Nonparametric Moving Average Control Chart for Process Variability

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***Abstract* - In this paper a nonparametric moving average control chart for monitoring process variability is proposed. The chart uses statistic based on the first and third quartiles. The proposed chart is simple to use. The performance of the chart is studied using Average Run Length criterion. The proposed chart performs better than the chart based on quartiles due to Amin et al. (1995) and synthetic chart due to Khilare and Shirke (2012) for various distributions. When underlying distribution is double exponential or Gamma, the Average Run Length performance of the chart is better than that of a well-known S^2 chart. An example is provided to illustrate the working of the chart.**

Keywords: moving average, nonparametric, sign statistic, average run length.

I. INTRODUCTION

Control charts are one of the primary tools of statistical process control. Usually control charts to monitor process characteristics are based on some distributional assumptions. Nonparametric control charts do not require the assumption of any specific probability distribution for the underlying process. It is observed that nonparametric charts are simple to use and therefore in the recent years nonparametric charts have attracted researchers' attention. Shewhart type nonparametric charts proposed to monitor process location or process variation are based on a suitable nonparametric statistic. It is well-known fact that Shewhart type control charts do not perform well for small shifts in the process parameter. To overcome this demerit, control charts with memory are used. Moving Average control charts are control charts with memory. They are found to be superior to the Shewhart chart in detecting the small process shifts because they use the information contained not only in the current sub group sample but also from previous samples. The present work is focused on nonparametric moving average control charts for monitoring increase in the process variability.

In quality control applications McGilchrist and Woodyer (1975) proposed a distribution-free cumulative sum technique for monitoring rainfall amounts. Bakir and Reynolds (1979) studied a nonparametric procedure for process control based on within-group ranking. Amin and Searcy (1991) studied the behavior of the EWMA control chart using the Wilcoxon signed-rank statistic. Amin et al. (1995) developed nonparametric quality control charts based on the sign statistic. Bakir (2006) developed distribution-free quality control charts based on signed-rank-like statistic. Bakir (2004) proposed a distribution-free Shewhart quality control chart based on signed-ranks. Chakraborti and Eryilmaz (2007) proposed control charts based on signed-rank statistic. Chakraborti and Van de Wiel (2008) proposed Mann-Whiteny statistic based control chart. Human et al. (2010) studied nonparametric Shewhart-type sign control charts based on runs. Ho and Costa (2011) proposed monitoring a wandering mean with an np chart and this chart also works with sign statistic. Pawar and Shirke (2010) proposed a nonparametric Shewhart-Type Synthetic Control Chart for location based on signed-rank statistic.

Amin et al. (1995) proposed a sign chart for process variation based on quartiles; however, performance of this chart is not up to the mark as compared to S^2 chart for normal and some non-normal distributions. Das (2008a) proposed nonparametric control chart for monitoring variability based on the two sample variability test due to Ansari and Bradley (1960). Its in-control performance is better than that of the S^2 chart where as the out-of-control performance is not up to the mark. Das and Bhattacharya (2008) proposed a chart based on nonparametric test on variability and shown that it has better in-control performance than the Shewhart S chart. Das (2008b) developed a nonparametric control charts for controlling process variability based on rank test. Khilare and Shirke (2012) developed a nonparametric synthetic chart for process variability based on sign statistic. In this article we propose a moving average control chart for process variability based on sign statistic. Rest of the paper is organized as follows:

Sign chart and synthetic sign chart for process variability are described briefly in section 2. In section 3 moving average control chart and proposed moving average control chart for process variability using sign statistic are described. The Average Run Length (ARL) performance of the proposed chart and its comparison with the other charts is studied in section 4. Section 5 illustrates the use of the proposed chart with numerical example.

II. SIGN CHART AND SYNTHETIC SIGN CHART FOR MONITORING PROCESS VARIABILITY

Lehmann (1975) studied a nonparametric statistical method based on ranks. Control charts using test statistic for comparing two variances would require to obtain an initial sample of size *m* when the process is in-control and then at each sample time *i*, a sample of size *n* is taken from the process. These two samples are then pooled to get a pooled sample of size (*m*+*n*). The observations in the pooled sample are then ranked from smallest to the largest element and charting statistic based on the ranks of the observations is calculated.

Another approach given by Bradley (1968) is based on Westenberg's two sample inter-quartile range test. The test is based on pooling two samples S_1 and S_2 into one sample and then counting the number of observations belonging to the S_1 sample that are above the third quartile (Q_3) or below the first quartile (Q_1), where Q_1 and Q_3 are based on the pooled sample. Amin et al.(1995) adapted the idea of this test to the one sample case and provided a chart based on quartiles. In the control chart applications Q_1 and Q_3 would need to be specified by process engineers or more likely estimated from the process data when the process is in-control. Let X_{ij} be the j^{th} observation in the i^{th} sample $(i=1, 2, 3, \dots ; j=1, 2, \dots, n)$. Define,

$$U_{ij} = \begin{cases} 1 & if\ X_{ij} < Q_1\ or\ X_{ij} > Q_3, \\ 0 & if\ X_{ij} = Q_1\ or\ X_{ij} = Q_3\ , \\ -1 & if\ Q_1 < X_{ij} < Q_3. \end{cases}$$

The sign statistic corresponding to the i^{th} sample is given by

$$U_i = \sum_{j=1}^{n} U_{ij}\ , \quad i=1,2,\dots\dots \tag{1}$$

The random variable $V_i = \frac{U_i+n}{2}$ follows binomial distribution with parameters n and p, where p is the probability of a nonconforming item which is given by $p = pr[X_{ij} < Q_1\ or\ X_{ij} > Q_3],$

which is function of process parameter(s).

While monitoring process variability, most of the times an increase in the process standard deviation, σ from its specified value σ_0 is of interest. In this case a one-sided control chart is desirable. A one-sided sign chart signals when $U_i \geq d$, where d is an upper control limit of the chart to be fixed suitably. This is called an upper one-sided sign chart for detecting an increase (or an upward shift) in σ.

The ARL of a sign chart for shift of magnitude $\delta = \sigma_1/\sigma_0$ in σ is given by,

$$ARL(\delta) = 1/pr(U_i \geq d), \tag{2}$$

where $pr(U_i \geq d) = \sum_{k=\frac{d+n}{2}}^{n} \binom{n}{k} p^k (1-p)^k$

with $p = pr(Q_1 < X_{ij}\ or\ Q_3 > X_{ij}\,|\sigma = \delta\sigma_0)$

The upper control limit *d* is determined using equation (2) for desired in-control *ARL=ARL(0)* when the process is in control ($\sigma = \sigma_0$). The synthetic control chart for monitoring process variability is proposed by Khilare and Shirke (2012). The synthetic control chart is described in the following.

A synthetic control chart for process variability is obtained by combining the sign chart based on quartiles and Conforming Run Length (*CRL*) chart due to Bourke (1991). The operations of the synthetic control chart are out-lined below:

1) Determine control limits *d* and *L* of sign chart and *CRL* chart respectively.

2) At time *i*, take a sample of n units for inspection and calculate U_i.

3) If *Ui* < *d*, a sample is a conforming one and control flow goes back to step (2). Otherwise, a sample is a nonconforming one and control flow continues to the next step.

4) Count the number of samples between the current and previous nonconforming samples. This number is taken as *CRL* value for synthetic chart.

5) If *CRL* > *L*, then the process is said to be under control and control flow goes back to the step (2). Otherwise the process is considered as out-of-control and control flow continues to next step.

6) Take action to locate and remove the assignable cause.

The *ARL* of Synthetic Sign chart for shift δ in σ is

$$ARL(\delta) = 1/(p_1 * (1-(1-p_1)^L)) \tag{3}$$

where $p_1 = pr(U_i \geq d)$

$$= \sum_{k=\frac{d+n}{2}}^{n} \binom{n}{k} p^k (1-p)^k$$

and $p = pr(Q_1 < X_{ij}\ or\ Q_3 > X_{ij}\,|\sigma = \delta\sigma_0)$

The parameters *d* and *L* of the synthetic sign chart are determined using equation (3) for desired in-control *ARL=ARL(0)* when the process is in control ($\sigma = \sigma_0$). One may refer to Khilare and Shirke (2012) for more details about the synthetic chart.

In the following section we propose a chart based on moving averages to monitor process standard deviation.

III. NONPARAMETRIC CONTROL CHART BASED ON MOVING AVERAGES

Let ($x_{i1}, x_{i2} \dots, x_{in}$) be a sample (sub group) of size *n* (>1) of independent observations from the process output at sampling instance *i*, *i* = 1, 2, ... The probability distribution of the process is assumed to be continuous with process standard deviation σ. We assume that, when the process is in control, $\sigma = \sigma_0$. Let $U_i = U(x_{i1}, x_{i2} \dots, x_{in})$ be a suitable nonparametric statistic for σ based on the subgroup sample. **C**harting statistic of Moving Average chart is based on the average of the most recent '*w*' (>1) nonparametric statistics defined as

$$\Psi_i = \begin{cases} (U_1 + U_2 + \cdots + U_i)/i & ; \ i < w \\ (U_i + U_{i-1} + \cdots + U_{i-w+1})/w & ; \ i \geq w. \end{cases} \quad (4)$$

Here '*w*' is called as the span of the moving average. If we are interested only in detecting shift in σ in positive direction, the control chart will have only upper control limit. Let *UCL* be the upper control limit of the chart. *UCL* is chosen so as to have the desired in-control *ARL*. If the interest is to detect shift in negative direction, the control chart will have only lower control limit (*LCL*). The control chart to detect shifts in both the directions, both *LCL* and *UCL* are required by the chart.

A Nonparametric Moving Average chart gives an out of control signal for the first sampling instant *i*, if

i) if $\Psi_i \geq UCL$ (when interest is to detect shift in σ in positive direction).

ii) if $\Psi i \leq LCL$ (when interest is to detect shift in σ in negative direction).

iii) if $\Psi i \leq LCL$ or $\Psi i \geq UCL$ (when interest is to detect shift in σ in both the directions).

In order to obtain *LCL/UCL*, we need to know the distribution of the charting statistic. Since the sequence of charting statistics, Ψ_i, $i = 1, 2, \ldots$. is a sequence of dependent variables, it is not easy to obtain exact distribution of Ψ_i , $i = 1, 2, \ldots$. In such cases simulation technique can be used to obtain control limits and the *ARL* values. Further, since the distribution of Ψ_i is discrete, many times, it will not be possible to get *LCL/UCL* that gives in-control *ARL* exactly equal to the desired value, say *ARL(0).* In such a case we choose that *LCL/UCL* for which in-control *ARL* is close to the desired *ARL(0).*

To construct a nonparametric moving average control chart for process variability based on nonparametric sign statistic, we assume that, when the process is in control, the process standard deviation $\sigma = \sigma_0$. For nonparametric moving average control chart for variability, our charting statistic Ψ_i defined in equation (4) is based on nonparametric sign statistic U_i defined in equation (1). The one-sided moving average chart signals when $\Psi_i \geq UCL$.

IV. PERFORMANCE STUDY OF THE MOVING AVERAGE CHART

The main task of a control chart is to detect change in the process as early as possible and to give an out-of-control signal. A control chart is more efficient if *ARL* of the chart is large when the process is in control and small when the process is out of control.

Since the exact distribution of the charting statistic Ψ_i is not known, the simulation technique is used to compute in-control and out-of-control *ARL* values. Computer programs are written in open source software R to study the performance of the control charts under different process distributions. We compared the performance of the moving average chart with sign chart, synthetic sign chart and S^2 chart for normal, double exponential Cauchy and gamma distributions. We do not consider S^2 chart for Cauchy distribution while comparing various charts. The *ARL* values are calculated under the assumption that the standard deviation of the distributions is one when the process is in-control. As in Bakir (2004), the scale parameter is set to be $\lambda = 1/\sqrt{2}$ for the double exponential distribution to achieve a standard deviation of 1. For the Cauchy distribution, $\lambda = 0.2605$ is chosen to achieve a tail probability of 0.05 above $\mu + 1.645$, the same as that of a normal distribution with a mean μ and a standard deviation of 1. The *ARL* values for sign chart and synthetic sign chart are exact. For S^2 chart under normal distribution, the *ARL* values are exact while under other distributions they are simulated. The *ARL* values of moving average sign chart are simulated based on 10,000 runs for samples of sizes n=7 and n=10.

Under normal distribution, the *ARL* of S^2 chart for shift δ in σ is

$$ARL(\delta) = 1/pr(S^2 \geq UCL) \quad , \quad (5)$$

where $pr(S^2 \geq UCL) = pr(\chi^2_{n-1} \geq \frac{(n-1)UCL}{\sigma^2} | \sigma = \delta\sigma_0)$

The upper control limit *UCL* is determined using equation (5) for desired in-control *ARL=ARL(0)* when the process is in control ($\sigma = \sigma_0$).

The performance of different charts is compared by equating their in-control *ARL*s. For that we computed the adjusted *ARL* of chart B to match with in-control ARL of the chart A, using following formula

$$[ARL(\delta)]_B = \frac{[ARL(\delta)]_B}{[ARL(0)]_B} \times [ARL(0)]_A.$$

In Table 1, we provide in-control ARL values for various values of Upper Control Limits (d), and span w, when n=7. These values remain same for all the continuous distributions as long as the process standard deviation is 1. It is clear that in-control ARL increases as d or w increases.

Table 2-Table 5 provide ARL comparison of four charts namely, Moving Average (MA), Synthetic sign, Sign and S^2 Charts for n=7. We consider three values of w; the span of MA chart. Table 6- Table 9 provide ARL comparison of the same charts when n=10.

Table 1: In control *ARL* values of Moving Average Chart under normal distribution when n=7.

w=2		w=3		w=4	
d	ARL(0)	d	ARL(0)	d	ARL(0)
4	40.64	3.00	30.35	3.0	70.03
5	160.63	3.67	96.86	3.5	209.5
6	1128.84	4.33	319.68	4.0	657.48

Table 2: *ARL* Performance of M. A., Synthetic, Sign and S^2 Chart under normal distribution (n=7, *ARL(0)* = 128).

$\frac{\sigma_1}{\sigma_0}$	MA Chart			Synthetic Chart	Sign Chart	S^2 Chart
	w=2 d=5	w=3 d=4.33	w=4 d=3.5	d=5 L=2	d=7	UCL= 2.906
1.0	128.00	128.00	128.00	128.00	128.00	128.00
1.1	58.72	47.49	47.50	58.72	74.92	39.39
1.2	33.18	22.83	21.95	31.94	48.67	16.77
1.3	19.99	12.69	12.40	19.68	34.15	8.93
1.4	13.54	8.32	8.13	13.29	25.40	5.57
1.5	9.91	5.61	5.75	9.64	19.76	3.89
1.6	7.50	4.19	4.31	7.38	15.93	2.95
1.7	6.09	3.28	3.45	5.90	13.22	2.38
1.8	5.01	2.61	2.87	4.88	11.23	2.02
1.9	4.26	2.21	2.44	4.16	9.72	1.77
2.0	3.70	1.91	2.14	3.61	8.55	1.59

Table 3: *ARL* Performance of M. A., Synthetic, Sign and S^2 Chart under double exponential distribution (n=7, *ARL(0)* = 128).

$\frac{\sigma_1}{\sigma_0}$	MA Chart			Synthetic Chart	Sign Chart	S^2 Chart
	w=2 d=5	w=3 d=4.33	w=4 d=3.5	d=5 L=2	d=7	UCL= 4.517
1.0	128.00	128.00	128.00	128.00	128.00	128.00
1.1	67.93	56.50	56.04	67.25	82.35	61.10
1.2	39.93	29.72	29.21	39.84	57.02	33.05
1.3	26.18	17.97	17.31	25.88	41.78	20.54
1.4	18.30	11.72	11.33	18.04	32.00	13.30
1.5	13.62	8.04	8.06	13.29	25.40	9.49
1.6	10.66	5.96	6.16	10.25	20.75	7.22
1.7	8.27	4.78	4.74	8.21	17.36	5.54
1.8	6.90	3.78	4.01	6.76	14.81	4.47
1.9	5.96	3.16	3.34	5.71	12.85	3.78
2.0	4.98	2.62	2.93	4.93	11.31	3.30

Table 4: *ARL* Performance of M. A., Synthetic, Sign and S^2 Chart under gamma distribution (n=7, *ARL(0)* = 128).

$\frac{\sigma_1}{\sigma_0}$	MA Chart			Synthetic Chart	Sign Chart	S^2 Chart
	w=2 d=5	w=3 d=4.33	w=4 d=3.5	d=5 L=2	d=7	UCL= 3.869
1.0	128.00	128.00	128.00	128.00	128.00	128.00
1.1	59.17	47.59	46.09	57.89	74.19	49.88
1.2	31.73	21.99	21.24	30.98	47.61	24.71
1.3	19.44	12.07	11.66	18.77	32.98	14.49
1.4	12.76	7.67	7.59	12.49	24.21	9.52
1.5	9.12	5.14	5.25	8.93	18.60	6.90
1.6	6.88	3.75	4.04	6.76	14.81	5.25
1.7	5.50	2.92	3.13	5.35	12.15	4.38
1.8	4.42	2.35	2.58	4.38	10.20	3.65
1.9	3.78	1.91	2.19	3.70	8.74	3.21
2.0	3.16	1.67	1.89	3.20	7.62	2.87

Table 5: *ARL* Performance of M. A., Synthetic, Sign and S^2 Chart under Cauchy distribution (n=7, *ARL(0)* = 128).

$\frac{\sigma_1}{\sigma_0}$	MA Chart			Synthetic Chart	Sign Chart
	w=2 d=5	w=3 d=4.33	w=4 d=3.5	d=5 L=2	d=7
1.0	128.00	128.00	128.00	128.00	128.00
1.1	70.74	59.78	59.53	70.17	84.80
1.2	43.29	31.73	31.69	42.39	59.58
1.3	28.21	19.63	18.69	27.71	43.91
1.4	19.59	12.68	12.42	19.29	33.65
1.5	14.47	8.79	8.71	14.15	26.64
1.6	11.11	6.37	6.53	10.84	21.68
1.7	8.88	4.93	5.10	8.60	18.04
1.8	7.32	4.02	4.19	7.03	15.31
1.9	6.00	3.30	3.43	5.90	13.22
2.0	5.11	2.72	2.96	5.06	11.57

Table 6: *ARL* Performance of M. A., Synthetic, Sign and S^2 Chart under Normal distribution (n=10, *ARL(0)* = 171.88).

$\frac{\sigma_1}{\sigma_0}$	MA chart			Synthetic Chart	Sign Chart	S^2 Chart
	w=2 d=6	w=3 d=4.33	w=4 d=4	d=6 L=2	d=10	UCL= 2.5748
1.0	171.88	171.88	171.88	171.88	171.88	171.88
1.1	69.24	61.38	50.28	66.54	79.97	41.78
1.2	33.73	29.30	21.92	32.11	43.18	15.39
1.3	19.47	16.37	11.44	18.16	26.03	7.52
1.4	12.72	10.79	7.23	11.56	17.05	4.48
1.5	8.72	7.46	4.98	8.04	11.91	3.06
1.6	6.60	5.78	3.82	5.98	8.76	2.31
1.7	5.22	4.68	3.05	4.69	6.71	1.88
1.8	4.33	3.87	2.58	3.84	5.31	1.61
1.9	3.64	3.35	2.29	3.25	4.32	1.43
2.0	3.17	2.96	2.03	2.82	3.60	1.32

Table 7: *ARL* Performance of M. A., Synthetic, Sign and S^2 Chart under double exponential distribution (n=10, *ARL(0)* = 171.88).

$\frac{\sigma_1}{\sigma_0}$	MA chart			Synthetic Chart	Sign Chart	S^2 Chart
	w=2 d=6	w=3 d=4.33	w=4 d=4	d=6 L=2	d=10	UCL= 4.022
1.0	171.88	171.88	171.88	171.88	171.88	171.88
1.1	80.55	72.22	62.63	78.42	91.53	71.15
1.2	43.64	38.29	29.79	41.78	54.14	35.73
1.3	26.53	22.37	16.24	25.03	34.72	20.23
1.4	17.14	14.97	10.33	16.42	23.72	12.53
1.5	12.27	10.51	7.31	11.56	17.05	8.49
1.6	9.55	8.32	5.33	8.61	12.78	6.12
1.7	7.40	6.60	4.28	6.71	9.90	4.65
1.8	5.97	5.31	3.55	5.43	7.89	3.76
1.9	4.97	4.55	2.95	4.53	6.45	3.06
2.0	4.27	3.93	2.63	3.87	5.37	2.60

Table 8: *ARL* Performance of M. A., Synthetic, Sign and S^2 Chart under gamma distribution (n=10, *ARL(0)* = 171.88).

$\frac{\sigma_1}{\sigma_0}$	MA chart			Synthetic Chart	Sign Chart	S^2 Chart
	w=2 d=6	w=3 d=4.33	w=4 d=4	d=6 L=2	d=10	UCL= 3.447
1.0	171.88	171.88	171.88	171.88	171.88	171.88
1.1	67.38	60.80	50.45	65.42	78.85	56.41
1.2	32.70	27.82	20.52	30.97	41.85	24.47
1.3	18.46	15.71	10.73	17.20	24.76	13.43
1.4	11.73	10.14	6.61	10.77	15.92	8.42
1.5	8.09	6.96	4.56	7.38	10.93	5.85
1.6	5.99	5.41	3.55	5.43	7.89	4.46
1.7	4.52	4.21	2.85	4.22	5.95	3.56
1.8	3.77	3.56	2.39	3.43	4.64	3.03
1.9	3.17	3.04	2.04	2.89	3.72	2.64
2.0	2.76	2.67	1.84	2.50	3.05	2.34

Table 9: *ARL* Performance of M. A., Synthetic, Sign and S^2 Chart under Cauchy distribution (n=10, *ARL(0)* = 171.88).

$\frac{\sigma_1}{\sigma_0}$	MA chart			Synthetic Chart	Sign Chart
	w=2 d=6	w=3 d=4.33	w=4 d=4	d=6 L=2	d=10
1.0	171.88	171.88	171.88	171.88	171.88
1.1	85.29	78.40	66.70	82.55	95.45
1.2	47.22	41.49	32.10	44.99	57.65
1.3	28.72	24.84	17.89	27.13	37.28
1.4	19.31	16.33	11.22	17.75	25.49
1.5	13.52	11.57	7.81	12.42	18.26
1.6	10.00	8.62	5.74	9.17	13.60
1.7	7.69	6.68	4.44	7.08	10.46
1.8	6.23	5.52	3.66	5.67	8.28
1.9	5.19	4.61	3.09	4.69	6.71
2.0	4.39	4.04	2.69	3.98	5.55

CONCLUSIONS

- The synthetic sign chart performs better than the sign chart for all the process distributions considered here. However the performance of the MA chart is better than the synthetic chart for all the process distributions when the span of the moving average $w \geq 3$. The ARL performance of the MA chart goes on increasing as the span of the MA chart increases.
- The MA chart with w=4 performs better than the S^2 chart except for the normal distribution. Though the performance of the S^2 chart is superior to all the other charts for the normal distribution, it is possible to
- choose an appropriate w so as to match the ARL performance of MA chart with the S^2 chart.
- The MA chart performs better even for small sample size like n=7 and the performance improves as sample size increases.

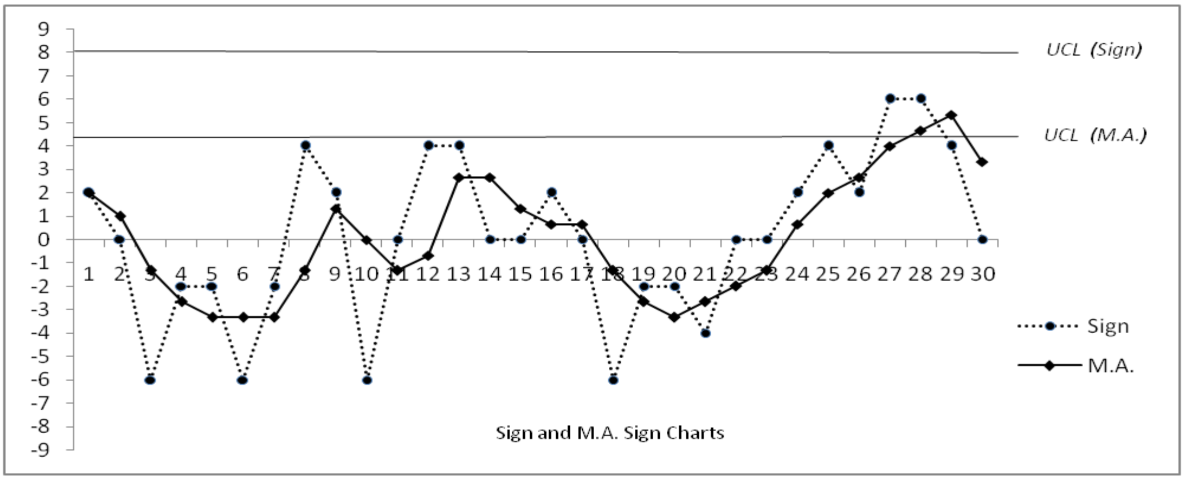
VI ILLUSTRATIVE EXAMPLE

To illustrate the construction of the proposed nonparametric moving average control chart for variability, we use the simulated data. Twenty samples of size ten each were randomly generated from normal distribution with mean zero and in-control standard deviation σ_0= 1 and then, ten samples of size ten each were randomly generated from normal distribution with mean zero and standard deviation σ_1= 1.2. Upper control limit of MA chart for i < w(=3) is not shown in Figure 1. These control limits decrease as the value of w increases. In the present illustration these limits have no effect as far as status of the process is concerned at the beginning.

The values of Moving Average Statistics were calculated with span of moving average *w =2, 3, 4* for each sample. In Figure 1, we plotted sign chart using upper control limit *d = 8* and moving average chart with span *w = 3* using upper control limit *UCL = 4.33*. It is observed that the sign chart does not signal at all but the moving average chart signals at sample numbers 28 and 29.

Table 10: Values of Sign and Moving Average Statistics for the samples in the example.

Sample No.	Sign	MA W=2	MA W=3	MA W=4	Sample No.	Sign	MA W=2	MA W=3	MA W=4
1	2	2	2.00	2.00	16	2	1	0.67	1.50
2	0	1	1.00	1.00	17	0	1	0.67	0.50
3	-6	-3	-1.33	-1.33	18	-6	-3	-1.33	-1.00
4	-2	-4	-2.67	-1.50	19	-2	-4	-2.67	-1.50
5	-2	-2	-3.33	-2.50	20	-2	-2	-3.33	-2.50
6	-6	-4	-3.33	-4.00	21	-4	-3	-2.67	-3.50
7	-2	-4	-3.33	-3.00	22	0	-2	-2.00	-2.00
8	4	1	-1.33	-1.50	23	0	0	-1.33	-1.50
9	2	3	1.33	-0.50	24	2	1	0.67	-0.50
10	-6	-2	0.00	-0.50	25	4	3	2.00	1.50
11	0	-3	-1.33	0.00	26	2	3	2.67	2.00
12	4	2	-0.67	0.00	27	6	4	4.00	3.50
13	4	4	2.67	0.50	28	6	6	**4.67**	**4.50**
14	0	2	2.67	2.00	29	4	5	5.33	4.50
15	0	0	1.33	2.00	30	0	2	3.33	4.00



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