

Multiloop Adaptive Controllers for a Nonlinear Interacting Coupled Tank Process

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Abstract— This paper presents the design and implementation of Direct Adaptive Controller (DAC) to control the level of liquid in a nonlinear two tank interacting process. Mean Square error is chosen as minimization criteria for the design of the controller. The objective of this work includes performance comparison of Adaptive MIT and Adaptive PI controller with two tank interacting process. The Constant PI, Adaptive MIT and Adaptive PI controllers are implemented for four different operating regions. White box model of the process is used in this work. The design and simulation studies are carried out in MATLAB/SIMULINK.

Keywords— Nonlinear system, Interacting two tank system, MRAC, MIMO, Mathematical modeling, White box model

I. INTRODUCTION

In process industries one of the major problem is to control the liquid level in tanks. Vital industries such as Petrochemical industries, Paper industries, Water treatment industries have tanks used for chemical treatment and/or mixing the process fluids. In two tank interacting process, the level of liquid in two tanks must be controlled to improve the quality of the product. The difficulty in level control of Multiple Input Multiple Output (MIMO) process is due to its complex dynamics and the interacting nature. Control of the nonlinear process is a difficult task by itself. Deepa et al.[1] have compared the performances of MRAC with fuzzy control for a Single Input Single Output (SISO) system. Anna Joseph et al.[2] and Rathikarani et al.[3] have used Model Reference Adaptive Controller (MRAC) to control nonlinear process. For a coupled tank process, a Model Predictive Controller is designed by Gireesh Kumar et al.[4].

First principle based model of the MIMO process is used in this work. The most widely used PI controllers in the industrial applications have simple structures and good dynamic performances. These Constant PI (CPI) controller are popular in industrial applications, as they are easy to install and reasonably robust. It is necessary to develop advanced PI controllers for controlling nonlinear processes. Adaptive controller's parameters are adjusted automatically to compensate the variation in the process characteristics. These controllers performs better when compared to Constant Gain PI controllers for Nonlinear processes. Hence Adaptive MIT and Adaptive PI controller are designed and implemented in this work. The MRAC based Adaptive MIT and Adaptive PI controllers are designed to control the liquid level in interacting tanks. When production rate changes, the dynamics

of the process along with the amount of interaction varies. The adaptive controller has to decrease the error vector between the reference model and plant to zero. The proposed method can adjust the controller parameters in response to changes in plant and disturbances by referring to the reference model that satisfies properties of the desired closed loop control system

II. DESCRIPTION OF LEVEL PROCESS

In this work, two tank interacting process available in the laboratory is considered. This process is a MIMO process with two controlled variables. Hence two controllers are designed and implemented to control the level in two tank interacting process. The level in tank1 and tank2 are the controlled variables. Pump1 and pump2 are used to feed inflow to the tank1 and tank2. The Hand Valves (HV) are adjusted so that the levels in both the tanks are brought to nominal condition initially. Disturbances are applied to the tanks by varying the position of the hand valves HV21 and HV26. When the flow to the tank 1 is varied, the inflow to tank 2 also varies. When the level and/or flow of tank2 varies, tank1 level change, due to interaction between the tanks (Fig. 1). The volumetric inflow rate into the tank1 and tank2 are q_{in1} and q_{in2} . The volumetric flow rate from the tank1 and tank2 are q_{01} and q_{02} . Flow rate between tank1 and tank2 is q_{12} . The height of the liquid level is h_1 in tank1 and h_2 in tank2.

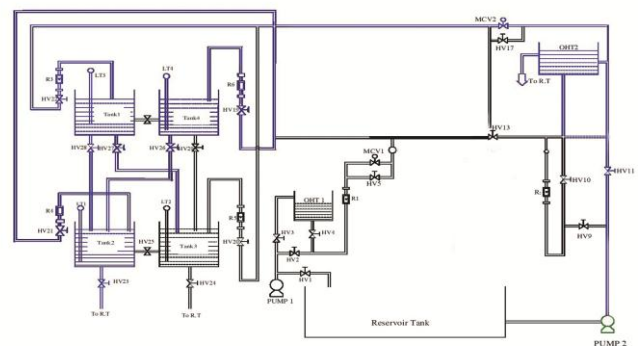


Fig.1 Piping and Instrumentation Diagram for two tank interacting level process

The schematic diagram of the two tank interacting level process is shown in Fig. 2. The controlled variables in the process are level in tank1 (h_1) and level in tank2 (h_2). The Manipulated variables to the process are q_{in1} (l/hr) for tank1, q_{in2} (l/hr) for tank2.

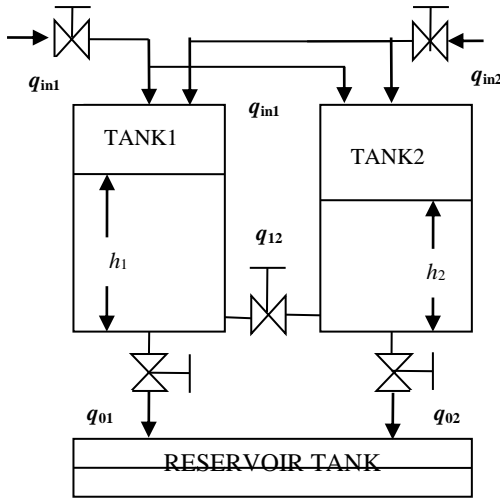


Fig. 2 Schematic diagram of Two tank interacting level process

The mathematical model of the process is obtained from mass balance equations, and are given below,

$$A_1 \frac{dh_1}{dt} = q_{in1} - q_{01} - q_{12} \quad (1a)$$

$$A_2 \frac{dh_2}{dt} = q_{in2} - q_{02} + q_{12} \quad (1b)$$

where $A_1 = A_2 = 1130.4 \text{ cm}^2$ are the cross sectional area of the tank1 and tank2. $h_1 = h_2 = 25 \text{ cm}$ are the height of the tank1 and tank2. $a_1 = 5.3 \text{ cm}^2$; $a_2 = 10.6 \text{ cm}^2$ are the restriction areas in the outlet pipes of tank1 and tank2. $g = 9.81 \text{ cm}^2/\text{s}$ acceleration due to gravity and $c_d = 0.8$ the discharge co-efficient.

A. Modelling of Level Process

The four models relating the two controlled outputs h_1 and h_2 with two manipulated inputs q_{in1} and q_{in2} are essential to design the multi-loop controllers [5]. The model transfer functions with the flow rates as manipulated inputs and the levels as controlled outputs can be written as follows:

$$G_{11}(s) = \left(\frac{h_1}{q_{in1}} \right)_{q_{in2}} ; G_{21}(s) = \left(\frac{h_2}{q_{in1}} \right)_{q_{in2}}$$

$$G_{12}(s) = \left(\frac{h_1}{q_{in2}} \right)_{q_{in1}} ; G_{22}(s) = \left(\frac{h_2}{q_{in2}} \right)_{q_{in1}}$$

Interacting two tank process is modeled from the process reaction curve [Fig. 3 and Fig. 4]. The levels in the tanks are initially maintained at nominal operating condition ($h_1 = 12.6 \text{ cm}$, $h_2 = 12.1 \text{ cm}$). In 800th sampling instant 10LPH change is given in q_{in1} . which causes h_1 to change from 12.6 to 20cm. Due to interaction between the tanks h_2 has reached the steady state value of 16cm from its nominal value.

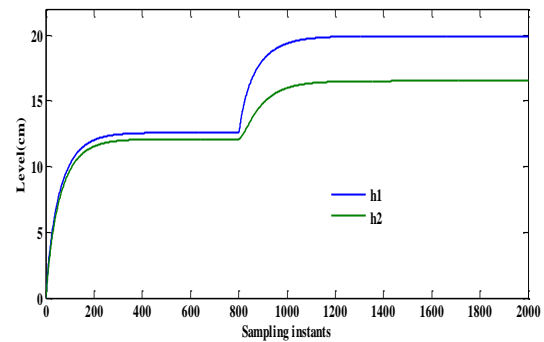


Fig. 3 Open loop responses of h_1 and h_2 for +10LPH change in q_{in1}

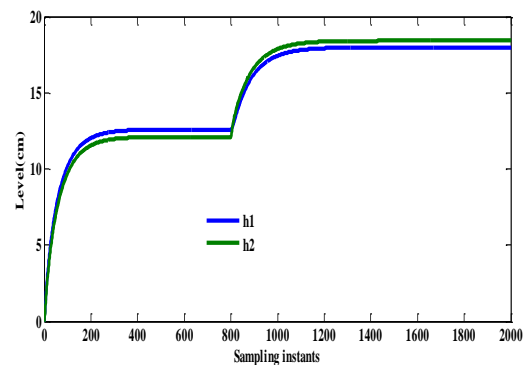


Fig. 4. Open loop responses of h_1 and h_2 for +10LPH change in q_{in2}

In the same manner, step change is given in q_{in2} maintaining q_{in1} in nominal condition [Fig. 4]. Using these responses the model of the process in continuous time domain for the change in q_{in1} and q_{in2} are computed using process reaction curve method and are tabulated in Table I.

TABLE I.
IDENTIFIED MODELS

Operating Regions	Δq_{in} (LPH)	Level (cm)	Models
1	-5	12.6 to 9.8	$M_{c_{11}} = \frac{0.5486e^{-5.5s}}{57s+1}$ $M_{c_{12}} = \frac{0.338e^{-4.5s}}{48s+1}$ $M_{c_{21}} = \frac{0.449e^{-10.5s}}{58.5s+1}$ $M_{c_{22}} = \frac{0.6084e^{-3s}}{27s+1}$
2	+5	12.6 to 16.02	$M_{c_{11}} = \frac{0.586e^{-4.5s}}{54s+1}$ $M_{c_{12}} = \frac{0.502e^{-8.5s}}{73.5s+1}$ $M_{c_{21}} = \frac{0.387e^{-6.5s}}{64s+1}$ $M_{c_{22}} = \frac{0.5806e^{-2.5s}}{71s+1}$
3	-10	12.6 to 7.45	$M_{c_{11}} = \frac{0.5635e^{-3.5s}}{34.5s+1}$ $M_{c_{12}} = \frac{0.3742e^{-14.5s}}{43.5s+1}$ $M_{c_{21}} = \frac{0.4607e^{-8s}}{52.5s+1}$ $M_{c_{22}} = \frac{0.6345e^{-4.5s}}{22.5s+1}$
4	+10	12.6 to 20	$M_{c_{11}} = \frac{0.737e^{-3.5s}}{69s+1}$ $M_{c_{12}} = \frac{0.508e^{-3s}}{75s+1}$ $M_{c_{21}} = \frac{0.4607e^{-7.5s}}{52.5s+1}$ $M_{c_{22}} = \frac{0.587e^{-2.5s}}{75s+1}$

B. Validation of the Models

Time domain validation of the models are shown in Fig. 5 and Fig. 6. To evaluate the degree of closeness of the model with actual process the validation is done. The actual response (white box) of the process is compared with model response (black box) for the same input.

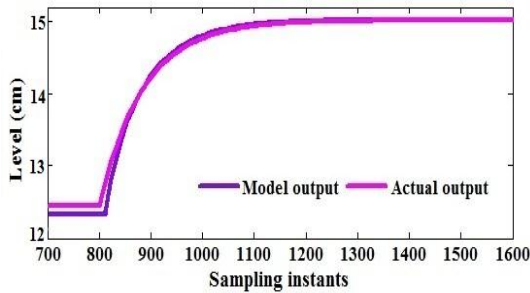


Fig. 5 Time domain validation for the Model (Tank1)

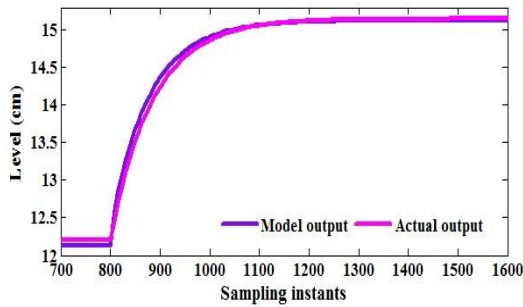


Fig. 6 Time domain validation for the Model (Tank2)

III. DESIGN OF CONTROLLERS

The process considered in this work is a Two Input Two Output (TITO) process. The constant PI, Adaptive MIT and Adaptive PI are designed and implemented to control level in the tanks. Hence two control loops are designed and implemented.

A. Constant PI (CPI) Controller

The CPI controller can be used to improve the dynamic response as well as reduce or eliminate the steady state error. The strategy used for controlling the interacting process with controllers are shown in Fig.7. The reference set points are h_{sp1} and h_{sp2} . Manipulated inflow rates to tank1 and tank2 are q_{in1} and q_{in2} . The process outputs from tank1 and tank2 are h_1 and h_2 . The variables e_1 and e_2 are the modeling errors to the controllers. G_{c1} and G_{c2} are the controllers in loop1 and loop2.

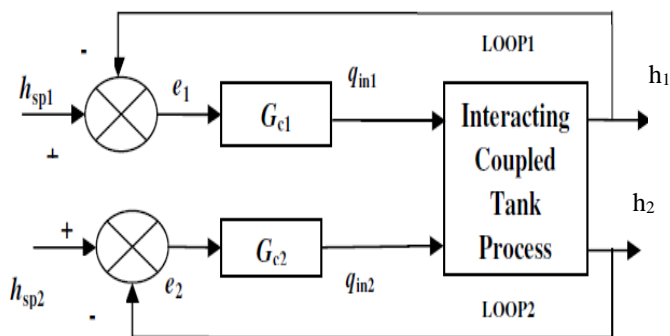


Fig. 7 Block Schematic representation of closed loop system

The controller parameters for various operating ranges of the taken up process using the Ziegler Nichols tuning method are presented in Table II.

Table II
 PI Controller Parameters

Operating Regions	PI Controller parameters	
	G_{c1}	G_{c2}
1	$K_{c1} = 14.084; T_{i1} = 4.05$	$K_{c2} = 12.344; T_{i2} = 3.86$
2	$K_{c1} = 7.771; T_{i1} = 5.65$	$K_{c2} = 7.728; T_{i2} = 3.90$
3	$K_{c1} = 12.450; T_{i1} = 2.54$	$K_{c2} = 15.567; T_{i2} = 1.22$
4	$K_{c1} = 9.056; T_{i1} = 2.89$	$K_{c2} = 7.997; T_{i2} = 1.08$

B. Adaptive Controller

To control level in each tank, MRAC is used. The structure of the MRAC system with MIT rule used in this work is shown in Fig. 8. Each control loop consists of a reference model, adjustment mechanism and controller. The reference model describes the desired input/output character of the closed loop system. The controller drives the control signal so that the plant's closed loop characteristics from the command signal; h_{spi} to the plant output h_i is equal to the dynamics of the reference model, h_m . The suffix 'i' in the variables represents the control loop, nos. 1 and 2.

Matching the plant and the reference model characteristics guarantees the convergence of the modeling error to zero for any given command signal (h_{spi}). The controller drives the difference between the process response and desired model output to zero asymptotically at a rate constrained by the adaptation gain [6,7].

The designed controller has a conventional inner loop followed by a separate adaptive outer loop to adjust the controller's feedback gains (θ_{1i}, θ_{2i}) based on equating the coefficients of closed loop plant to coefficients of the desired model.

The advantage is that the proposed technique can deal with the nonlinear nature of the process and also retain the designer's intuition and insight through the relatively simple design scheme that is proposed. This controller design is based on "grey box" model which combines both black and white box models.

The outer loop adjusts the controller parameter in such a way that the model error (e_i), the difference between process output h_i ($i=1,2$) and model output h_m is small.

$$e_i = h_i - h_m \quad (2)$$

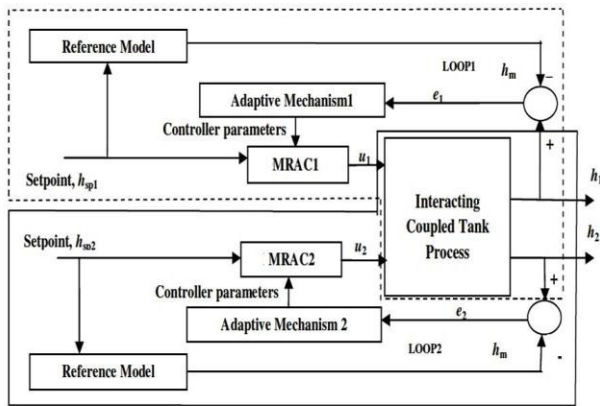


Fig. 8 Block Schematic diagram of the system with adaptive control

C. Adaptive MIT Controller

The controller parameters (θ) may be adjusted with the following loss function,

$$J(\theta_{ni}) = \frac{1}{2} e_i^2 \tag{3}$$

where $n=1,2$ represents controller parameter number. In order to minimize the loss function J , the parameters can be changed in the direction of negative gradient of J .

The control law is

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t) \tag{4}$$

The closed loop transfer function is given by equation

$$\frac{y}{u_c} = \frac{b}{s^2 + a_1 s + a_2} \tag{5}$$

where u_c is the command signal (input). The controllers parameters are to be adapted such that the process output (equation 5) follows the model output (equation 6)

$$\frac{y_m}{u_c} = \frac{b_m}{s^2 + 2\delta\omega_n s + \omega_n^2} \tag{6}$$

The modeling error is as follows

$$e = y - y_m$$

Substituting $u(t)$ in equation (5)

$$y = \frac{b\theta_1}{s^2 + a_1 s + (b\theta_2 + a_2)} u_c$$

$$y_m = \frac{b_m}{s^2 + 2\delta\omega_n s + \omega_n^2} u_c$$

For perfect model following the controller parameters are chosen as (when $e=0$)

$$\theta_1 = \frac{b_m}{b}$$

$$\theta_2 = \frac{a_m - a}{b}$$

The sensitivity derivatives are obtained by the partial derivatives of modeling error with respect to the controller parameters

$$\frac{\partial e}{\partial \theta_1} = \frac{b_m}{s^2 + 2\delta\omega_n s + \omega_n^2} u_c$$

$$\frac{\partial e}{\partial \theta_2} = \frac{b_m}{s^2 + 2\delta\omega_n s + \omega_n^2} \times (-y) \tag{7}$$

The controller parameters are obtained

$$\theta_1 = -\frac{\gamma}{s} \left[\frac{1}{s^2 + 2\delta\omega_n s + \omega_n^2} \times u_c \right] e$$

$$\theta_2 = \frac{\gamma}{s} \left[\frac{1}{s^2 + 2\delta\omega_n s + \omega_n^2} \times y \right] e$$

D. Adaptive PI (API) Controller

The design of API controllers leads to large improvement in industries. API controllers are simple and easy to implement [8,9]. Hence an API based on MRAC is designed and implemented in this work. The API algorithm used in this work is given by equation (8).

$$u = k_p (u_c - y) + \frac{k_i}{s} (u_c - y) \tag{8}$$

where k_p and k_i are the proportional and integral gains of the controller [10,11]. Based on apriori knowledge the process considered for control is represented by equation (5). The closed-loop transfer function is given by

$$\frac{y}{u_c} = \frac{bk_p + k_i}{s^2 + (a + bk_p)s + bk_i} \tag{9}$$

For perfect model matching

$$s^2 + s(a + bk_p) + bk_i = s^2 + 2\delta\omega_n s + \omega_n^2$$

The adapted PI Controller parameters based on MIT

algorithm are shown in equations (10) and (11).

$$k_p = -\frac{\gamma}{s} e \frac{s}{s^2 + 2\delta\omega_n s + \omega_n^2} [u_c - y] \tag{10}$$

$$k_i = -\frac{\gamma}{s} e \frac{1}{s^2 + 2\delta\omega_n s + \omega_n^2} [u_c - y] \tag{11}$$

IV. SIMULATION RESULTS

The servo and regulatory responses of interacting tank (white box model) are plotted in Fig. 9. The damping ratio (δ) of the reference model is 0.7.

The set point tracking for level h_1 with Conventional PI, Adaptive MIT and Adaptive PI are presented in Fig. 9. The h_{1sp} is the set point for tank1 at nominal operating condition. The set point variation of h_1 from 12.6 to 17.6cm is applied at 950th sampling instant. Due to interaction the level of tank 2 increases. The h_1 -API is the level of the tank1 using API controller. The h_1 -CPI is the level of tank1 with CPI controller. The h_1 -MIT is the level of tank1 when Adaptive MIT controller is used.

The set point tracking for level h_2 with Conventional PI, Adaptive MIT and Adaptive PI are shown in Fig.10. Due to set point variation in tank1, the level in tank 2 varies at 950th

sampling instant. Due to controllers in loop2, the level variations are nullified and brought to nominal operating condition (12.1cm). A set point variation for level h_2 from 12.1 to 17.1cm is applied at 1800th sampling instant, due to interaction there is considerable rise in h_1 .

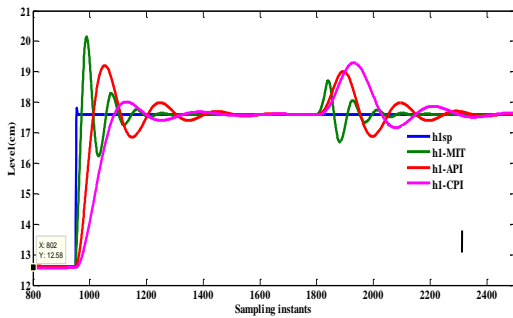


Fig. 9 Servo and Regulatory responses of the Interacting Coupled tank process for h_1 ($\delta = 0.7$)

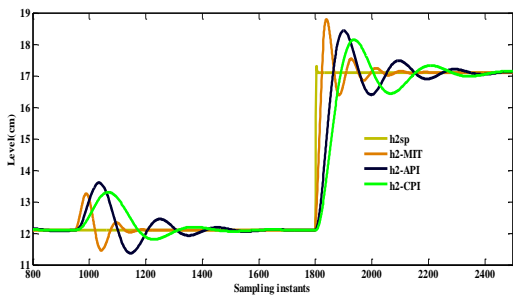


Fig. 10 Servo and Regulatory responses of the Interacting Coupled tank process for h_2 ($\delta = 0.7$)

Fig. 11 shows the response of the controllers for tank1 and tank2. At 950th sampling instant, the inflow rate to the tank 1 increases as well as inflow rate to tank2 decreases. In tank 2, the 1800th sampling instant the inflow rate of tank 2 increases due to this change the flow rate of the tank 1 decreases in order to bring back the level h_1 to set point.

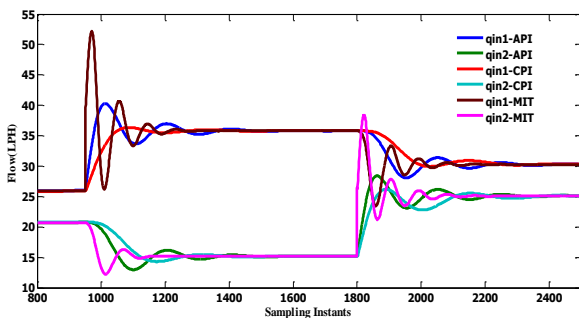


Fig. 11 Response of controllers for tank1 and tank 2

The adaptation of Proportional and Integral gains (K_c, K_i) for CPI and API can be visualized from Fig. 12 and Fig. 13.

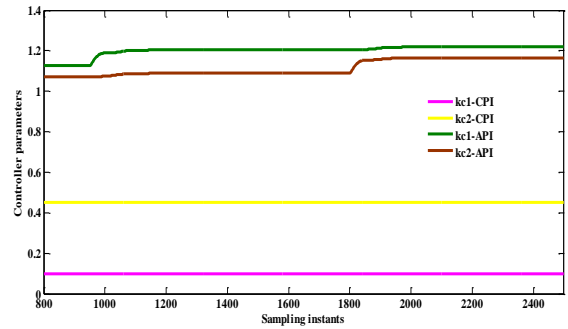


Fig. 12 Adaptation of Proportional Gains

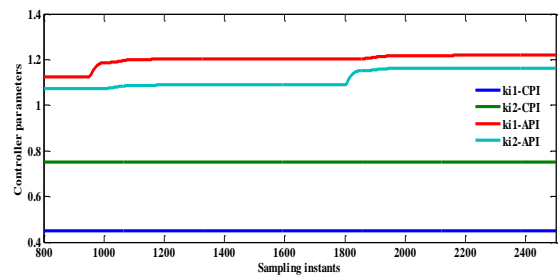


Fig. 13 Adaptation of Integral Gains

The vanishing nature of adapted controller parameters (θ_1, θ_2) of MIT1 and MIT2 can be visualized from Fig. 14.

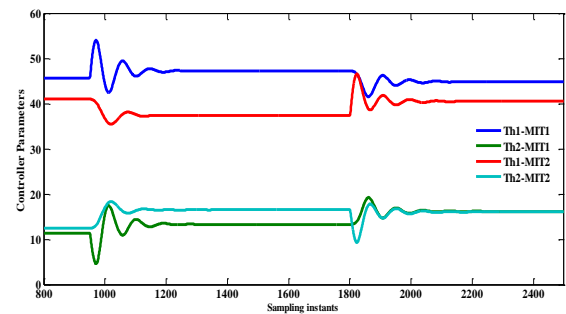


Fig. 14 Adaptation of the controller parameters (MIT)

The reference model's damping ratio (δ) is changed from 0.7 to 1 and 0.7 to 2. The set point tracking for level h_1 with Conventional PI, Adaptive MIT and Adaptive PI for ($\delta = 1$) are presented in Fig. 15. The set point tracking for level h_2 with Conventional PI, Adaptive MIT and Adaptive PI for ($\delta = 1$) are shown in Fig.16. Fig.17 shows the corresponding response of the controllers for tank1 and tank2. The adaptation of Proportional and Integral gains (K_c, K_i) for CPI and API can be visualized from Fig. 18 and Fig. 19. The vanishing nature of adapted controller parameters (θ_1, θ_2) of MIT1 and MIT2 can be visualized from Fig. 20.

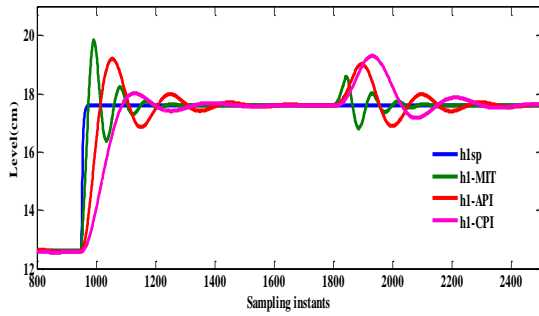


Fig. 15 Servo and Regulatory responses of the Interacting Coupled tank process for h_1 ($\delta = 1$)

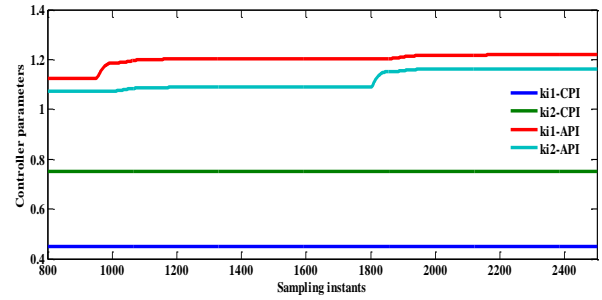


Fig. 19 Adaptation of Integral Gains

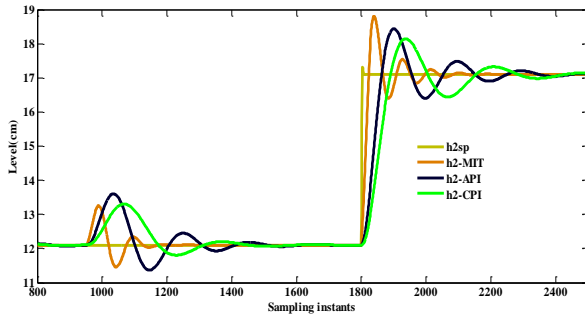


Fig. 16 Servo and Regulatory responses of the Interacting Coupled tank process for h_2 ($\delta = 1$)

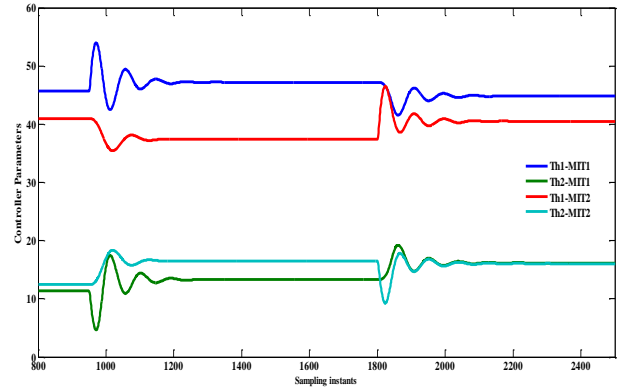


Fig. 20 Adaptation of the controller parameters(MIT)

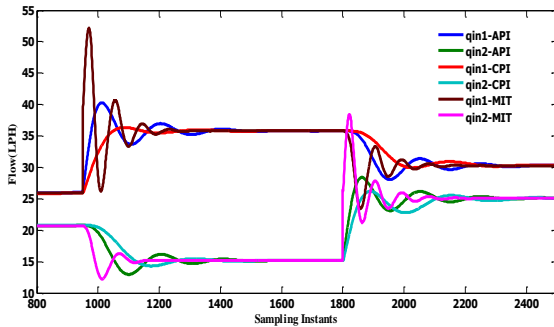


Fig. 17 Response of controllers for tank1 and tank2

Table IIIA and IIIB shows the Time Integral Criteria of the process for various controllers with various reference model parameters

Table IIIA
 Performance comparison of Adaptive controllers

Parameters		IAE			ISE		
		CPI	API	MIT	CPI	API	MIT
$\delta = 0.7$	Tank1	2025	2165	911.5	18550	19450	7673
	Tank2	7829	2361	883.8	115700	2142000	6923
$\delta = 1.0$	Tank1	2024	2167	908.7	18540	19310	7520
	Tank2	7817	2367	867.4	117400	2134000	7626
$\delta = 2.0$	Tank1	2022	2175	904	18070	18920	7091
	Tank2	7795	2388	804	116900	2121000	7268

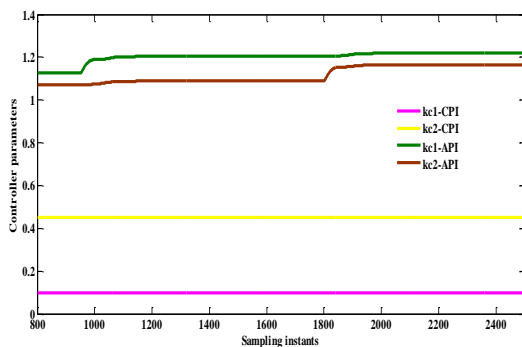


Fig. 18 Adaptation of Proportional Gains

Table IIIB
 Performance comparison of Adaptive controllers

Parameters		ITAE		
		CPI	API	MIT
$\delta = 0.7$	Tank1	5063000	5413000	2279000
	Tank2	19650000	5902000	2038000
$\delta = 1.0$	Tank1	5062000	5418000	2268000
	Tank2	19540000	5814000	20136000
$\delta = 2.0$	Tank1	506000	5438000	2254000
	Tank2	19480000	5468000	1934000

V. CONCLUSION

This paper has presented a Gradient approach based MRAC to control level in the interacting tanks. Identification of the first principle based process is done in simulation. The ISE and IAE values of the process with Adaptive MIT controller has lesser values compared to the process with conventional and Adaptive PI controller. From the implementation of MRAC based MIT, it is inferred that by increasing damping factor the time integral absolute error are minimized. Peak overshoot and undershoot are minimum in conventional PI controller. Hence Adaptive MIT controller is suitable for control of the Interacting coupled tank process when compared to Conventional and Adaptive PI controllers.

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