International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 1 Issue 9, November- 2012

Modified 3D SCN Transmission Line Matrix Method For Metamaterials Frequency Selective Surfaces

Mahmoud A. Abdalla1 and Mohammed H. Abd El-Azeem2

1Electronic Engineering Department, MTC University, Cairo, Egypt,

[2]*Electronics and Communications Engineering Department, Arab Academy for Science, Technology* and *Maritime Transport, Cairo, Egypt*

Abstract

In this paper we introduce a modified 3-Dimensional Symmetrical Condensed Node (SCN) transmission line matrix and its application in the metamaterial frequency selective surfaces. Through the work, we explain the modification of the 3D SCN TLM algorithm to simulate efficiently dispersive and non linear metamaterial frequency selective structures. The theoretical code modification principles have been discussed in details. As a study case for our modified code, it has been applied to compute the reflection and transmission coefficients for an incident electromagnetic wave upon a dielectric slab loaded by a metamaterial split ring resonator. These calculations have been compared to others computed using other computation methods computed using commercial packages. There is good agreement between the results obtained using our modified TLM code compared to those obtained using the commercial code.

1. Introduction

Electromagnetic numerical computations techniques are used extensively to figure out the electromagnetic propagation through complex structures. There are several electromagnetic numerical techniques such as Finite Difference Time Domain method (FDTD) [1], Finite Element Method (FEM) [2], Method of Moment (MoM) [3] and Transmission line (TL) Matrix modeling (TLM) [4,5]. Comparison between the different numerical computation methods illustrate that none of the techniques is well-suited to all or even most electromagnetic applications. In other words, each method has its advantages and its limitations to be applicable with certain applications. Therefore, the selection of the electromagnetic numerical technique is based on the suitability for the applications.

The numerical techniques used for electromagnetic simulations can be classified depending on the requested solution domain (time or frequency). They can be classified according to the formulation of Maxwell's equations being solved numerically; they either use the Differential form or Integral form of Maxwell's equations.

The FEM and MoM methods are based on frequency domain Maxwell’s equations. FEM is based on descretizising the solution region into finite elements in which the partial differential equations formulated of Maxwell's equations. [6]. FEM is a powerful numerical technique for handling wider range of problems involving complex geometries [6]. However, it can’t efficiently model large radiation problems.

MoM is based on the integral formulation of Maxwell's equations which are transformed into matrix equation and easily solved using matrix inversion [7]. MoM can exclude the air around the objects in the descretization, but it won’t model inhomogeneous and nonlinear dielectrics. MoM and the FEM usually employ harmonic form of Maxwell’s equations. Hence, these methods are largely confined to linear systems. FDTD and TLM methods are based on time domain Maxwell’s equations to provide the transient electric and magnetic field components. Because of this, nonlinear elements and dielectric can be incorporated into both methods [8]. In the FDTD method [1,8], finite difference equations are used to solve Maxwell's equations for a restricted computational domain divided into volume elements. TLM is based on the analogy between propagation of electromagnetic waves described by Maxwell’s equations and the equations that describe electric impulses travelling in a network of transmission lines [9-11].

Transmission line matrix method was proven to be a powerful method in modeling complex geometries [12]. A three dimensional (3-D) symmetrical condensed node (SCN) in TLM computer code has an advantage over other methods in modeling non-homogeneous media in which the position of precise transitions between different homogeneous materials is required [13]. As a results SCN TLM representation of fields has been applied with good accuracy on dealing with dispersive and non linear media [14,15]. Moreover, SCN TLM is an accurate method especially for structure with metallic sharp edges [16].

Metamaterial can be defined as effective structure media that consists of a periodic structure that can demonstrate non nature electric permittivity and permeability. These parameter values are frequency dependent and may be positive or negative. Thus metamaterials are mainly considered as non linear and dispersive media [17].

The first introduced and most common metamaterial structure is based on employing periodic array of Split Ring Resonators (SRRs) [18]. SRR consists of two concentric metal rings separated by a gap, each with a split at opposite sides. These structures when excited by suitable electromagnetic fields have resonance behavior and show unusual properties. Metamaterial SRR structures have been employed in different guided and radiated wave applications [17], frequency selective applications [19]. This encourages the researchers for using TLM for modeling of metamaterial Frequency selective surfaces (FSS) [20]. The accuracy and efficient modeling is the goal in this challenging research.

In this paper, we introduce the modification of 3D symmetrical SCN TLM code to be applicable with non linear and dispersive metamaterial structure. Also, we introduce the modified code application on the case of metamaterial frequency selective surface formed using periodic array of split ring resonators. The reflection and transmission coefficients are computed using our modified code and compared to their values computed using electromagnetic numerical techniques employing standard commercial software packages (MoM and FEM). The obtained results assure the visibility and powerful of the proposed modified code to be applicable in case of metamaterial frequency selective surface applications.

2. Theory

To simulate inhomogeneous materials a 3D TLM node which consists of three shunt nodes connected with three series nodes as shown in Figure 1 (a) [21]. On the other hand, a 3D-SCN TLM node, used with dispersive and non linear media, consists of three shunt nodes connected with three series nodes Figure 1 (b).

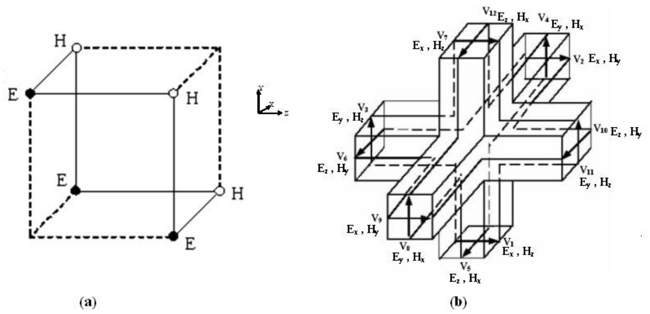

Figure 1. (a) The 3D TLM node basic structure, (b) Ports & field distribution in homogenous, lossless SCN

The voltages at the three shunt nodes represent the three components of the electric field while the currents of the series nodes represent the three components of the magnetic field. Thus, the dielectric properties of the media can be simulated by three open circuited stubs, one for each direction of propagation and the magnetic properties by three short circuited stubs. Losses are simulated by three terminated stubs [22].

The characteristic impedance Z_0 and admittance Y0 = 1/Z0 of the main node lines are chosen to model the free space permeability (μ_0) and permittivity (ε_0), respectively. The variations on the medium permittivity (ε_x) can be achieved by connecting capacitive stub, with an admittance equals to (Y_yY_0). On the other hand, the variations on the medium permeability (μy) can be achieved by inductive stubs, with an impedance equals to (Z_xZ_0).

The distributed inductance (L_d) and capacitance (C_d) of the stub per unit length can be expressed in terms of the non-normalized impedance (Z_x) and admittance (Y_x), respectively, as

$$C_d = Y_x \varepsilon_o \tag{1}$$

$$L_d = Z_x \mu_o \tag{2}$$

In the modified algorithm the non-linearity of the dielectric media is taken care of by making the characteristic admittance of the open circuited stubs at each node a function of the electric field at that node. Therefore, assuming the relative dielectric constant $\varepsilon_r(E(t))$ is now a function of the instantaneous electric field at each node, the characteristic admittance of each open circuited stub will be given by

$$Y(E(t)) = 4\left(\varepsilon_r\ (E(t)) - 1\right) \tag{3}$$

The scattering matrix at each node is now also a function of the electric field E(t) and following the notation in [21,22], the scattering and connecting processes are given by

$$V_k^r = S\,(E(t))V_k^i \tag{4}$$

$$V_{k+1}^i = C\,V_k^r \tag{5}$$

where S(E(t)) is the scattering matrix, V^r and V^i are the reflected and incident voltages, respectively, k is the solution time point, and C is the connection matrix. Similarly, for dispersive material in which the relative dielectric constant εr is a function of the time derivative of the electric field, the characteristic admittance of the open circuited stub will be given by

$$Y\frac{dE(t)}{dt} = 4\left(\varepsilon_r\frac{dE(t)}{dt} - 1\right) \tag{6}$$

$$V_k^r = S\left(\frac{dE(t)}{dt}\right)V_k^i \tag{7}$$

$$V_{k+1}^i = C\;V_k^r \tag{8}$$

Modeling non-linear magnetic materials is done similar to above procedure except that short circuited stubs are used and the magnetic fields are used to update the permeability. For non-linear conductivity, a similar procedure is used with three terminated transmission lines. The conductivity could either be a function of the electric or magnetic fields. Moreover, anisotropic materials can be dealt by setting different characteristic admittances for the three open circuited stubs representing the dielectric constant in all three directions.

3. Metamaterial FSS Structure

Frequency Selective Surfaces is defined as an array of periodic structure that has two configurations; perforated apertures on a conducting sheet or metallic patches on a substrate. These structures resonate at a given frequency and attain spectral selectivity. FSS structures are widely used for electromagnetic waves filtration either for civilian or military applications.

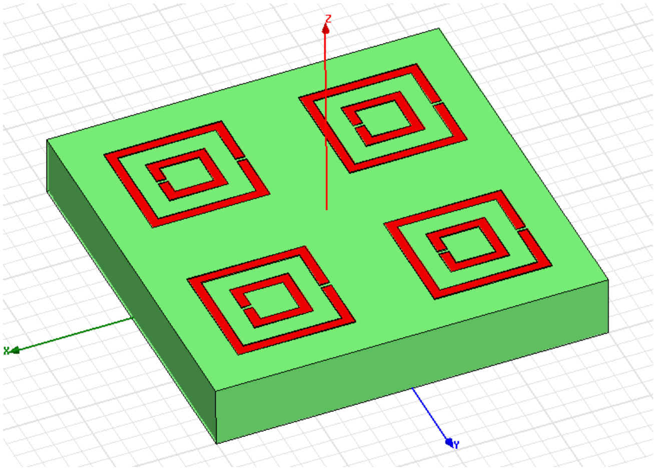

Figure 2. The 3D geometry of the metamaterial FSS structure

Since FSS structure is symmetrical, Babinet's principle can be employed to produce band stop FSS or band pass FSS [23]. A metamaterial FSS structure was formed by using a dielectric layer loaded by a periodic SRR array.

The accuracy of the modified code has been tested in this section for the case of incident electromagnetic waves upon a metamaterial frequency selective structure. The 3-D structure geometry of the tested structure is shown in Figure 2.

The metamaterial SRR layer is a symmetric of periodic distance 5 mm. The inner rectangular has an inner diameter (2a=1mm), the two rings separation is (d=0.3 mm), all the lines thickness are (W=0.25 mm), and the gap in both inner and outer ring is (g= 0.5 mm). The circuit is printed over a lossy FR4 substrate with relative dielectric constant εr= 4.4, a dielectric loss tangent, tan δ = 0.02, and thickness = 1.6 mm. The designed dimensions were selected so that the structure demonstrates a FSS at X- band.

The incident electric field is perpendicular to the gap between the two rings and hence produces a large capacitance in the small gap region between the rings and hence reducing the resonant frequency of the SRR.

It is worth to mention that the split rings structure acts as a resonant structure in the case of incident electric field along the gap between the two rings. The splits in the rings allow the resonant frequency to occur at lower frequency compared to the case of the complete rings. The opposite direction of the split rings is to concentrate.

4. Results

The objective of this section is to investigate the transmitted and the reflected electromagnetic waves from the aforementioned periodic array of the SRRs over dielectric substrate. The transmission and reflection coefficients were computed using MoM (employing the commercial Ansoft-design software), the FEM (employing the commercial Ansoft-HFSS software), and the TLM (employing our modified 3D SCN TLM code).

Thanks to the symmetry in the SRR array, one SRR particle was examined and hence the SRR array periodicity was simulated by inserting the metamaterial structure into a 3D model of perfect electric and magnetic conductors and surrounded in direction of propagation by perfect matching layer (PML) [24] as shown in Figure 3. It is worth to mention that the code has been tested for 80,000 times to ensure stabilization and accuracy of the solution. Similar boundary conditions were assigned in each other methods according to the constraints of the used commercial software packages.

The calculated reflected and transmission coefficients, assuming normal incident wave, using the aforementioned modified 3D SCN TLM code is shown in Figure 4. As shown, the structure demonstrates the selected stop band to be centered at 10.1 GHz. At this frequency, the transmission coefficient (S_{21}) goes below -20 dB whereas the reflection coefficient (S_{11}) is about -2 dB.

For validation of the calculated FSS properties, the same problem was calculated using MoM and employing the commercial software (Ansoft-Design) whose results are plotted in Figure 5. As shown, the results predict the FSS stopband to be centered at 10 GHz. At this frequency, the transmission coefficient (S_{21}) is around -13 dB whereas the reflection one (S_{11}) is -3 dB approximately. By comparing the results in Fig. 3 and these results in Figure 4, we can conclude the good agreement between our modified 3D SCN TLM code and MoM results. This confirms the code capabilities to model metamaterial FSS structures.

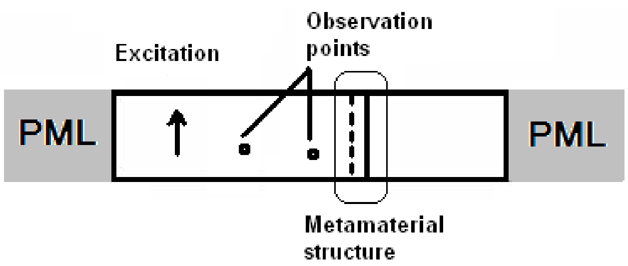

Figure 3. SRR inside a 3D PML TLM structure

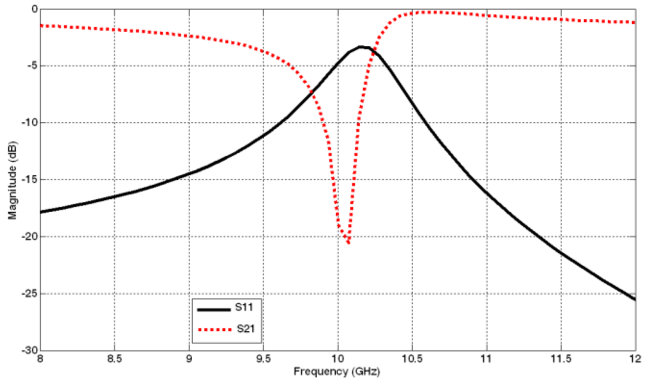

Figure 4.Simulated transmission and reflection coefficient for SRR metamaterial FSS using modified

For more validation, the transmission and reflection coefficients were computed using FEM employing the commercial software (HFSS) and plotted in Figure 6. The stopband in this case is centered at 9.7 GHz at which the transmission coefficient (S_{21}) is -14 dB and the reflection coefficient (S_{11}) is -3 dB. These results demonstrate about 0.4 GHz frequency shift compared our modified 3D SCN TLM code results in Figure 4 and 0.4 GHz frequency shift compared with MoM results in Figure 5. However, this can be claimed due to the non efficient of FEM at radiation problems as mentioned above.

Therefore, we can claim that our modified 3D SCN TLM can predict the stopband/passband properties of metamaterial FSS structures and agree with high degree with other computation methods, especially the MoM method.

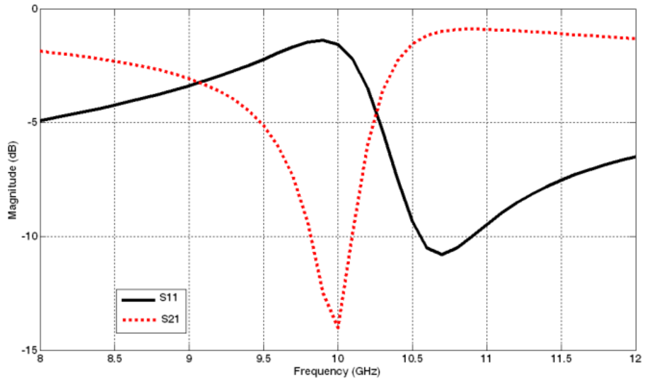

Figure. 5.Simulated transmission and reflection coefficient for SRR metamaterial FSS using MoM (Ansoft Design)

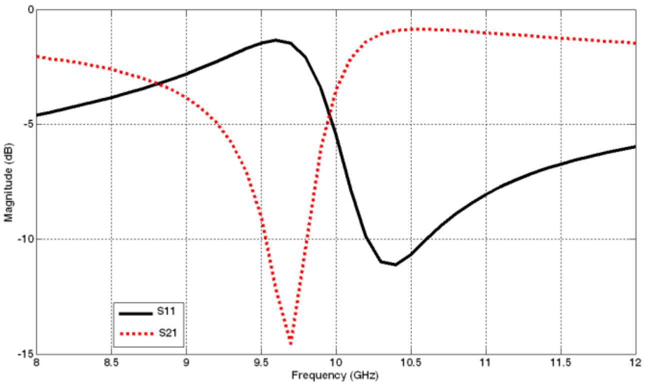

Fig. 6.Simulated transmission and reflection coefficient for SRR metamaterial FSS using FEM (Ansoft HFSS).

5. Conclusion

A powerful and accurate modeling of metamaterial frequency selective surface using a modified 3D SCN TLM code has been presented. The modified 3D SCN TLM code was explained mathematically to deal with non linear and dispersive metamaterial structures. The modified code was applied to the case of metamaterial frequency selective surface based on using of periodic array of standard SRRs array. The results were compared to those obtained by FEM and MoM standard commercial software packages. The results confirmed accuracy of the transmitted and reflected coefficient computed using our modified TLM code compared to other computation methods.

10. References

[1] Kane S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media", *IEEE Trans. Antennas and Propagation*, Vol. 14, No. 3, pp. 302-307, May 1966.

[2] P. P. Silvster, "Finite element analysis of planar microwave networks", *IEEE Trans. on Microwave Theory and Techniques*, Vol. 21 No. 2, pp. 104 – 108, Feb 1973.

[3] Roger F. Harrington, *Field Computation by Moment Methods*, Wiley-IEEE Press, April 1993.

[4] Johns, P.B. Beurle, R.L., "Numerical solution of 2-dimensional scattering problems using a transmission-line matrix", *in Proc. IEE* Vol. 118, No. 9, pp. 1203 – 1208, Sep. 1971

[5] M.N.O. Sadiku and L.C. Agba, "A simple introduction to the transmission-line modeling", *IEEE Transactions on Circuits and Systems*, vol. 37, nO. 8, pp. 991 - 999, Aug 1990

[6] P. P. Silvester, "*Finite elements for electrical engineers*", Cambridge University Press, 3rd Edition, 1996.

[7] Matthew N.O. Sadiku, "*Numerical Techniques in Electromagnetics*",2nd Ed., CRC press, 2001.

[8] Karl S. Kunz and Raymond J. Luebbers, "*The Finite Difference Time Domain Method for Electromagnetics*", CRC press, 2001.

[9] W.J.R. Hoefer, "The Transmission-Line Matrix Method--Theory and Applications", *IEEE Trans on Microwave Theory and Techniques*, Vol. 33 No. 10, pp. 882 – 893, Oct 1985

[10] S. Akhtarzad and P.B. Johns, "Generalized elements for TLM. method of numerical analysis", in Proc. IEE vol. 122, no. 12, pp. 1349 - 1352, Dec. 1975.

[11] S. Akhtarzad and P. B. Johns, "The solution of Maxwelf's equations in three space dimensions and time by the TLM method of numerical analysis," *Proc. Inst. Elec. Eng*., vol. 122, no. 12, pp. 1344-1348, Dec. 1975.

[12] C. Christopoulos, "*The transmission-line modeling (TLM) method in electromagnetics*," USA: Morgan & Claypool Publishers series, 2006.

[13] Mallik, A.R.A. Johns P., "Numerical results for the symmetrical condensed TLM node" IEEE Transaction on Microwave Theory and techniques, Vol. MTT-35, No.4 Apri1,1987.

[14] M I Sobhy, E A Hosny, M H Abd El-Azeem, and K W Royer, "Simulation of nonlinear and anisotropic structures in 3D TLM", *in Proc. 25th European Microwave Conference*, 1995, pp. 830 - 833, Italy.

[15] M.H. Abd El-Azeem, M.I. Sobhy, K.W. Royer, and E.A. Hosny, "Using 3D-(SCN) TLM for the analysis of anisotropic and non-linear structures", *in proc. Third International Conference on Computation in Electromagnetics*, UK Apr 1996, pp. 215 - 219,

[16] N.R.S. Simons, R. Siushansian, J. LoVetri, M. Cuhaci,, "Comparison of the transmission-line matrix and finite-difference time-domain methods for a problem containing a sharp metallic edge", IEEE Trans. on Microwave Theory and Techniques, Vol.47, No.10, pp. 2042 - 2045, Oct 1999.

[17] C. Caloz, T. Itoh, *Electromagnetic metamaterials: transmission line theory and microwave applications*, John Wiley & Sons, Inc., 2006.

[18] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," *IEEE Transactions on Microwave Theory and Techniques*, vol. 47, pp. 2075-84, 1999.

[19] F. Bayatpur, K. Sarabandi, "A Metamaterial Frequency-Selective Superstrate for Phased-Array Applications," *Radar Conference, IEEE*, May 2009, PP. 1-4.

[20] Blanchard, C., J. A. Port, J. A. Morente, and A. Salinas, "Dispersion inherent to TLM nodes for modeling of metamaterials," *Electron. Lett*, Vol. 46, No. 2, 110-112, 2010.

[21] P.B. Johns, "A symmetrical condensed node for the TLM method", *IEEE Transaction on Microwave Theory and Techniques*, pp. 370-377, Vol. 35, 1987.

[22] C.E. Tong and Y.Fujino," An efficient algorithm for TLM analysis of electromagnetic problems using the symmetrical condensed node", pp. 1420-1424, IEEE Transaction MTT, Vol.39, August 1997.

[23] Ben A. Munk, "*Frequency Selective Surfaces: Theory and Design*", John Wiley & Sons Inc., 2000.

[24] M.H. Abd El-Azeem,, "New Techniques for Simulation of Frequency selective surfaces", *in proc. 11th International Conference on aerospace sciences & Aviation Technology*, ASAT-11, May 2