Monopole-Radiating Dyon Solution in Anti-De Sitter Space Time

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Abstract

We analyze here the structure of space-time singularities formed during the radial in-fall of a coherent stream of charged "photons"-a piece of the monopole radiating dyon metric. we study the nature of singularities which develop in the space time on the anti-de-sitter background. It is shown that the singularities formed in gravitational collapse of monopole-radiating dyon solution in anti-de-sitter background are not hidden inside the event horizon. It is also shown that final outcome of collapse depends sensitively on the electric and magnetic charge parameters. Further it is found that naked singularities are strong in Tripler's sense.

I Introduction.

It would be interesting to investigate whether the singularity forming at the end of gravitational collapse is observable. There is an important conjecture related to the singularities known as cosmic censorship hypothesis (CCH) given by Penrose [1]. This states that the collapse of a physically reasonable initial data yields a space-time singularity which is always hidden behind the event horizon. It has two versions, i.e., weak and strong. According to the weak version, singularity formed by gravitational collapse is not visible to a far away observer. The strong cosmic censorship hypothesis states that the singularity cannot be observed even by an observer who is very close to it. Wald [2] discussed some examples to justify the validity of weak form of CCH.

When a massive star is on the verge of completing its nuclear cycle, then the thermonuclear reactions in the interior of the star cannot counter balance the immense gravitational pull of the star. Under most general conditions general relativity predicts that such a collapse must end in a singularity, which may or may not be clothed by an event horizon. A singularity may be physically described as a region in the space-time with extreme curvature, vanishing volume and unbounded gravitational forces. However, general relativity remains silent on the nature (BH or NS) or physical properties of such a singularity. This is basically due to the fact that mathematical structure breaks down preventing analysis at and beyond the singularity. This has triggered extensive research on Gravitational collapse during the past few decades. After all one would always like to know whether, and under what conditions gravitational collapse leads to the formation of a black hole (BH). A few decades back R. Penrose (1969) proposed the cosmic censorship hypothesis (CCH), which states that the singularities formed in gravitational collapse of physically reasonable matter cannot be seen by any distant observer in the universe. It implies that the singularities formed in asymptotically flat space-times are always bounded by event horizons and hence are destined to be black holes. With the announcement of this proposal, study of gravitational collapse has gained special importance, because one would always like to know that whether there exists any physical collapse solutions that lead to naked singularities (NS), which will serve as counterexamples of CCH [3]. Important cases of naked singularities analyzed so for include dust collapse [4-9], radiation collapse [10-17], collapse of perfect fluid [18,19] and strange quark matter [20-21].

A.Chamorro and K.S.Virbhadra have obtained an exact solution of the Einstein- Maxwell equations which are a magnetic charge generalization to the Bonnor-Vaidya solution and describe the gravitational and electromagnetic fields of a non-radiating massive radiating dyon [22]. The paper is based on the composite charges i.e. an electric charge and a magnetic charge bound together by their gravitational interaction. Hence it would be interesting to study the nature of the singularities formed in the gravitational collapse of such composite space-time [23]. In this paper, we study the Monopole-Radiating Dyon solution in anti-de-sitter space-time. We also show that gravitational constant does affect the nature of singularity.

We conclude the paper in V section by some concluding remarks.

II Monopole-Radiating Dyon Solution.

The metric, which describes the gravitational field of non-rotating massive radiating dyon as found by Chamorro and Virbhadra [21] is

$$ds^2 = -\left(1 - \frac{2m(u,r)}{r}\right) du^2 + 2dudr + r^2(d\theta^2 + sin^2\theta d\emptyset^2), \quad (1)$$

Where

$$m(u,r) = f(u) + g(u,r) - \frac{\left(q_e^2(u)+q_m^2(u)\right)}{2r} - \frac{\Lambda r^3}{3} \quad (2)$$

Here $f(u)$ is the standard Vaidya mass, $g(u,r)$ is monopole function, $q_e(u)$ and $q_m(u)$ are electric and magnetic charge parameters respectively. These parameters depend on the Eddington advanced time coordinate u.

The model considered in this paper is obtained from an energy-momentum tensor of the form

$$G_i^k = R_i^k - \frac{1}{2} R g_j^k = 8\pi(E_i^k + N_i^k) \quad (3)$$

Where

$$\{x^\mu\} = \{u,\ r,\ \theta,\ \emptyset\}, \qquad (\mu = 0,\ 1,\ 2,\ 3).$$

E_i^k is related to the electromagnetic tensor F_{ki} in the familiar way

$$E_i^k = \frac{1}{4\pi}\left[-F_{im}F^{km} + \frac{1}{4} g_i^k F_{mn}F^{mn}\right] \quad (4)$$

$$N_i^k = V_i V^k \quad (5)$$

is the energy-momentum tensor of the null fluid. V^k is the null fluid current vector satisfying $g_{ik}V^iV^k = 0$. Electric current vector $J_{(e)}^i$ and magnetic current vector $J_{(m)}^i$ are given by

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}\left(\sqrt{-g}F^{ik}\right) = 4\pi J_{(e)}^i \quad (6)$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}\left(\sqrt{-g}\,{}^*F^{ik}\right) = 4\pi J_{(m)}^i \quad (7)$$

Where ${}^*F^{ik}$ is the dual of the electromagnetic field tensor F^{ik} and is given by

$${}^*F^{ik} = \frac{1}{2\sqrt{-g}} \in^{iklm} F_{lm} \quad (8)$$

Where $\in^{iklm}$ is the Levi-Civita tensor density.

The non-vanishing components of the Einstein tensor for the above metric are given by

$$G_0^0 = G_1^1 = -G_2^2 = -G_3^3 = \frac{\left(q_e^2(u)+q_m^2(u)\right)}{r^4}, \quad (9)$$

$$G_0^1 = k^2 , \tag{10}$$

Where

$$k^2 = \frac{2(q_e \dot{q_e} + q_m \dot{q_m} - \dot{f} r)}{r^3} \tag{11}$$

Here the over dot denotes the derivative with respect to the retarded coordinate u.

Energy-momentum tensor of the electromagnetic field and null fluids are given by

$$E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = \frac{\left(q_e^2(u) + q_m^2(u)\right)}{8\pi r^4} , \tag{12}$$

$$N_0^1 = \frac{k^2}{8\pi} \tag{13}$$

III Nature of the Singularities in Monopole-Radiating Dyon Solution.

The physical solution is for < 0 **,** the space-time becomes flat with $f(u) = 0$, $q_e(u) = 0$, $q_m(u) = 0$ **.** At $u = T$, Say, the radiation is turned off. For$u > T$, $\dot{f}(u) = \dot{q_e}(u)\, \dot{q_m}(u) = 0$, i.e. $f(u), q_e^2(u)\ and\ q_m^2(u)$ are positive definite . Thus the metric for $u = 0\ to\ u = T$ is radiating dyon solution and for $u > T$ it becomes a static dyon solution [24].

To investigate the structure of the collapse, we need to consider the radial null geodesics define by $ds^2 = 0$. $(k^\theta = k^\emptyset = 0)$. The equation for outgoing radial null geodesic for metric (1) is given by

$$\left(1 - \frac{2m(u,r)}{r}\right) du^2 - 2du dr = 0$$

i.e.

$$\frac{dr}{du} = \frac{1}{2}\left(1 - \frac{2m(u,r)}{r}\right).$$

Using the mass function (2) , above equation becomes

$$\frac{dr}{du} = \frac{1}{2}\left(1 - \frac{2f(u)}{r} - \frac{2g(u,r)}{r} + \frac{q_e^2(u) + q_m^2(u)}{r^2} + \frac{\Lambda r^2}{3}\right) \tag{14}$$

In general, Eq. (14) does not yield an analytic solution. However, if $f(u) \propto u$, $q_e^2(u) \propto u^2$, $q_m^2(u) \propto u^2$,Eq. (14) becomes homogeneous and can be solved in terms of elementary functions[25].

In particular, we take

$$f(u) = \lambda u, \qquad g(u,r) = ar \tag{15}$$

and

$$q_e^2(u) = \begin{cases} 0 , & u \le 0 \\ \frac{\mu^2 u^2}{2} , & 0 < u \le T \\ \mu^2 T^2 (const) , & u > T \end{cases} \tag{16}$$

$$q_m^2(u) = \begin{cases} 0 , & u \le 0 \\ \frac{\delta^2 u^2}{2} , & 0 < u \le T \\ \delta^2 T^2 (const) , & u > T \end{cases} \tag{17}$$

Where λ, a, μ^2 and δ^2 are some positive constants. Inserting the expressions for $f(u)$, $g(u,r)$, $q_e^2(u)\ and\ q_m^2(u)$ into Eq.(2) we obtain the mass function for monopole radiating dyon solution as

$$m(u,r) = \lambda u + ar - \frac{\mu^2 u^2 + \delta^2 u^2}{4r} - \frac{\Lambda r^3}{6} \tag{18}$$

It follows that with the choice of above mass function, the metric (1) becomes self-similar [1] (a spherically symmetric space-time is a self similar if $g_u(ct, cr) = g_u(t, r)$ and $g_{rr}(ct, cr) = g_{rr}(t, r)$ for every > 0) admitting a homothetic killing vector ξ^a given by

$$\xi^a = u\frac{\partial}{\partial u} + r\frac{\partial}{\partial r} \tag{19}$$

and satisfies

$$L_\xi g_{ab} = \xi_{a;b} + \xi_{b;a} = 2g_{ab} , \tag{20}$$

Where L denote the Lie derivative.

Defining $k^a = dx^a/dk$ as a tangent to radial null geodesics, where k is an affine parameter, it follows that $\xi^a k_a$ is constant along radial null geodesics. Thus

$$\xi^a k_a = uk_u + rk_r = C, \tag{21}$$

Where C is a constant. Radial null geodesic equations of metric (1), on using the null condition $k^a k_a = 0$, takes the simple form

$$\frac{dk^u}{dk} - \left(\frac{m'}{r} - \frac{m^2}{r^2}\right)(k^u)^2 = 0, \tag{22}$$

$$\frac{dk^r}{dk} + \left(\frac{\dot{m}}{r} - \frac{m'}{r} + \frac{m^2}{r^2} + \frac{2mm'}{r^2} - \frac{2m^2}{r^3}\right)(k^u)^2 + 2\left(\frac{m'}{r} - \frac{m^2}{r^2}\right) k^u k^r = 0 \tag{23}$$

Let

$$k^u = \frac{du}{dk} = \frac{P(u,r)}{r}, \tag{24}$$

Then from the null condition $k^a k_a = 0$ we obtain

$$k^r = \left(1 - \frac{2m}{r}\right)\frac{P}{2r}, \tag{25}$$

Also

$$k_r = k_1 = g_{1j} k^j = g_{10} k^0$$

$$k_r = \frac{P}{r} \tag{26}$$

And

$$k_u = k_0 = g_{0j}k^j = g_{00}k^0 + k^1$$

Therefore

$$k_u = -\frac{P}{2r}\left(1 - \frac{2\lambda u}{r} - 2a + \frac{\mu^2u^2+\delta^2u^2}{2r^2} + \frac{\Lambda r^2}{3}\right) \qquad (27)$$

Eq. (22), (27) and (28) yields

$$P = \frac{12C}{12+(12-2\Lambda r^2-6)X+12\lambda X^2-3(\mu^2+\delta^2)X^3} \qquad (28)$$

Where X is a self-similarity variable defined by $X = u/r$. The singularity occurring at $r = 0$ is naked if the outgoing radial null geodesic equation has atleast one real positive root [26]. In the case of pure Vaidya space-time it has been shown that for a mass function $m(u) = \lambda u/2$, the central singularity is naked for $\lambda \leq 1/8$, and the collapse ends into black hole if $\lambda > 1/8$ [27].

Hence it would be interesting to investigate whether the gravitational collapse of Vaidya space-time could yield a naked singularity under the influence of the gravitational constant and composite field produced by electric and magnetic charges.

With the help of Eq. (18), the equations of the outgoing radial null geodesics for the metric (1) are given by

$$-\left(1-\frac{2m}{r}\right)du^2 + 2dudr = 0$$

Therefore

$$\frac{dr}{du} = \frac{1}{2}\left(1-\frac{2m}{r}\right) \qquad (29)$$

i.e.

$$\frac{dr}{du} = \frac{1}{2}\left(1 - \frac{2\lambda u}{r} - 2a + \frac{\mu^2u^2+\delta^2u^2}{2r^2} + \frac{\Lambda r^2}{3}\right) \qquad (30)$$

Let $$X_0 = \lim_{\substack{u\to 0\\ r\to 0}} \frac{u}{r} = \lim_{\substack{u\to 0\\ r\to 0}} \frac{du}{dr} \qquad (31)$$

Hence Eq. (31) can be written as

$$X_0 = \lim_{\substack{u\to 0\\ r\to 0}} \frac{du}{dr} = \frac{2}{1-2\lambda X-2a+\frac{\mu^2}{2}X^2+\frac{\delta^2}{2}X^2+\frac{\Lambda r^2}{3}}$$

i.e. $$X_0 = \frac{12}{6-12\lambda X-12a+3(\mu^2+\delta^2)X^2+2\Lambda r^2}$$

$$3(\mu^2+\delta^2)X^3 - 12\lambda X^2 + (6-12a+2\Lambda r^2)X - 12 = 0 \qquad (32)$$

The above equation governs the nature of the singularity. If this equation has at least one real and positive root, then the singularity will be naked. If the equation has no positive root, then the collapse ends into a black hole.

In particular, for $\lambda = 0.1,\ \mu = 0.1,\ \delta = 0.4,\ a = 0,\ \Lambda = 0.1,\ r = 0.1$, one of the roots of Eq. (32) is $X_0 = 2.0954$, indicating that the gravitational collapse in this case ends into a naked singularity.

We calculated the value of X_0 for different values of $\lambda,\ \mu,\ \delta,\ a,\ \Lambda$ *and* r

For fixed $a = 0$, $\delta = 0.4$, $\Lambda = 0.1$, $r = 0.1$ then Eq.(33) becomes

$$3(\mu^2+\delta^2)X^3 - 12\lambda X^2 + (6+2\Lambda r^2)X - 12 = 0 \qquad (33)$$

Table 1 Values of X_0 for different values of λ, *and* μ

λ	X_0					
	μ=0.	0.2	0.3	0.4	0.5	0.6
0.1	2.09	1.999	1.877	1.752	1.63	1.53
0.2	3.30	2.919	2.533	2.22	1.97	1.78
0.3	5.72	4.782	3.802	3.044	2.52	2.16
0.4	8.33	6.977	5.471	4.220	3.31	2.70
0.5	10.8	9.145	7.196	5.528	4.27	3.38
0.6	13.3	11.26	8.903	6.853	5.27	4.13
0.7	15.8	13.36	10.58	8.172	6.30	4.92
0.8	18.2	15.43	12.25	9.479	7.32	5.71
0.9	20.6	17.49	13.90	10.77	8.33	6.51

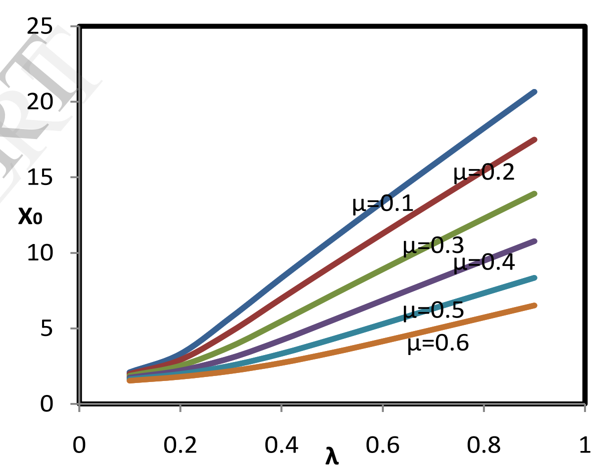

Figure 1: Graph of the values of X_0 against the value of λ.

From the graph we may observe that the value of X_0 have positive real roots. Also it is seen that the value of X_0 decreases as increase the value of μ. Also we notice that the lower value of X_0 shifted towards the peak.

For fixed value of μ and different value of λ, a we have different positive real root of X_0.

For fixed $\mu = 0.1,\ \delta = 0.4,\ \Lambda = 0.1,\ r = 0.1$ then the Eq. (32) becomes,

$$3((0.1)^2+\delta^2)X^3 - 12\lambda X^2 + (6-12a+2\Lambda r^2)X - 12 = 0 \qquad (34)$$

Table 2: Values of X_0 for different values of λ *and* a .

λ	X_0				
	a=0	0.25	0.5	0.75	0.85
0.1	2.0954	3.0025	3.8994	4.6819	4.9652
0.2	3.3002	4.549	5.4868	6.2491	6.5214
0.3	5.7206	6.7043	7.479	8.1367	8.3768
0.4	8.3388	9.0486	9.6633	10.2129	10.4186
0.5	10.8819	11.4298	11.9297	12.3923	12.5686
0.6	13.369	13.8148	14.2335	14.6294	14.7822
0.7	15.8207	16.1969	16.5562	16.9008	17.0349
0.8	18.2493	18.5748	18.8893	19.1937	19.3128
0.9	20.662	20.9491	21.2285	21.5008	21.6078

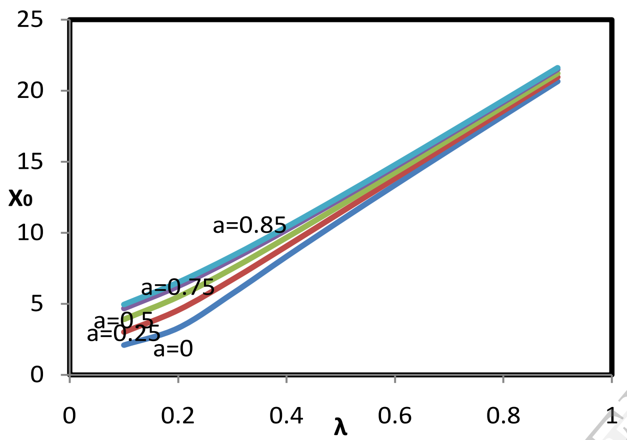

Figure 2: Graph of the values of X_0 against the value of λ for μ=0.1.

From the graph we may observe that the value of X_0 increases as increasing the value of λ.

The lower value shifted towards peak of the graph and the value of X_0 for a=0.75 and a=0.85 have the nearly same value.

For fixed value of λ and different value of μ, a we have different value of X0.

For fixed $\lambda = 0.1$, $\delta = 0.4$, $\Lambda = 0.1$, $r = 0.1$ then Eq. (32) becomes,

$$3(\mu^2 + \delta^2)X^3 - 12(0.1)X^2 + (6 - 12a + 2\Lambda r^2)X - 12 = 0 \qquad (35)$$

Table 3: Values of X_0 for different values of μ, a and $= 0.1$.

μ	X_0				
	a=0	0.25	0.5	0.75	0.85
0.1	2.0954	3.0025	3.8994	4.6819	4.9652
0.2	1.9995	2.7837	3.5691	4.2684	4.5239
0.3	1.8772	2.5247	3.1807	3.7786	3.9996
0.4	1.7525	2.2812	2.8205	3.3224	3.5102
0.5	1.6366	2.0713	2.5161	2.9368	3.0959
0.6	1.5329	1.8953	2.2663	2.6216	2.7571
0.7	1.4416	1.7482	2.062	2.3653	2.4819
0.8	1.3612	1.6246	1.8938	2.1556	2.2568
0.9	1.2904	1.5195	1.7534	1.982	2.0707

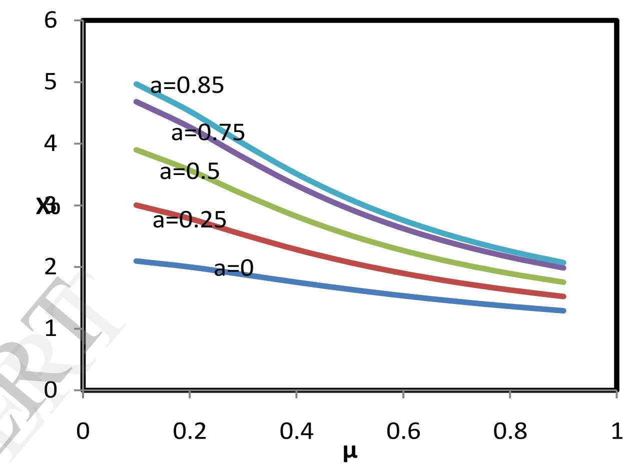

Figure 3: Graph of the values of X_0 against the value of μ for λ =0.1.

From the graph it is clear that the value of X_0 decreases for increase the value of μ. It is also note that the value of X_0 for μ=0.1 have much more difference than the value of μ=0.9 and high value shifted towards lower value.

IV Strength of the naked singularity.

It has seen the nakedness of the singularity in the previous section; in this section we study the strength of singularity. The Clark and Krolak Criterion the strength of singularities has been analyzed and shown that these naked singularities are gravitationally strong. If the naked singularity is not strong then it cannot be considered as a physically reliable singularity and hence such naked singularities may not be considered as counter examples to CCH. A naked singularity is said to be strong if at least along one radial null geodesic with affine parameter k, with k = 0 at the singularity [27], one should have

$$\Psi = \lim_{k\to 0} k^2 R_{ab} k^a k^b > 0 \qquad (36)$$

Where k^a is tangent to the null geodesics and R_{ab} is the Ricci tensor using Eq. (24) and (25) we write

$$\Psi = \lim_{k\to 0} k^2 R_{ab} k^a k^b = \lim_{k\to 0} k^2 \frac{2\dot{m}}{r^2}(k^u)^2 \quad (37)$$

$$= [4\lambda - X(\mu^2+\delta^2)\lim_{k\to 0}\left(\frac{kP}{r^2}\right)^2 \quad (38)$$

as singularity is approached, $k \to 0, r \to 0$ and $X \to X_0$ and using L-Hospital's rule, we find that

$$\Psi = \frac{4\lambda-(\mu^2+\delta^2)X_0}{1-2a+\frac{\Lambda r^2}{3}-2\lambda X_0+\frac{1}{2}(\mu^2+\delta^2){X_0}^2} \quad (39)$$

Thus the singularity is strong if $4\lambda - (\mu^2+\delta^2)X_0 > 0$
For our particular case (i.e. λ=0.1, μ=0.1, δ=0.4, a=0, Λ=0.1, r=0.1, $X_0 = 2.0954$)
We have $4\lambda - (\mu^2+\delta^2)X_0 = 0.043782$. Thus the naked singularity arising in the monopole-radiating dyon solution in the anti-de-sitter space-time is a strong curvature singularity.

V Concluding Remarks.

Cosmic censorship conjecture has become a challenging and most significant open problem in a general relativity. Many possible counter examples to this conjecture have been proposed over the past four decades, although none of them have proved to be sufficiently generic. In this work, there appears a singularity that is not hidden by horizon this singularity is called a naked singularity.

In the present work we have studied monopole radiating dyon solution in anti-de-sitter space time. Here we examine the structure of space time singularities formed during the radial in fall of coherent stream of charged "photons" – a piece of the monopole radiating dyon metric.

It has been shown that the singularities formed in gravitational collapse of monopole radiating dyon solution in anti-de-sitter background are not hidden inside the event horizon. Thus one can argue that composite charged field (electric and magnetic charges) and gravitational constant does not affect to gravity and cannot prevent a naked singularity from forming completely, so that CCH actually violets.

Also, using the clark and krolak criteria [28] the strength of singularities has been analyzed and shown that the naked singularities in the composite solution in anti-de-sitter background are gravitationally strong.

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