation (2) are referred in [7].

$$\omega_i = \frac{\lambda_i^2}{2\pi l^2}\sqrt{\frac{EI}{m}} \qquad (2)$$

According to boundary conditions proposed in this work, the values for λ_i^2 were taken from [6]. The results are shown in the table below:

Mode	Natural frequencies (Hz) Euler-Bernoulli Equation
1	14.1813
2	56.7252
3	127.6317
4	226.9008
5	354.5326
6	510.5269

Table.3. Natural frequencies of master leaf spring obtained by applying Euler-Bernoulli equation.

Equations and mode shapes for the first three natural frequencies of a pinned-pinned beam are shown in fig. 10.

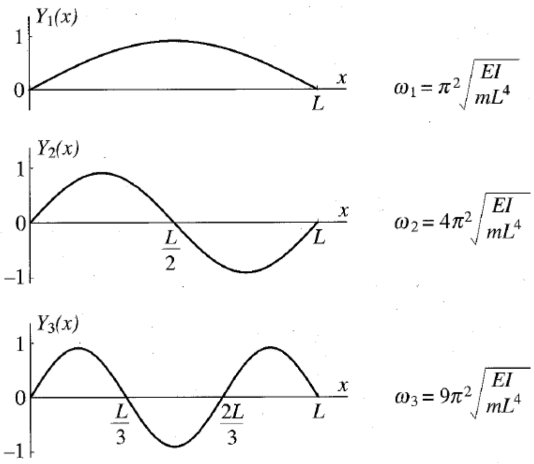

Fig.10 Mode shapes of a pinned-pinned beam. [7]

From the fourth natural frequency there is a variation of the values obtained by FEA and those obtained by the Euler-Bernoulli equation. This variation is mainly produced because the Euler-Bernoulli equation does not take into account the existence of eyes at both ends of the master leaf spring. In order to prove the approximation of the first three natural frequencies, a modal analysis of master leaf spring without eyes was done. Boundary conditions proposed in this work were applied.

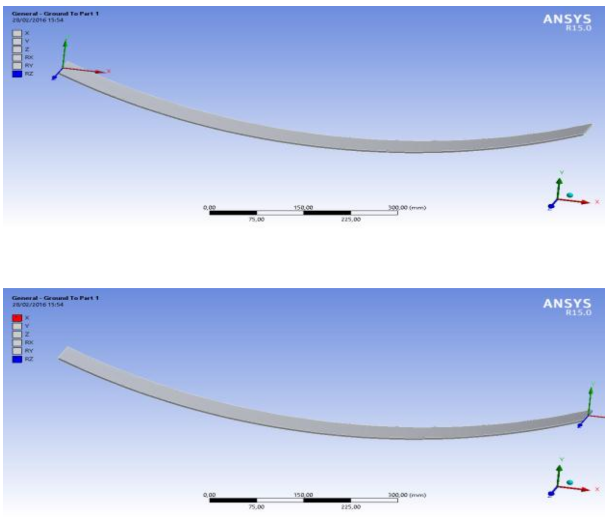

Fig.11 and 12. Boundary conditions of master leaf spring without eyes.

Natural frequencies obtained by FEA are shown in the table below:

Mode	Natural frecuency (Hz) master leaf spring without eyes. FEA
1	13.78
2	55.215
3	129.37
4	228.63
5	282.81
6	361.0
7	518.35

Table.4. Natural frequencies obtained by FEA of master leaf spring without eyes.

Harmonic analysis of the master leaf spring was done by using ANSYS Workbench. Considering that the average weight of a commercial vehicle is around one ton, a 3000 Newton force was applied.

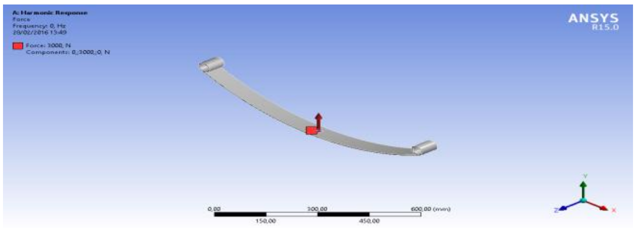

Fig.13 3000 Newton force applied.

Displacement (in millimeters) was the response quantity chosen. The displacement-frequency graphs obtained are shown below:

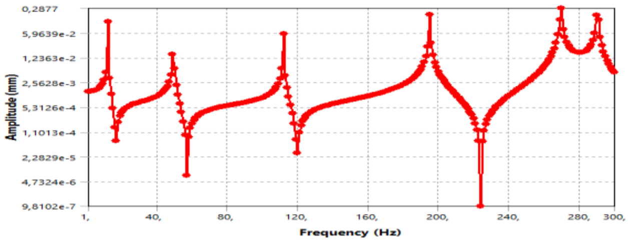

Fig.14. Deformation response in a range of 300 Hz.

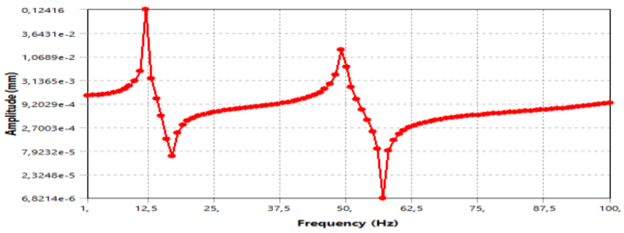

Fig.15. Deformation response in a range from 0-100 Hz.

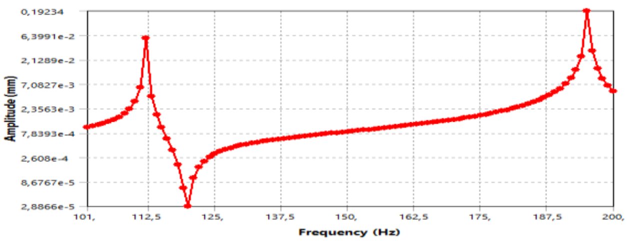

Fig.16. Deformation response in a range from 100-200 Hz.

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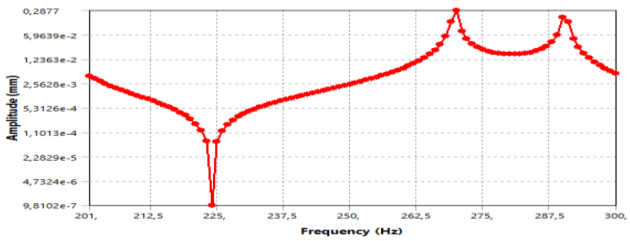

Fig.17. Deformation response in a range from 200-300 Hz.

The responses for the master leaf spring at the corresponding resonance frequencies are shown in the table below:

Mode	Natural frequencies (Hz)	Displacement (mm)
1	12.034	0.12416
2	49.324	6.369e-003
3	111.91	5.5994e-002
4	195.14	0.19234
5	269.66	0.2877
6	290.45	0.18171

Table.5.Displacements and resonance frequencies of master leaf spring

V. RESULTS AND DISCUSSION

Natural frequencies of the master leaf spring were determined by applying the described boundary conditions in [4] and subsequently with those proposed in this work based on [3].

Mode	Natural frequencies (Hz) with suggested boundary conditions in [4]	Natural frequencies (Hz) with proposed boundary conditions
1	29.643	12.034
2	87.151	49.324
3	174.53	111.91
4	269.67	195.14
5	288.85	269.66
6	435.19	290.45

Table.6.Comparison between natural frequencies obtained by FEA and different boundary conditions.

Euler-Bernoulli equation can be used to get an approximation of first natural frequencies of a master leaf. Since the equation does not take into account the existence and effect of the eyes, there are minimum variations in the values.

Comparison of natural frequencies of master leaf spring without eyes obtained by FEA and those obtained by Euler-Bernoulli equation are shown in table.7.

Mode	Natural frequencies (Hz) Euler-Bernoulli Equation	Natural frecuency (Hz) FEA
1	14.1813	13.78
2	56.7252	55.215
3	127.6317	129.37
4	226.9008	228.63
5	--	282.81
6	354.5326	361.0
7	510.5269	518.35

Table.7.Comparison between natural frequencies of master leaf spring without eyes obtained analytically and those obtained by FEA

The fifth natural frequency obtained by the Euler-Bernoulli equation belongs to the value of the sixth natural frequency obtained by FEA. This occurs because the Euler-Bernoulli equation only takes into account bending vibrations whereas FEA considers bending, torsional vibrations, and the effect of their combination.

According to the previous consideration, values of the first natural frequencies obtained by FEA can be confirmed by applying the Euler-Bernoulli equation. An analytical analysis of the complete master leaf spring was carried out. Results are shown in table.8:

Mode	Natural frequencies (Hz) Euler-Bernoulli equation.	Natural frequencies (Hz) FEA
1	14.1813	12.034
2	56.7252	49.324
3	127.6317	111.91
4	226.9008	195.14

Table.8.Comparison between first natural frequencies of master leaf spring obtained analytically and those obtained by FEA

Finally, low natural frequencies of first mode shapes that complete those obtained in [4] are shown in the table below:

Low natural frequencies (Hz) FEA	Natural frequencies (Hz) FEA obtained in [4]
12.034	----
49.324	----
111.91	----
195.14	----
----	207.75
----	517.3
----	1158.1
----	1277.8
----	1589
-----	2455.55

Table.9. Low natural frequencies that complete those obtained in [4]

VI. CONCLUSION

Natural frequencies of mode shapes obtained by modal and harmonic analyses are similar. Natural frequencies of first mode shapes were corroborated with the Euler-Bernoulli equation.

It was proved that the master leaf spring curvature does not have a significant effect in the natural frequencies of mode shapes when Euler-Bernoulli equation is applied. The existence of the eyes at both ends of the master leaf spring is not considered in Euler-Bernoulli equation nonetheless it provides approximate values of low natural frequencies in first mode shapes.

The knowing of the natural frequencies of first mode shapes of vibration is important. This importance relies on the fact that these frequencies have a high probability to excite a system and cause the phenomenon of resonance. Resonance at low frequencies can liberate a large amount of energy. This energy originates the appearance of noise, stress concentration, fatigue and a significant reduction of the life cycle of structures or systems.

VII. REFERENCES

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