

# Nonlinear Stability Analysis of Flexibly Supported Finite ISO VISCOUS Oil Journal Bearings Including Bearing Surface Deformation

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**Abstract:** This paper analyses the stability characteristics of a rigid rotor mounted in flexibly supported hydrodynamic oil journal bearings considering pressure depended viscosity including the effect of elastic distortion on the surface of bearing liner. This theoretical analysis is intended to show how the effect of elastic distortion along with the flexible support on the journal bearing performance considering pressure depended viscosity can be calculated for three-dimensional bearing geometries. The deformation equations for bearing surface will be solved simultaneously with hydrodynamic equations considering constant viscosity. A Non-linear time transient method is used to simulate the journal and bearing centre trajectory and thereby to estimate the stability parameter. In this analysis forth order Runge-Kutta method is used to determine the locus of the journal and the bearing centre for the various operating conditions. The stability of the system is determined from the combined stability effect in journal and bearing centre. It has been found that stability decreases with increase of of the elasticity parameter of the bearing.

**Keyword -** Journal bearing, surface deformation, variable viscosity, eccentricity ratio, Reynolds equation

## I. INTRODUCTION

Journal bearings are widely used in rotating machineries to support large loads at mean speed of rotation. Regardless of significant advancement in lubrication technology, these bearings fail due to metal to metal contact when they operate below certain minimum speed especially during starting and stopping operations. In order to save cost of replacing the bearing, these bearings are provided with flexible liner. But the deformation of liner, affects the performance characteristics of the bearing particularly at high values of eccentricity ratio.

Many investigators notably O'Donoghue et al. [10], Brighton et al. [11] and Majumder et al. [9], Jain et al. [13], Chandrawat and Sinhasan [5], Oh and Huebner [19] solved the journal bearing problem considering the effect of elastic distortion of the bearing liner. O'Donoghue et al. [10] dealt the analysis with the infinitely long bearing approximation. Brighton et al.[11] described the methods of solution for finite journal bearing considering the effect of elastic distortions. Majumder et al. [9] used the numerical methods to determine the effects of elastic

distortion in the bearing liner on bearing performance. Majumder [9] had done the stability analysis also by linearised method. The displacement equations thus derived were compared with those of O'Donoghue et al. [10] for two dimensional elasticity problem with axial displacements reduced to zero. The displacement equations and form of film pressure tallied completely with Mazumder et al. [9] and stability performance analysis is done considering liner deformation through parametric study of the various variables like eccentricity ratio, slenderness ratio, Poisson ratio, liner thickness to radius ratio with variation in pressure depended viscosity.

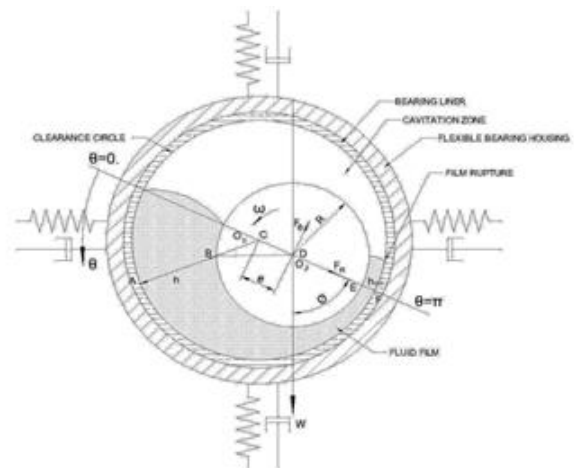


Fig. 1: shows the schematic diagram of the flexibly supported oil journal bearing with flexible liner:

## II. BASIC THEORY

Using the normal assumptions in the theory of hydrodynamic lubrication, modified Reynolds equation for dynamic conditions with fluid in rotating coordinate systems derived from Navier-Stokes equations and continuity equation in the bearing clearance of an oil-lubricated bearing as follows:

$$\frac{\partial}{\partial x} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial z} \right) = 6 \omega R \left( 1 - \frac{2}{\omega} \frac{\partial \phi}{\partial t} \right) \frac{\partial h}{\partial x} + 12 \frac{\partial h}{\partial t} \quad (1)$$

Where  $h$  is the oil film thickness,  $p$  is the oil film pressure,  $\eta$  is the oil viscosity,  $\omega$  is the angular velocity of journal and  $R$  is the journal radius.

The arrangement of journal bearing system with bearing liner is shown in a schematic diagram (figure 1) above. Equation (1) when non-dimensionalised with the following substitutions,

$$\theta = \frac{x}{R}, \bar{h} = \frac{h}{c}, \bar{z} = \frac{z}{L/2}, \bar{p} = \frac{p c^2}{\eta \omega R^2}, \tau = \omega_p t, \Omega = \frac{\omega_p}{\omega}$$

The following modified Reynolds equation considering variable viscosity is obtained in non-dimensional form:

$$\frac{\partial}{\partial \theta} \left( \bar{h}^3 \frac{\partial \bar{p}}{\partial \theta} \right) + \frac{\partial}{\partial \bar{z}} \left( \bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{z}} \right) = 6 \left( 1 - \Omega \frac{\partial \phi}{\partial \tau} \right) \frac{\partial \bar{h}}{\partial \theta} + 12 \Omega \frac{\partial \bar{h}}{\partial \tau} \quad (2)$$

Boundary conditions for equation (1) are as follows

1. The pressure at the ends of the bearing is assumed to be zero (ambient):  $\bar{p}(\theta, \pm 1) = 0$

2. The pressure distribution is symmetrical about the mid-plane of the bearing:  $\frac{\partial \bar{p}}{\partial \bar{z}}(\theta, 0) = 0$

3. Cavitation boundary condition is given by:

$$\frac{\partial \bar{p}}{\partial \theta}(\theta_2, \bar{z}) = 0 \text{ and } \bar{p}(\theta, \bar{z}) = 0 \text{ for } \theta \geq \theta_2$$

Where  $\theta_2$  is the angular coordinate at which the film cavitates.

The oil fluid film thickness,  $h$ , in the case of flexible bearing can be written as,

$$h = c + e \cos \theta + \delta \quad (3)$$

where  $\delta$  is the deformation of the bearing surface and it is a function of  $\theta$  and  $z$ .

$$\text{Therefore, } \bar{h} = 1 + \varepsilon \cos \theta + \bar{\delta} \quad (4)$$

$$\text{Where } \bar{h} = \frac{h}{c}, \varepsilon = \frac{e}{c} \text{ and } \bar{\delta} = \frac{\delta}{c}$$

Before finding solution to equation (1) satisfying the appropriate boundary conditions, the elastic deformation  $\bar{\delta}_0$  is obtained by a method similar to that of Majumder et al. [9] and Brighton et al. [11]. In present calculation the three displacement components  $u, v$  &  $w$  in  $r, \theta$  &  $z$  directions are solved simultaneously satisfying the boundary conditions with an approximate method, as Brighton et al. [11], to evaluate the displacements.

The pressure distribution in the bearing clearance of the rigid bearing is first calculated by solving two dimensional steady state Reynold's equation. The film pressure is then expressed in double Fourier series of the form:

$$p = \sum_m^I \sum_n^I p_{m,n} \cos \frac{2m\pi z}{L} \cos(n\theta + \alpha_{m,n}) \quad (5)$$

Where  $\sum^I$  indicates that the first term of the series is halved.

$p_{m,n}$  and  $\alpha_{m,n}$  as follows,

$$p_{m,n} = \frac{4}{\pi L} \left[ \left\{ \int_0^{2\pi/2} \int_0^{2\pi/2} p \cos \frac{2m\pi z}{L} \cos n\theta dz d\theta \right\}^2 + \left\{ \int_0^{2\pi/2} \int_0^{2\pi/2} p \cos \frac{2m\pi z}{L} \sin n\theta dz d\theta \right\}^2 \right]^{1/2} \quad (6)$$

$$\alpha_{m,n} = \tan^{-1} \frac{\int_0^{2\pi/2} \int_0^{2\pi/2} p \cos \frac{2m\pi z}{L} \sin n\theta dz d\theta}{\int_0^{2\pi/2} \int_0^{2\pi/2} p \cos \frac{2m\pi z}{L} \cos n\theta dz d\theta} \quad (7)$$

$$p_{0,0} = \frac{2}{\pi L} \int_0^{2\pi} \int_0^{L/2} p d\theta dz \quad (8)$$

These displacements are substituted in the stress-strain relationships using Lamé's constants. The six components of stresses are then used in the equations of equilibrium to obtain the following three displacement equations,

The first term of the right-hand side of equation (5) is  $\frac{1}{2} p_{0,0}$ .

Using the end condition of the bearing (i.e  $p=0$  at  $z=L/2$ ) we can obtain  $p_{0,0}$ . This term does not contribute any deformation at  $z=L/2$ . Its effect for the other values of  $z$  is included in the total deformation. The boundary conditions of the inner radius are

$$\sigma_r = -p, \tau_{r\theta} = 0, \tau_{rz} = 0 \quad (9)$$

After non-dimensionalisation, the equation (6), (7) and (8) becomes

$$\bar{p}_{m,n} = \frac{2}{\pi} \left[ \left\{ \int_0^{2\pi/2} \int_0^{2\pi/2} \bar{p} \cos m\pi \bar{z} \cos n\theta d\bar{z} d\theta \right\}^2 + \left\{ \int_0^{2\pi/2} \int_0^{2\pi/2} \bar{p} \cos m\pi \bar{z} \sin n\theta d\bar{z} d\theta \right\}^2 \right]^{1/2} \quad (10)$$

$$\alpha_{m,n} = \tan^{-1} \frac{\int_0^{2\pi/2} \int_0^{2\pi/2} \bar{p} \cos m\pi \bar{z} \sin n\theta d\bar{z} d\theta}{\int_0^{2\pi/2} \int_0^{2\pi/2} \bar{p} \cos m\pi \bar{z} \cos n\theta d\bar{z} d\theta} \quad (11)$$

$$\text{and } \bar{p}_{0,0} = \frac{1}{\pi} \int_0^{2\pi} \int_0^{L/2} \bar{p} d\theta d\bar{z} \quad (12)$$

$$\text{where } \bar{p}_{m,n} = \frac{p_{m,n} c^2}{\eta_0 \omega R^2}, \bar{p} = \frac{p c^2}{\eta_0 \omega R^2}, \bar{z} = \frac{z}{L/2}$$

The outer surface of the bearing is rigidly enclosed by the housing, preventing any displacement of the outer surface. The ends of the bearing are prevented from expanding axially, but are free to move circumferentially or radially.

The displacement components in  $r, \theta$  and  $z$  directions are found from the pressure distribution, which has been expressed in a Fourier series. It is apparent that the displacements will also be harmonic functions.

These displacements were substituted in the stress-strain relationships using Lamé's constants. The six components of stresses were then used in the equations of equilibrium to obtain the following three displacement equations.

$$C^* \frac{d^2 u^*}{d y^2} + \frac{C^*}{y} \frac{d u^*}{d y} - (C^* + n^2) \frac{u^*}{y} + (C^* - 1) \frac{n}{y} \frac{d v^*}{d y} - (C^* + 1) \frac{n}{y} v^* - k^2 u^* + (C^* - 1) k \frac{d w^*}{d y} = 0 \quad (13)$$

$$\frac{d^2 v^*}{d y^2} + \frac{1}{y} \frac{d v^*}{d y} - (1 + C^* n^2) \frac{v^*}{y} - k^2 v^* - (C^* - 1) \frac{n}{y} \frac{d u^*}{d y} - (C^* + 1) \frac{n}{y} u^* - n k (C^* - 1) \frac{w^*}{y} = 0 \quad (14)$$

$$\frac{d^2 w^*}{d y^2} + \frac{1}{y} \frac{d w^*}{d y} - \frac{n^2}{y} w^* - C^* k^2 w^* - k (C^* - 1) \frac{d u^*}{d y} - k (C^* - 1) \frac{u^*}{y} - n (C^* - 1) k \frac{v^*}{y} = 0 \quad (15)$$

Where,  $\lambda = \frac{E \nu}{(1+\nu)(1-2\nu)}$ ,  $C^* = 2 + \frac{\lambda}{\mu}$ ,  $k = \frac{2m\pi r_i}{L}$  &  
 $\mu = \frac{E}{2(1+\nu)}$

The boundary conditions are, at  $\bar{y} = 1$ ,

$$C^* \frac{d u^*}{d \bar{y}} = -\frac{1}{\mu} p_{m,n} - (C^* - 2) \left( \frac{n v^*}{y} + \frac{u^*}{y} + k w^* \right) \quad (16)$$

$$\frac{d v^*}{d \bar{y}} = \frac{n u^*}{y} + \frac{v^*}{y} \quad (17)$$

$$\frac{d w^*}{d \bar{y}} = u^* k \quad (18)$$

$$\text{and at } \bar{y} = \frac{b}{a}, \quad u^* = v^* = w^* = 0 \quad (19)$$

The equations (13), (14) and (15) expressed first in finite difference form solving the displacement equations with the boundary conditions (16-19) the values of the distortion coefficient  $d_{m,n}$  were obtained and expressed as,

$$d_{m,n} = \frac{\mu u^*}{R p} \quad (20)$$

The radial deformation  $\delta$  of the bearing surface will be  $\delta = u^*$

$$\text{or, } \delta = d_{m,n} \frac{R p}{\mu} r_i \cos(n\theta + \alpha_{m,n}) \cos \frac{2m\pi z}{L}$$

Considering the bearing clearance is very small in compare to the diameter of the journal, the total radial deformation will be

$$\delta = \frac{R p_{0,0} d_{0,0}}{2 \mu} + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} d_{m,n} \frac{R}{\mu} p_{m,n} \cos(n\theta + \alpha_{m,n}) \cos \frac{2m\pi z}{L} \quad (21)$$

(m,n) ≠ (0,0)

After non-dimensionalisation, the equation (21) becomes

$$\bar{\delta} = 2(1+\nu) F \left[ p_{0,0} d_{0,0} + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} p_{m,n} d_{m,n} \cos(n\theta + \alpha_{m,n}) \cos m\pi \bar{z} \right] \quad (22)$$

(m,n) ≠ (0,0)

Where,  $\bar{\delta} = \frac{\delta}{c}$  and  $F = \frac{\eta_0 \omega R^3}{E c^3}$  and  $\mu$  is replaced by  $\frac{E}{2(1+\nu)}$

Knowing the distortion coefficient  $d_{m,n}$  and using the expressions for  $p_{m,n}$  &  $\alpha_{m,n}$  from equations (10), (11) and also for  $p_{0,0}$  from equation (12) the radial deformation in the inner bearing surface  $\bar{\delta}$  at any point  $(\theta, \bar{z})$  was computed.

### III. METHOD OF ANALYSIS

The modified film pressure  $p$  was first obtained from equation (2) in finite difference form assuming a constant film shape and using Gauss-Seidel method with successive over relaxation scheme. The convergence criterion adopted for pressure is  $\left| 1 - \left( \frac{\sum p_{new}}{\sum p_{old}} \right) \right| \leq 10^{-5}$ .

Then this pressure distribution was expressed as a double Fourier series as given by equation (5). The deformation equation (22) was then calculated for a given  $F$  using distortion coefficients from equation (20). The film thickness equation was then modified using equation (4). The process was repeated until a compatible film shape and pressure distribution was determined.

**A. Fluid film forces:** At any point on the journal the film pressure is  $p$  and the film force is  $p R d\theta dz$ , where  $R d\theta dz$  is any small segment at an angle  $\theta$  with the line of centres. This will have components  $p R d\theta dz \cos\theta$  in the direction along the line of centres and  $p R d\theta dz \sin\theta$  in the direction normal to the line of centres.

Component  $F_r$  of the oil fluid film forces along the line of centres is given by,

$$F_r = 2 \int_0^{\frac{L}{2}} \int_{\theta_1}^{\theta_2} p \cos\theta (R d\theta) dz \quad (23)$$

where  $\theta_1$  and  $\theta_2$  are angular coordinates at which the fluid film commences and cavitates respectively.

Component  $F_\phi$  of the oil fluid film forces perpendicular the line of centres is given by

$$F_\phi = 2 \int_0^{\frac{L}{2}} \int_{\theta_1}^{\theta_2} p \sin\theta (R d\theta) dz \quad (24)$$

Using  $\bar{p} = \frac{p c^2}{\eta_0 \omega R^2}$ ,  $\bar{z} = \frac{z}{L/2}$ ,  $\bar{F}_r = \frac{F_r c^2}{\eta_0 \omega R^3 L}$ , and  $\bar{F}_\phi = \frac{F_\phi c^2}{\eta_0 \omega R^3 L}$ ,

the non-dimensional form is given by,

$$\bar{F}_r = \int_0^1 \int_{\theta_1}^{\theta_2} \bar{p} \cos\theta d\theta d\bar{z} \quad (25)$$

$$\bar{F}_\phi = \int_0^1 \int_{\theta_1}^{\theta_2} \bar{p} \sin\theta d\theta d\bar{z} \quad (26)$$

**B. Steady state load**

From the film forces in  $r$  and  $\phi$  directions, neglecting the time dependent term in Reynolds equation, the resultant film force which is balanced by the load applied to the shaft can be calculated and the angle between load  $\bar{W}_0$  and the line of centres (i.e attitude angle  $\phi_0$ ) are determined by

$$\bar{W}_0 = \sqrt{\left( \bar{F}_r^2 + \bar{F}_\phi^2 \right)} \quad (27)$$

$$\phi_0 = \tan^{-1} \left( -\frac{\bar{F}_\phi}{\bar{F}_r} \right) \quad (28)$$

where  $\bar{F}_r$  and  $\bar{F}_\phi$  are the dimensionless steady state hydrodynamic forces in  $r$  and  $\phi$  directions respectively.

Since the film pressure has been obtained numerically for all the mesh points, integrations in equations (25) and (26) can

be easily performed numerically by using Simpson's 1/3 rd. rule to get  $\bar{F}_r$  and  $\bar{F}_\phi$ . The steady state load ( $\bar{W}_0$ ) and the attitude angle ( $\phi_0$ ) are then calculated by using equations (27) and (28).

C. Equation of Motion:

The equation of motion for a rigid rotor supported on four identical flexibly supported bearings are given by,

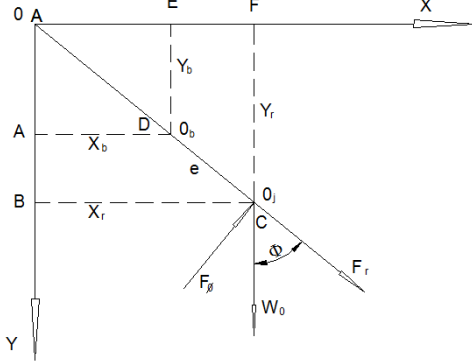


Fig. 2: Coordinate system of hydrodynamic fluid film forces in circumferential & radial direction

$$M_r \cdot \frac{d^2 X_r}{dt^2} = F_r \sin \phi + F_\phi \cos \phi \quad (29)$$

$$M_r \cdot \frac{d^2 Y_r}{dt^2} = F_r \cos \phi - F_\phi \sin \phi + W_0 \quad (30)$$

$$M_b \cdot \frac{d^2 X_b}{dt^2} = -F_\phi \cos \phi - F_r \sin \phi - B \cdot \frac{dX_b}{dt} - KX_b \quad (31)$$

$$M_b \cdot \frac{d^2 Y_b}{dt^2} = F_\phi \sin \phi - F_r \cos \phi - B \cdot \frac{dY_b}{dt} - KY_b \quad (32)$$

The relation between rotor & bearing motion are given by,

$$X_r = X_b + e \sin \phi \quad (33)$$

$$Y_r = Y_b + e \cos \phi \quad (34)$$

The above two equations are substituted in equations of motion. Finally the equations of motion are expressed in non-dimensional form as follows,

$$\ddot{\bar{X}}_b = \frac{d\bar{X}_b}{d\tau} \quad (35)$$

$$\ddot{\bar{Y}}_b = \frac{d\bar{Y}_b}{d\tau} \quad (36)$$

$$\ddot{\bar{X}}_b = \frac{1}{m \cdot \bar{M} \cdot \bar{W}_0 \cdot \Omega^2} \left[ -\bar{F}_\phi \cos \phi - \bar{F}_r \sin \phi - \Omega \bar{W}_0 \cdot \bar{B} \cdot \bar{X}_b - \bar{W}_0 \cdot \bar{K} \cdot \bar{X}_b \right] \quad (37)$$

$$\ddot{\bar{Y}}_b = \frac{1}{m \cdot \bar{M} \cdot \bar{W}_0 \cdot \Omega^2} \left[ \bar{F}_\phi \sin \phi - \bar{F}_r \cos \phi - \Omega \bar{W}_0 \cdot \bar{B} \cdot \bar{Y}_b - \bar{W}_0 \cdot \bar{K} \cdot \bar{Y}_b \right] \quad (38)$$

$$\dot{\varepsilon} = \frac{d\varepsilon}{d\tau} \quad (39)$$

$$\dot{\phi} = \frac{d\phi}{d\tau} \quad (40)$$

$$\ddot{\varepsilon} = \frac{A_3 \cdot F - A_4 \cdot E}{A_2 \cdot A_3 - A_1 \cdot A_4} \quad (41)$$

$$\ddot{\phi} = \frac{A_2 \cdot E - A_1 \cdot F}{A_2 \cdot A_3 - A_1 \cdot A_4} \quad (42)$$

where,

$$C = -2 \dot{\varepsilon} \cdot \dot{\phi} \cos \phi + \varepsilon \cdot \sin \phi \cdot \left( \dot{\phi} \right)^2 + \frac{1}{\bar{M} \cdot \bar{W}_0 \cdot \Omega^2} \left[ \bar{F}_r \sin \phi + \bar{F}_\phi \cos \phi \right]$$

$$D = 2 \dot{\varepsilon} \cdot \dot{\phi} \sin \phi + \varepsilon \cdot \cos \phi \cdot \left( \dot{\phi} \right)^2 + \frac{1}{\bar{M} \cdot \bar{W}_0 \cdot \Omega^2} \left[ \bar{F}_r \cos \phi - \bar{F}_\phi \sin \phi + \bar{W}_0 \right]$$

$$A_1 = \sin \phi, \quad A_2 = \cos \phi, \quad A_3 = \varepsilon \cdot \cos \phi, \quad A_4 = -\varepsilon \cdot \sin \phi$$

$$G = \ddot{\bar{X}}_b$$

$$H = \ddot{\bar{Y}}_b$$

$$E = C - G$$

$$F = D - H$$

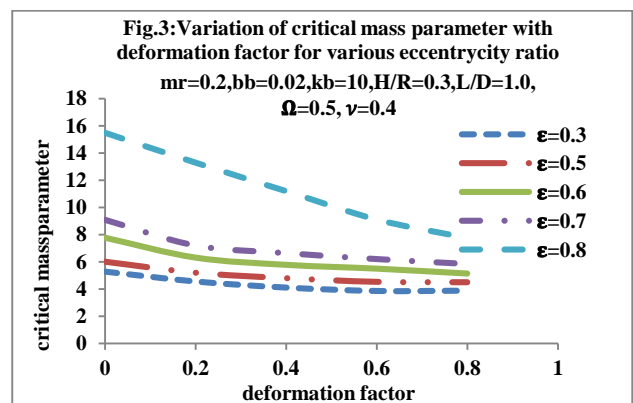
D. Solution scheme:

For stability analysis, a non-linear time transient analysis is carried out using the equations of motion [equations (29) to (32)] to compute a new set of  $\varepsilon, \phi, X_b, Y_b$  and their derivatives for the next time step for a given set of  $L/D$ , steady state eccentricity ratio  $\varepsilon_0$ , deformation factor  $F$ , mass parameter  $\bar{M}$ . The fourth order Runge-Kutta method is used for solving the equations of motion. The hydrodynamic forces are computed every time step by solving the partial differential equation for pressure satisfying the boundary conditions.

E. Stability Analysis

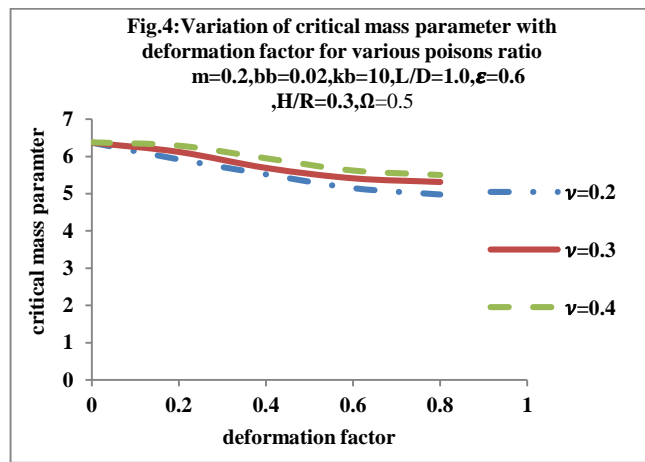
To study the effect of bearing surface deformation on journal centre trajectory of flexibly supported bearings a set of trajectories of journal and bearing centre has been studied and it is possible to construct the trajectories for numbers of complete revolution of the journal the plots shows the stability of the journal when the trajectory of journal and bearing centre ends in a limit cycle. Critical mass parameter for a particular eccentricity ratio, slenderness ratio and deformation factor is found when the trajectories end with limit cycle (Fig. 10 & Fig. 11) or it changes its trend from stable to unstable.

IV. RESULTS AND DISCUSSION



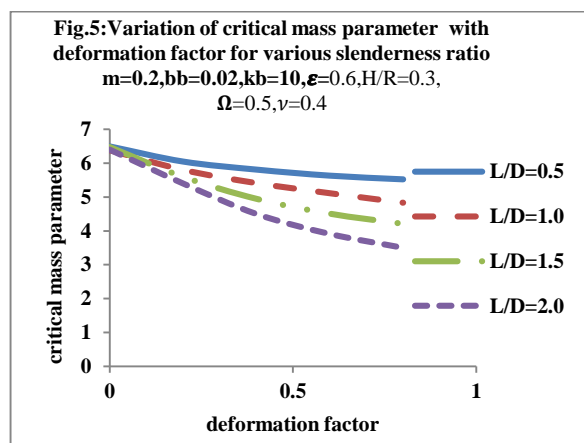
**A Effect of Eccentricity ratio ( $\epsilon$ ):**

Figure.3 shows that the critical mass parameter of journal bearings as a function of deformation factor ( $F$ ) for  $L/D=1.0, H/R=0.3, \nu=0.4$  when eccentricity ratio ( $\epsilon_0$ ) is considered as a parameter. From the figure it is found that when other parameters remain same as eccentricity ratio increases the critical mass parameter increases. Further, for the eccentricity ratio beyond 0.6 the family of the curves shows drooping trend which becomes more significant up to  $F=0.4$ . For the eccentricity ratio  $\epsilon_0 \leq 0.6$  the characteristics are more or less horizontal meaning that the mass parameter  $\bar{M}$  remains unaffected with a change in  $F$ . The stability threshold falls rapidly with  $F$  at higher eccentricity ratio.



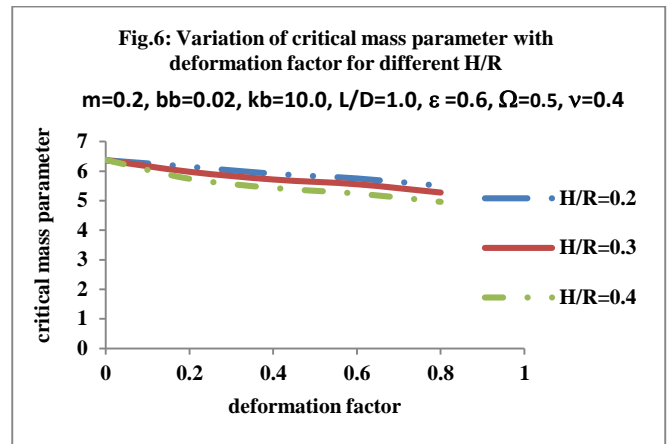
**B Effect of Poisson's ratio ( $\nu$ ):**

Figure.4 is the plot of dimensionless critical mass parameter of journal bearing as a function of deformation factor ( $F$ ) for  $L/D=1.0, H/R=0.3, \epsilon_0=0.6$  when Poisson's ratio ( $\nu$ ) is considered as a parameter. A scrutiny of the figure reveals that as Poisson's ratio increases, the dimensionless critical mass parameter increases. Further, the family of the curves shows declining trend i.e., critical mass parameter decreases with increase in deformation factor. The decreasing trend of the curve is very slow.



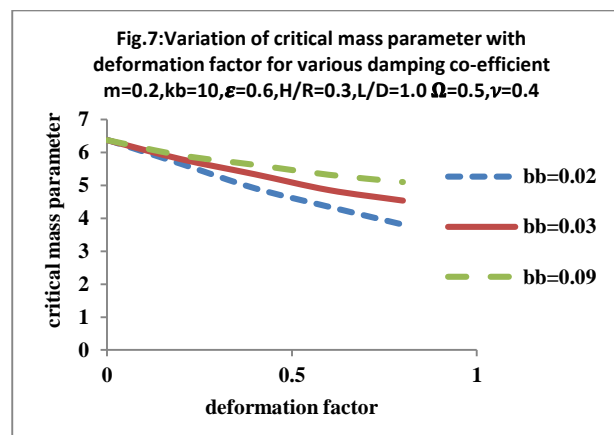
**C. Effect of slenderness ratio ( $L/D$ ):**

Effect of slenderness ratio  $L/D$  on the critical mass parameter of the bearing can be studied from figure 5. Here, dimensionless critical mass parameter of journal bearings is shown as a function of deformation factor ( $F$ ) for  $\epsilon_0=0.6, H/R=0.3, \nu=0.4$ . It is found that when other factors remain unaltered, an increase in  $L/D$  decreases the critical mass parameter.



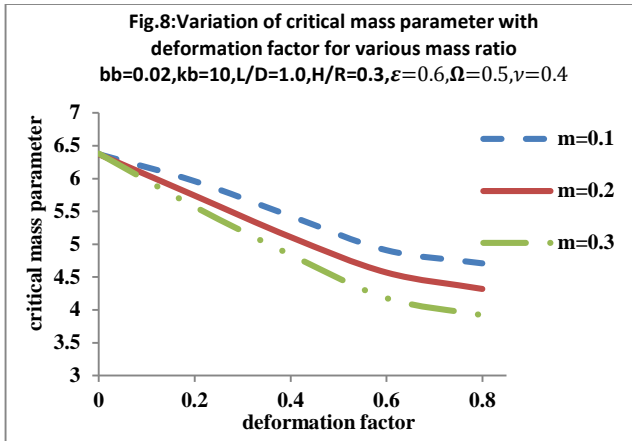
**D. Effect of liner thickness to journal radius ratio ( $H/R$ ):**

In figure.6 the dimensionless critical mass parameter of journal bearings is shown as a function of deformation factor ( $F$ ) for  $L/D=1.0, \nu=0.4, \epsilon_0=0.6$ , liner thickness to journal radius ratio ( $H/R$ ) is considered as a parameter. It is observed from the figure that as ( $H/R$ ) increases the dimensionless critical mass parameter decreases.



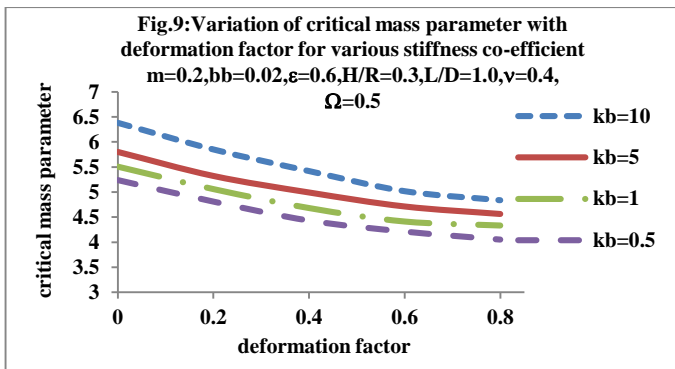
**E. Effect of support damping co-efficient ( $bb$ ):**

Effect of damping co-efficient ( $bb$ ) on dimensionless critical mass parameter of the bearing can be studied from figure 7. Here, dimensionless critical mass parameter of journal bearings is shown as a function of deformation factor ( $F$ ) for  $\epsilon_0=0.6, H/R=0.3, \nu=0.4$ . It is found that when other factors remain unaltered, an increase in  $bb$  increases the critical mass parameter.



F. Effect of mass ratio ( $m$ ) :

Figure.8 is the plot of dimensionless critical mass parameter of journal bearing as a function of deformation factor ( $F$ ) for  $L/D = 1.0$ ,  $H/R=0.3$ ,  $\epsilon_0=0.6$  when mass ratio is considered as a parameter. A scrutiny of the figure reveals that as mass ratio increases, the dimensionless critical mass parameter decreases.



G. Effect of support stiffness co-efficient ( $kb$ ):

In figure.9 the dimensionless critical mass parameter of journal bearings is shown as a function of deformation factor ( $F$ ) for  $L/D=1.0$ ,  $\nu=0.4$ ,  $\epsilon_0=0.6$ , stiffness co-efficient is considered as a parameter. It is observed from the figure that as  $kb$  increases the dimensionless critical mass parameter increases.

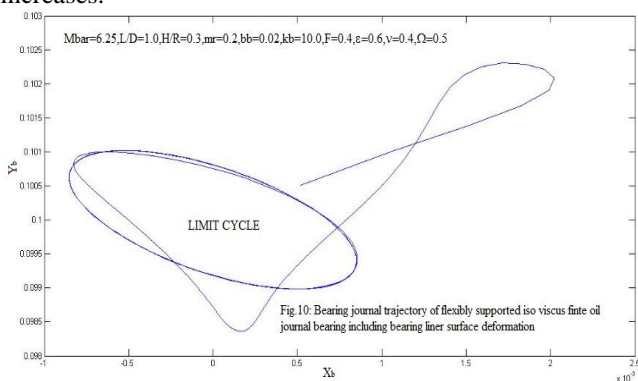


Fig.10: Bearing journal trajectory of flexibly supported iso viscous finite oil journal bearing including bearing liner surface deformation

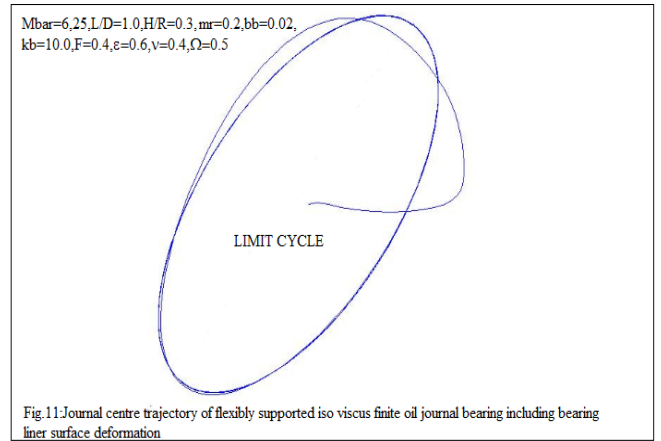


Fig.11: Journal centre trajectory of flexibly supported iso viscous finite oil journal bearing including bearing liner surface deformation

## V. CONCLUSION

Numerical methods are used to determine the effects of elastic distortions in the bearing liner on bearing stability of finite journal bearing:

1. The stability decreases as the bearing liner is made more flexible for high eccentricity ratios (i.e.,  $\epsilon_0 > 0.8$ ). For  $\epsilon_0 < 0.5$ , the flexibility of the bearing liner had little or no effect on stability.
2. Bearing is highly stable when  $L/D$  is small but drops as  $L/D$  increases from 0.5 to 2.0. This stability drops as deformation factor increases.
3. The hydrodynamic pressure and hence the stability is reduced as the bearing liner becomes more flexible, especially at eccentricities greater than 0.8.
4. As the Poisson ratio increases the stability increases but drop sharply when bearing liner is made more flexible.
5. As the liner thickness to radius ratio increases the stability decreases but drop when bearing liner is made more flexible

## NOMENCLATURE

$a = r_i$	Inner radius of the bearing liner [ $m$ ]
$b = r_o$	Outer radius of the bearing liner [ $m$ ]
$c$	Radial clearance [ $m$ ]
$R$	Journal radius [ $m$ ]
$D$	Journal diameter [ $m$ ]
$d_{m,n}$	Distortion coefficient of $m,n$ harmonic
$m,n$	Axial and circumferential harmonics
$e$	Eccentricity [ $m$ ]
$e_0$	Steady state eccentricity [ $m$ ]
$E$	Young's modulus [ $N / m^2$ ]
$F$	Elasticity parameter or deformation factor, $= \frac{\eta_0 R^3 \omega}{c^3 E}$
$F_s$	Shear force on journal surface [ $N$ ]
$F_r$	Nondimensional fluid film force along the line of centers

$\bar{F}_\phi$	Nondimensional fluid film force perpendicular the line of centres
$\bar{F}_{r_0}, \bar{F}_{\phi_0}$	Non-dimensional steady state fluid film forces
$h$	Oil film thickness [ $m$ ]
$h_0$	Steady state oil film thickness [ $m$ ]
$\bar{h}$	Non-dimensional oil film thickness
$H$	Thickness of bearing liner [ $m$ ]
$L$	Length of bearing [ $m$ ]
$p$	Oil film pressure [ Pa ]
$p_0$	Steady state film pressure [ Pa ]
$\bar{p}$	Dimensionless oil pressure
$Q$	End flow of oil [ $m^3 / s$ ]
$\bar{Q}$	Nondimensional End flow
$u, v, w$	Components of fluid velocity in the x, y, and z direction, respectively. [ $m / s$ ]
$U$	Shaft peripheral speed [ $m / s$ ]
$W_0$	Steady state load [ N ]
$\bar{W}_0$	Dimensionless steady state load
$x, y, z$	Circumferential, radial and axial coordinates
$\bar{\theta}, \bar{y}, \bar{z}$	Dimensionless coordinates in circumferential, radial and axial directions
$\eta_0$	Viscosity at inlet condition [ Pa s ]
$\bar{\eta}$	Non-dimensional viscosity of oil
$\rho$	Density [ $kg / m^3$ ]
$\nu$	Poisson's ratio
$\varepsilon$	Eccentricity ratio
$\varepsilon_0$	Steady state eccentricity ratio
$\phi$	Attitude angle [ rad ]
$\phi_0$	Steady state attitude angle [ rad ]
$\theta_1$	Angular coordinates at which the fluid film commences [ rad ]
$\theta_2$	Angular coordinates at which the fluid film cavitates [ rad ]
$\omega$	Angular velocity of journal [ rad / s ]
$\Omega$	Whirl ratio. [ $\omega_p / \omega$ ]
$\delta$	Deformation of bearing surface. [ $m$ ]
$\delta_0$	Steady state deformation of bearing surface. [ $m$ ]
$\bar{\delta} = \delta / c$	Non-dimensional deformation of bearing surface
$\lambda, \mu$	Lame's constants
$X_b$	Coordinate of bearing centre in x-direction
$Y_b$	Coordinate of bearing centre in y-direction
$X_r$	Coordinate of rotor centre in x-direction
$Y_r$	Coordinate of rotor centre in y-direction
$M_r$	Mass of rotor or journal

$M_b$	Mass of bearing
$m = \frac{M_b}{M_r}$	Mass ratio Critical Mass Parameter
$\bar{M} = \frac{M_r.c.\omega^2}{W_0}$	Viscosity Parameter
$B = \frac{\eta_0\alpha\omega R^2}{c^2}$	Bearing support stiffness coefficient Bearing support damping coefficient
$\bar{K} = kb$	
$\bar{B} = bb$	

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