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Nijenhuis Tensor in Para Almost F-3 Structure Manifold

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Abstract: **In this Paper, I have defined Nijenhuis tensor in Para Almost F-3 Structure Manifold and decompositions of Nijenhuis tensor have also been done in this manifold.**

1. INTRODUCTION

Let F_x, $x = 1,2,3$ be three structures on V_n, such that

$$\text{rank}\,((F_x)) = r, \text{ everywhere,} \tag{1.1}$$

$$(F_x)^2 F_y = -\in_{xyz} F_x F_z + \delta_{xy} F_x, \quad \text{x not summed,} \tag{1.2 a}$$

$$F_x (F_y)^2 = -\in_{xyz} F_z F_y + \delta_{xy} F_y, \quad \text{y not summed.} \tag{1.2 b}$$

Then F_x is called Para almost F-3 structure.

Consequently,

$$F_x^3 - F_x = 0 \text{ in Para almost F-3 structure manifold.} \tag{1.3}$$

2. NIJENHUIS TENSOR

The Nijenhuis tensor with respect to F_x is defined as

$$N_x(X,Y) = [F_x X, F_x Y] + F_x^2[X,Y] - F_x[F_x X, Y] - F_x[X, F_x Y]. \tag{2.1}$$

Theorem (2.1). In Para almost F-3 structure manifold, we have

$$N_1(F_1^2 X, Y) = [F_1 X, F_1 Y] + F_1^2[F_1^2 X, Y] - F_1[F_1 X, Y] - F_1[F_1^2 X, F_1 Y], \tag{2.2 a}$$

$$N_1(X, F_1^2 Y) = [F_1 X, F_1 Y] + F_1^2[X, F_1^2 Y] - F_1[F_1 X, F_1^2 Y] - F_1[X, F_1 Y], \tag{2.2 b}$$

$$N_1(F_1^2 X, F_1^2 Y) = [F_1 X, F_1 Y] + F_1^2[F_1^2 X, F_1^2 Y] - F_1[F_1 X, F_1^2 Y]$$
$$- F_1[F_1^2 X, F_1 Y], \tag{2.2 c}$$

$$N_1(F_2^2 X, Y) = [F_3 F_2 X, F_1 Y] + F_1^2[F_2^2 X, Y] - F_1[F_3 F_2 X, Y] - F_1[X, F_3 F_2 Y], \tag{2.2 d}$$

$$N_1(X, F_2^2 Y) = [F_1 X, F_3 F_2 Y] + F_1^2[X, F_2^2 Y] - F_1[F_1 X, F_2^2 Y] - F_1[X, F_3 F_2 Y], \tag{2.2 e}$$

$$N_1(F_2^2 X, F_2^2 Y) = [F_3 F_2 X, F_3 F_2 Y] + F_1^2[F_2^2 X, F_2^2 Y] - F_1[F_3 F_2 X, F_2^2 Y]$$
$$- F_1[F_2^2 X, F_3 F_2 Y], \tag{2.2 f}$$

$$N_2(F_1^2 X, Y) = [F_3 F_1 X, F_2 Y] + F_2^2[F_1^2 X, Y] - F_2[F_3 F_1 X, Y] - F_2[F_1^2 X, F_2 Y], \tag{2.2 g}$$

$$N_2(F_1^2 X, F_1^2 Y) = [F_3 F_1 X, F_3 F_1 Y] + F_2^2[F_1^2 X, F_1^2 Y] - F_2[F_3 F_1 X, F_1^2 Y]$$

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$$-F_2[F_1^2 X, F_3 F_1 Y], \qquad (2.2)\ h$$

$$N_2(X, F_3^2 Y) = [F_2 X, F_2 F_3^2 Y] + F_2^2[X, F_3^2 Y] - F_2[F_2 X, F_3^2 Y]$$

$$-F_2[X, F_2 F_3^2 Y], \qquad (2.2)\ i$$

$$N_2(F_3^2 X, F_2^2 Y) = -[F_1 F_3 X, F_2 Y] + F_2^2[F_3^2 X, F_2^2 Y] + F_2[F_1 F_3 X, F_2^2 Y]$$

$$-F_2[F_3^2 X, F_2 Y], \qquad (2.2)\ j$$

$$N_3(F_1^2 X, Y) = -[F_2 F_1 X, F_3 Y] + F_3^2[F_1^2 X, Y] + F_3[F_2 F_1 X, Y] - F_3[F_1^2 X, F_3 Y], \qquad (2.2)\ k$$

$$N_3(F_2^2 X, F_3^2 Y) = [F_1 F_2 X, F_3 Y] + F_3^2[F_2^2 X, F_3^2 Y] - F_3[F_1 F_2 X, F_3^2 Y]$$

$$-F_3[F_2^2 X, F_3 Y]. \qquad (2.2)\ l$$

Proof. Operating F_1^2 on X (or Y or X and Y together) in equation (2.1) and using the equation (1.3) in the resulting equations then we obtain the equations (2.2)a, (2.2)b and (2.2)c. Operating F_2^2 on X for $x = 1$ in equation (2.1), then using the equation (1.2)b, we get

$$N_1(F_2^2 X, Y) = [F_1 F_2^2 X, F_1 Y] + F_1^2[F_2^2 X, Y] - F_1[F_1 F_2^2 X, Y] - F_1[F_2^2 X, F_1 Y]$$

$$= -[F_3 F_2 X, F_1 Y] + F_1^2[F_2^2 X, Y] + F_1[F_3 F_2 X, Y] - F_1[F_2^2 X, F_1 Y],$$

which is the equation (2.2)d. Similarly, we can obtain the equations (2.2)e and (2.2)f.

Now, operating F_1^2 on X for $x = 2$ in equation (2.1) and using the equation (1.2) b, we get (2.2) g. Proof of the equations (2.2) h,………….., (2.2) l follows similarly.

Theorem (2.2). If we put

$$P_x(X, Y) = [F_x X, F_x Y] - F_x[F_x X, Y]. \qquad (2.3)$$

Then

$$P_1(F_1^2 X, Y) = [F_1 X, F_1 Y] - F_1[F_1 X, Y], \qquad (2.4)\ a$$

$$P_1(X, F_1^2 Y) = [F_1 X, F_1 Y] - F_1[F_1 X, F_1^2 Y], \qquad (2.4)\ b$$

$$P_1(F_1^2 X, F_1^2 Y) = [F_1 X, F_1 Y] - F_1[F_1 X, F_1^2 Y], \qquad (2.4)\ c$$

$$P_1(F_2^2 X, Y) = -[F_3 F_2 X, F_1 Y] + F_1[F_3 F_2 X, Y], \qquad (2.4)\ d$$

$$P_1(X, F_2^2 Y) = -[F_1 X, F_3 F_2 Y] - F_1[F_1 X, F_2^2 Y], \qquad (2.4)\ e$$

$$P_1(F_2^2 X, F_2^2 Y) = [F_3 F_2 X, F_3 F_2 Y] + F_1[F_3 F_2 X, F_2^2 Y], \qquad (2.4)\ f$$

$$P_1(F_1^2 X, F_2^2 Y) = -[F_1 X, F_3 F_2 Y] - F_1[F_1 X, F_2^2 Y], \qquad (2.4)\ g$$

$$F_1 P_1(F_1^2 X, F_2^2 Y) = -F_1[F_1 X, F_3 F_2 Y] - F_1^2[F_1 X, F_2^2 Y], \qquad (2.4)\ h$$

$$F_1 P_1(F_1 X, F_2^2 Y) = -F_1[F_1^2 X, F_3 F_2 Y] - F_1^2[F_1^2 X, F_2^2 Y], \qquad (2.4)\ i$$

$$P_2(X, F_1^2 Y) = [F_2 X, F_3 F_1 Y] - F_2[F_2 X, F_1^2 Y], \qquad (2.4)\ j$$

$$P_2(F_2^2 X, F_1^2 Y) = [F_2 X, F_3 F_1 Y] - F_2[F_2 X, F_1^2 Y], \qquad (2.4)\ k$$

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$$P_2(X, F_3^2 Y) = -[F_2 X, F_1 F_3 Y] - F_2[F_2 X, F_3^2 Y], \qquad (2.4)\ l$$

$$P_2(F_2^2 X, F_3^2 Y) = -[F_2 X, F_1 F_3 Y] - F_2[F_2 X, F_3^2 Y], \qquad (2.4)\ m$$

$$P_3(X, F_1^2 Y) = -[F_3 X, F_2 F_1 Y] - F_3[F_3 X, F_1^2 Y], \qquad (2.4)\ n$$

$$P_3(F_3^2 X, F_1^2 Y) = -[F_3 X, F_2 F_1 Y] - F_3[F_3 X, F_1^2 Y], \qquad (2.4)\ o$$

$$P_3(X, F_2^2 Y) = [F_3 X, F_1 F_2 Y] - F_3[F_3 X, F_2^2 Y], \qquad (2.4)\ p$$

$$P_3(F_3^2 X, F_2^2 Y) = [F_3 X, F_1 F_2 Y] - F_3[F_3 X, F_2^2 Y]. \qquad (2.4)\ q$$

Consequently

$$P_1(X,Y) = P_1(F_1^2 X, Y) = [F_1 X, F_1 Y] - F_1[F_1 X, Y], \qquad (2.5)\ a$$

$$P_1(X, F_1^2 Y) = P_1(F_1^2 X, F_1^2 Y) = [F_1 X, F_1 Y] - F_1[F_1 X, F_1^2 Y], \qquad (2.5)\ b$$

$$P_1(X, F_2^2 Y) = P_1(F_1^2 X, F_2^2 Y) = -[F_1 X, F_3 F_2 Y] - F_1[F_1 X, F_2^2 Y], \qquad (2.5)\ c$$

$$P_2(X, F_1^2 Y) = P_2(F_2^2 X, F_1^2 Y) = [F_2 X, F_3 F_1 Y] - F_2[F_2 X, F_1^2 Y], \qquad (2.5)\ d$$

$$P_2(X, F_3^2 Y) = P_2(F_2^2 X, F_3^2 Y) = -[F_2 X, F_1 F_3 Y] - F_2[F_2 X, F_3^2 Y] \qquad (2.5)\ e$$

$$P_3(X, F_1^2 Y) = P_3(F_3^2 X, F_1^2 Y) = -[F_3 X, F_2 F_1 Y] - F_3[F_3 X, F_1^2 Y], \qquad (2.5)\ f$$

$$P_3(X, F_2^2 Y) = P_3(F_3^2 X, F_2^2 Y) = [F_3 X, F_1 F_2 Y] - F_3[F_3 X, F_2^2 Y]. \qquad (2.5)\ g$$

Proof. Operating F_1^2 on X (or Y or X and Y together) for $x = 1$ in equation (2.3) and using the equation (1.3), we get (2.4)a, (2.4)b and (2.4)c. Similarly, operating F_2^2 on X (or Y or X and Y together) for $x = 1$ and using (1.2)b, we obtain the equations (2.4)d, (2.4)e and (2.4)f. Equation (2.4)g is obtained by operating F_1^2 on X and F_2^2 on Y for $x = 1$ then using the equation (1.2)b in the resulting equation. Proof of the equations (2.4)h, …………………, (2.4)o follows similarly.

Comparing the equations (2.4)a and (2.4)b, we get (2.5)a. Equation (2.5)b is obtained by comparing the equations (2.4)c and (2.4)d. Similarly, we can obtain the remaining equations.

Theorem (2.3). If we put

$$Q_x(X,Y) = F_x^2[X,Y] - F_x[X, F_x Y]. \qquad (2.6).$$

Then

$$Q_1(F_1^2 X, Y) = F_1^2[F_1^2 X, Y] - F_1[F_1^2 X, F_1 Y], \qquad (2.7)\ a$$

$$Q_1(F_1^2 X, F_1^2 Y) = F_1^2[F_1^2 X, F_1^2 Y] - F_1[F_1^2 X, F_1 Y], \qquad (2.7)\ b$$

$$F_1^2 Q_1(X,Y) = F_1^2[X,Y] - F_1[X, F_1 Y], \qquad (2.7)\ c$$

$$F_1^2 Q_1(X, F_1^2 Y) = F_1^2[X, F_1^2 Y] - F_1[X, F_1 Y], \qquad (2.7)\ d$$

$$Q_1(F_1^2 X, F_2^2 Y) = F_1[F_1^2 X, F_3 F_2 Y] + F_1^2[F_1^2 X, F_2^2 Y], \qquad (2.7)\ e$$

$$Q_1(F_2^2 X, F_2^2 Y) = F_1^2[F_2^2 X, F_2^2 Y] + F_1[F_2^2 X, F_3 F_2 Y], \qquad (2.7)\ f$$

$$Q_1(F_2^2 X, F_3^2 Y) = F_1^2[F_2^2 X, F_3^2 Y] - F_1[F_2^2 X, F_2 F_3 Y], \qquad (2.7)\ g$$

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$$Q_2(F_2^2 X, F_1^2 Y) = F_2^2[F_2^2 X, F_1^2 Y] - F_2[F_2^2 X, F_3 F_1 Y], \quad (2.7)\text{ h}$$

$$Q_2(F_1^2 X, F_3^2 Y) = F_2^2[F_1^2 X, F_3^2 Y] + F_2[F_1^2 X, F_1 F_3 Y], \quad (2.7)\text{ i}$$

$$Q_3(F_3^2 X, F_1^2 Y) = F_3^2[F_3^2 X, F_1^2 Y] + F_3[F_3^2 X, F_2 F_1 Y], \quad (2.7)\text{ j}$$

$$Q_3(F_3^2 X, F_2^2 Y) = F_3^2[F_3^2 X, F_2^2 Y] - F_3[F_3^2 X, F_1 F_2 Y], \quad (2.7)\text{ k}$$

$$F_1^2 Q_1(F_2^2 X, F_3^2 Y) = F_1^2[F_2^2 X, F_3^2 Y] - F_1[F_2^2 X, F_2 F_3 Y], \quad (2.7)\text{ l}$$

$$F_2^2 Q_2(F_2^2 X, F_1^2 Y) = F_2^2[F_2^2 X, F_1^2 Y] - F_2[F_2^2 X, F_3 F_1 Y], \quad (2.7)\text{ m}$$

$$F_2^2 Q_2(F_1^2 X, F_3^2 Y) = F_2^2[F_1^2 X, F_3^2 Y] + F_2[F_1^2 X, F_1 F_3 Y], \quad (2.7)\text{ n}$$

$$F_3^2 Q_3(F_3^2 X, F_1^2 Y) = F_3^2[F_3^2 X, F_1^2 Y] + F_3[F_3^2 X, F_2 F_1 Y], \quad (2.7)\text{ o}$$

$$F_3^2 Q_3(F_3^2 X, F_2^2 Y) = F_3^2[F_3^2 X, F_2^2 Y] - F_3[F_3^2 X, F_1 F_2 Y]. \quad (2.7)\text{ p}$$

Consequently

$$Q_1(F_1^2 X, Y) = F_1^2 Q_1(F_1^2 X, Y) = F_1^2[X, F_1^2 Y] - F_1[X, F_1 Y], \quad (2.8)\text{ a}$$

$$Q_1(X, F_1^2 Y) = F_1^2 Q_1(X, F_1^2 Y) = F_1^2[X, F_1^2 Y] - F_1[X, F_1 Y], \quad (2.8)\text{ b}$$

$$Q_1(F_1^2 X, F_1^2 Y) = F_1^2 Q_1(F_1^2 X, F_1^2 Y) = F_1^2[F_1^2 X, F_1^2 Y] - F_1[F_1^2 X, F_1 Y], \quad (2.8)\text{ c}$$

$$Q_1(F_2^2 X, F_3^2 Y) = F_1^2 Q_1(F_2^2 X, F_3^2 Y) = F_1^2[F_2^2 X, F_3^2 Y] - F_1[F_2^2 X, F_2 F_3 Y], \quad (2.8)\text{ d}$$

$$Q_2(F_2^2 X, F_1^2 Y) = F_2^2 Q_2(F_2^2 X, F_1^2 Y) = F_2^2[F_2^2 X, F_1^2 Y] - F_2[F_2^2 X, F_3 F_1 Y], \quad (2.8)\text{ e}$$

$$Q_2(F_1^2 X, F_3^2 Y) = F_2^2 Q_2(F_1^2 X, F_3^2 Y) = F_2^2[F_1^2 X, F_3^2 Y] + F_2[F_1^2 X, F_1 F_3 Y], \quad (2.8)\text{ f}$$

$$Q_3(F_3^2 X, F_1^2 Y) = F_3^2 Q_3(F_3^2 X, F_1^2 Y) = F_3^2[F_3^2 X, F_1^2 Y] + F_3[F_3^2 X, F_2 F_1 Y], \quad (2.8)\text{ g}$$

$$Q_3(F_3^2 X, F_2^2 Y) = F_3^2 Q_3(F_3^2 X, F_2^2 Y) = F_3^2[F_3^2 X, F_2^2 Y] - F_3[F_3^2 X, F_1 F_2 Y]. \quad (2.8)\text{ h}$$

Proof. The proof follows the pattern of the proof of the theorem (2.2).

Corollary (2.1). In the Para almost F-3 structure manifold, we have

$$Q_1(F_1 X, F_2^2 Y) = -F_1 P_1(F_1^2 X, F_2^2 Y) = F_1[F_1 X, F_3 F_2 Y] + F_1^2[F_1 X, F_2^2 Y], \quad (2.9)\text{ a}$$

$$Q_1(F_1^2 X, F_2^2 Y) = -F_1 P_1(F_1 X, F_2^2 Y) = F_1[F_1^2 X, F_3 F_2 Y] + F_1^2[F_1^2 X, F_2^2 Y], \quad (2.9)\text{ b}$$

$$F_1^2 P_1(F_1 X, F_1 Y) = Q_1(F_1^2 X, F_1^2 Y) = F_1^2[F_1^2 X, F_1^2 Y] - F_1[F_1^2 X, F_1 Y], \quad (2.9)\text{ c}$$

$$P_1(F_1^2 X, Y) + Q_1(F_1^2 X, Y) = N_1(F_1^2 X, Y) \quad (2.9)\text{ d}$$

$$P_1(X, F_1^2 Y) + Q_1(X, F_1^2 Y) = N_1(X, F_1^2 Y), \quad (2.9)\text{ e}$$

$$P_1(F_1^2 X, F_1^2 Y) + Q_1(F_1^2 X, F_1^2 Y) = N_1(F_1^2 X, F_1^2 Y), \quad (2.9)\text{ f}$$

$$\underset{1}{P}(\underset{2}{F}^2 X, Y) + \underset{1}{Q}(\underset{2}{F}^2 X, Y) = \underset{1}{N}(\underset{2}{F}^2 X, Y), \qquad (2.9)\text{ g}$$

$$\underset{1}{P}(\underset{2}{F}^2 X, \underset{2}{F}^2 Y) + \underset{1}{Q}(\underset{2}{F}^2 X, \underset{2}{F}^2 Y) = \underset{1}{N}(\underset{2}{F}^2 X, \underset{2}{F}^2 Y). \qquad (2.9)\text{ h}$$

Proof. Comparing the equations (2.4)h and (2.7)h, we get (2.9)a. Similarly comparing (2.4)i and (2.7)i, we get (2.9)b. Operating $\underset{1}{F}$ on X and Y in (2.3) for $x = 1$, we get

$$\underset{1}{P}(\underset{1}{F} X, \underset{1}{F} Y) = [\underset{1}{F}^2 X, \underset{1}{F}^2 Y] - \underset{1}{F}[\underset{1}{F}^2 X, \underset{1}{F} Y]. \qquad (2.10)$$

Operating $\underset{1}{F}^2$ in (2.10) throughout and using (1.3) then comparing the resulting equation with (2.7)c, we obtain (2.9)c. Adding the equations (2.4)a and (2.7)a then comparing the resulting equation with (2.2)g, we get (2.9)d. Proof of the remaining equations follows similarly.

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