Numerical Analysis of Cam Follower Mechanism And Effect of its Physical Parameter

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Abstract— **Cam and follower mechanism is modelled numerically. The contact force is analysed with considering the asperity interaction. Reynolds equation is applied for analysing the film thickness and the effect of physical parameter like cam radius, nose flank, cam angle is studied. How the film associated with the speed and frequency of the cam follower mechanism is analysed.**

Keywords— Contact force; Reynolds equation; hydrodynamic pressure; fluid film thikness; asperity interaction

1. INTRODUCTION

We know the availability of the organic fuel is limited so manufacture always tries to improve the efficiency of the IC engine. There are many losses in the IC engine. To reduce the power loss a fundamental understanding of the lubrication model is necessary. Theoretical lubrication model is known as mixed lubrication which includes the effect of surface elastic deformation, asperity interaction and squeeze film action.

Theoretical results say that when engine speed and load increased the noise level also increased. Bishop in 1950-51 given a theoretical model. He said that a rate of change of velocity is responsible for the noise. He proposed an improved cam profile model by considering the discontinuity of acceleration. He defined the profile by sine wave. Due to discontinuity of acceleration the efficiency of engine increased. The type of failure usually found in cam and follower mechanism is pitting, scuffing and wear of polish. Due to variation of load the Hertzian stress also developed in the cam which causes the failure of nose of cam. Hertizian stress in the cam should be less than the critical stress. It also depends upon the property of material. S. Carra, R. Garziera and M. Pellegrini[1] mentioned in their paper that by using cam mechanism, a simple machine can be designed with maximum force and negative roller-follower radius. They represented that the pressure angle is easily evaluated and because of pressure angle, the mechanism is practical and economic with restricting the space. Hua Qiu, Chang-Jun Lin, Zi-Ye Li, Hiroaki Ozaki, Jian Wang and Yong Yue[2] were proposed optimal technique for designing the cam curve. Both dynamic and static optimization was done at the same time because of the newly proposed technique which can be used to control the vibration. Long-Iong Wu, Wen-Tung Chang and Chun-Hsien Liu[3] were mentioned that the follower motion and other cam parameter were analyzed by giving velocity to the cam-follower mechanism. They concluded that the cam profile may concave, convex and flat in the upper portion because of the variation of the velocity. Zhiliang Qian[4] was represented that the analysis and calculation of the value of all the angles are in the cam-follower mechanism with considering of transmission angle having constant cam diameter were done. Livija Cveticanin[5] was done the analysis of the dynamic behavior in cam-follower mechanism with the consideration of the non-linear properties. It was investigated through numerical examples. The stability of the cam-follower mechanism was checked. E.E. Zayas, S. Cardona and L. Jordi[6] were represented that how the displacement is calculated in case of constant breadth cam mechanism. The displacement may translational or oscillating. it is obtained by some numerical analysis. T.K. Naskar and S. Acharyya[8] were analyzed experimentally the dynamic behavior of the mechanism and they compared the result experimentally and theoretically. They concluded that the experimental and theoretical results are approximately equal and it shows that the system was elastically deformed due to the stress developed. Yan-an Yao, Ce Zhang and Hong-Sen Yan[13] were investigating the properties to control the motion because of the speed. They concluded that the motion properties of the follower are performed better with the increase in the input speed. Wen-Tung Chang and Long-Iong Wu[15] were represented that how the tools are dealt for analyzing and synthesizing the error for designing the linkages. By analysis, it is concluded that the tools are used to design or manufacture of the linkage with minimum error. Hong-Sen Yan and Wen-Teng Chang[16] were defining the surface for the system. The curvature of the mechanism was analyzed with considering limit conditions. After the analysis, they concluded that the hyperbolical and globoidal surfaces are suitable for the roller-follower mechanism.

2. MOTION MECHANISM OF CAM AND FOLLOWER:

We assume that the situation in which the contact rising flank of cam. This is part number 1. Cam rotate about O and O' is the centre of curvature of cam surface at point C.

Therefore the absolute velocity in the X, Y direction will be,

$$u_2 = 0 \qquad \text{……………. (1)}$$

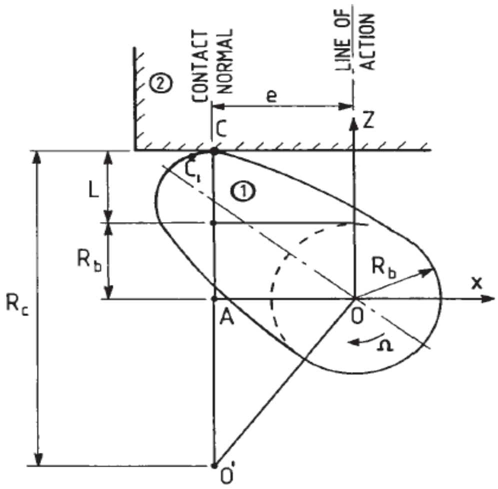

Fig.1. schematic diagram of cam and follower

$$v_2 = \frac{dL}{dt} = c\Omega \quad \text{……………(2)}$$

If 'a' is the acceleration then,

$$a = \frac{d^2 L}{dt^2} = \Omega \frac{dc}{dt} \quad \text{…………(3)}$$

Velocity in X-direction in contact with follower,

$$u_1 = (R_b + L)\Omega \quad \text{……….(4)}$$

Velocity of contact point along cam surface (ds/dt),

$$\frac{ds}{dt} = u_1 + \frac{de}{dt} = (R_b + L)\Omega + \frac{de}{dt} \quad \text{……….(5)}$$

$$R_c = \frac{ds}{dt} \cdot \frac{dt}{d\gamma} = \frac{ds}{d\gamma}$$

If the surface of follower is flat,

$$R_c = \frac{1}{\Omega} \cdot \frac{ds}{dt} \quad \text{………………….(6)}$$

Combining equation (3),(5) and (6),

$$R_c = R_b + L + \frac{de}{dt} = R_b + L + \frac{a}{\Omega^2} \quad \text{……………………. . (7)}$$

3. LOAD IN CONTACT MECHANISM:

Experimentally the calculation of load carried out by the cam and follower is very complex. Simply we can say that the force generated in the cam follower is the inertia force. The spring load generated because of relative motion the parts which are in contact. This force affects the stiffness, damping co-efficient and also try to deform the structure. There is certain assumption for the calculation of contact force like

(1) Friction force is neglected.
(2) Mechanism is rigid.
(3) And damping co-efficient, stiffness is omitted.

Product of mass and acceleration gives the inertia force of the mechanism. If the total mass is 'M_e' then,

$$M_e = M + \frac{m}{3} \quad \text{……….(8)}$$

I= inertia force

$$I = M_e \times a \quad \text{…………… (9)}$$

Spring force is 's'

$$s = Kd \quad \text{At stationary}$$

$$s = K(L + d)$$

When the deflection is equal to (L)
d= initial compression of the spring.
Therefore the total force on the cam is

$$W = I + S + M_e \cdot g \quad \text{…………..(10)}$$

4. REYNOLDS EQUATION:

The lubricated cam mechanism can be treated as cylinder acting on a plane where the cylinder is infinitely wide.
Assumption:
(1) Neglect side leakage.
(2) Lubricant density does not change by temperature and pressure.
1-D Reynolds equation for incompressible fluid:

$$\frac{d}{dx}\left[\frac{h^3}{12\eta}\frac{dp}{dx}\right] = u\frac{dh}{dx} + v_1 + v_2 + u_2 - \frac{dz_2}{dx} = -u_1\frac{dz_1}{dx} \quad \text{……………..(11)}$$

Where,

$$u = \frac{u_1 + u_2}{2}$$

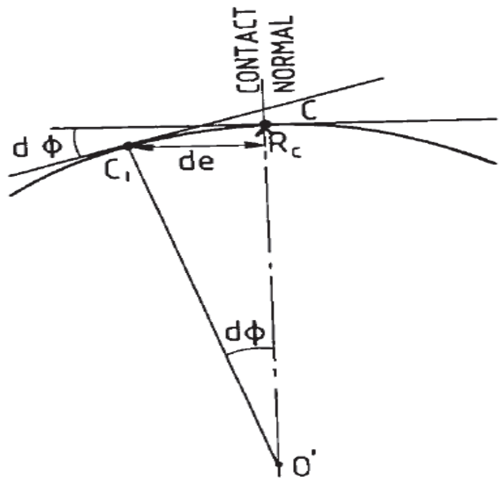

Fig.2.cam and follower as a cylinder and plane

$$v_1 = u_1\frac{dh}{dx} + \frac{\partial h}{\partial t} \qquad v_2 = 0$$

$$\frac{dz_1}{dx} = \frac{dh}{dx} \qquad \frac{dz_2}{dx} = 0$$

Equation (11) became,

$$\frac{d}{dx}\left[\frac{h^3}{12\eta}\frac{dp}{dx}\right] = u\frac{dh}{dx} + \frac{\partial h}{\partial t}$$
………(12)

Boundary condition for the damaging of film,

$p(-\infty) = 0$ and $\frac{dp}{dx} = (x_m) = p(x_m) = 0$

The assumption is valid when the profile is parabolic in nature and the magnitude of $\frac{x}{R_c}$ is small compared to unity.

$$h = h_0 + R_c - R_c\left[1 - \left(\frac{x}{R_c}\right)^2\right]^{\frac{1}{2}}$$

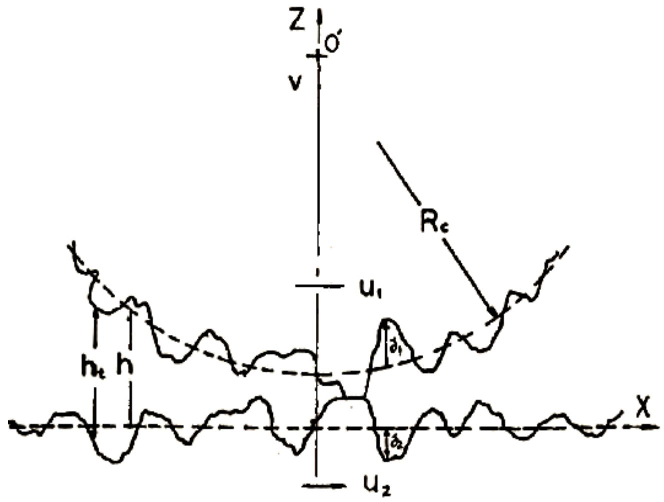

Fig.3. asperity interaction of cam follower

Expand the bracket and neglect the fourth and higher power term,

$$h = h_0 + \frac{x^2}{2R_c}$$ …………..(13)

5. ANALYSIS OF CAM AND FOLLOWER BASED ON ELASTICITY APPROACH:

In operating condition the cam and follower surface exerts the heavy load. Due to heavy load the surface of cam and follower deformed elastically.

From fig (2), Elastic deformation at a point at distance x from origin on the surface subjected to non linear pressure p(s) between $s = -s_1$ and $s = s_1$

$$v(x) = -\frac{2}{\Pi E'}\int_{-s_1}^{s_1} p(s)\ln(x-s)^2 \cdot ds + C$$
……..…..(14)

6. RELATIONSHIP BETWEEN PRESSURE AND VISCOSITY:

Viscosity changes as change in pressure. For isothermal condition the pressure and viscosity relationship,

$$\eta = \eta_0 \exp(\alpha p)$$ ……………….(15)

α =pressure viscosity constant

η_0 = reference viscosity at atm.

When the pressure is very high,

$$n = \begin{cases} \eta_0 \exp(\alpha p) \\ \eta_0 \exp[\alpha p_1 + \beta(p - p_1)] \end{cases}$$ ……..(16)

for $P > P_1$ and $P \le P_1$

β= pressure viscosity co-efficient

When we apply the Reynolds equation in equation (16), the result non linear which cannot be solved by simple integration, so an another parameter taken into consideration known as reduced pressure (q)

$$q = \begin{cases} \frac{1}{\alpha}[1 - \exp(-\alpha p)] \\ \frac{1}{\beta}[1 - \exp(-\{\alpha p_1 + \beta(p - p_1)\}] + \frac{\beta - \alpha}{\alpha\beta}[1 - \exp(-\alpha p_1)] \end{cases}$$

for $P \le P_1$ and $P > P_1$

Replacing 'p' in equation (12)

$$\frac{d}{dh}\left(h^3\frac{dq}{dx}\right) = 12\eta_0\left(u\frac{dh}{dx} + \frac{\partial h}{\partial t}\right)$$
…….(17)

For isoviscous lubricant,

'q' in the solution of (17) became ,

$$p = \begin{cases} -\frac{1}{\alpha}\ln[1 - \alpha q] \\ p_1 - \frac{1}{\beta}\ln[1 - \beta(q - q_1)\exp(\alpha p_1)] \end{cases}$$ ……….(18)

For $q \le q_1$ and $q > q_1$

Where, $q_1 = \frac{1}{\alpha}[1 - \exp(-\alpha p_1)]$

7. AVERAGE REYNOLDS EQUATION:

In operating condition the cam follower is treated as the rough cylinder with rough plane. For rough surface the Reynolds equation becomes,

$$\frac{d}{dx}\left(\phi_x\frac{h^3}{12\eta}\frac{d\bar{p}}{dx}\right) = \frac{(u_1 + u_2)}{2}\frac{d\bar{h}_1}{dx} + \frac{u_2 - u_1}{2}\sigma\frac{d\phi_s}{dx} + \frac{\partial\bar{h}}{\partial t}$$
…………(19)

Boundary condition

$$\frac{d\bar{p}}{dx}(x_m)=\bar{p}(x_m)=0, \bar{p}(-\infty)=0$$

And $h_1 = h + \delta_1 + \delta_2$

$\phi_x, \phi_s =$ Empirical pressure and shear flow factor.

ϕ_s Can be calculated by pressure gradient $\frac{\partial p}{\partial x}$

Pressure flow of bearing,

$$\frac{\partial}{\partial x}\left(\frac{h_t^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{h_t^3}{12\eta}\frac{\partial p}{\partial y}\right)=0 \quad \text{...........(20)}$$

Boundary condition

$$p(0,y)=p_a, p(L_x,y)=p_b$$

$$\frac{\partial p}{\partial y}(x,o)=\frac{\partial p}{\partial y}(x,L_y)=0$$

And no flow at contact.

Solving equation (20) numerically,

$$\phi_s=\left(\frac{1}{L_y}\int_0^l y\left(\frac{h_t^3}{12\eta}\frac{\partial p}{\partial x}\right)dy\right)+\left(\frac{h^3}{12\eta}\frac{\partial\bar{p}}{\partial x}\right)$$

Where $\frac{\partial\bar{p}}{\partial x}=\frac{p_b-p_a}{L_x}$

For two parallel surface,

$$\frac{\partial}{\partial x}\left(\frac{h_t^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{h_t^3}{12\eta}\frac{\partial p}{\partial y}\right)=\frac{\partial h_t}{\partial t}$$

Boundary condition,

$$p(0,y)=p(L_x,y)=0$$

$$\frac{\partial p}{\partial y}(x,0)=\frac{\partial p}{\partial y}(x,L_y)=0$$

8. EFFECT OF ROUGHNESS AND SLIDING IN NET FLOW:

$$\phi_x=\left\langle-\frac{h_t^3}{12\eta}\frac{\partial p}{\partial x}\right\rangle=\frac{1}{L_xL_y}\int_0^{L_y}\int_0^{L_x}\left(-\frac{h_t^3}{12\eta}\frac{\partial p}{\partial x}\right)dxdy$$

Mean flow Q_x due to sliding,

$$\phi_s=\frac{2}{\sigma U_s}E\left\langle-\frac{h_t^3}{12\eta}\frac{\partial p}{\partial x}\right\rangle$$

Where $U_s=\frac{u_2-u_1}{2}$

9. ASPERITY HEIGHT BETWEEN THE SURFACE OF CAM AND FOLLOWER IN CONTACT

Asperity contact force,

$$W_a=\frac{16}{15}\sqrt{2}\Pi(\eta\beta\sigma)^2E'\left(\frac{\sigma}{\beta}\right)^{\frac{1}{2}}AF_{\frac{5}{2}}\left(\frac{h}{\sigma}\right)$$

A_t=A= apparent area ,

$$AF_{\frac{5}{2}}\left(\frac{h}{a}\right)=W\int_{-\infty}^{\infty}F_{\frac{5}{2}}\left[\frac{h}{\sigma}\right]dx \quad \text{..........(26)}$$

$$A_t=\Pi^2(\eta\beta\sigma)^2AF_2\left[\frac{h}{\sigma}\right]$$

$$AF_2\left[\frac{h}{\sigma}\right]=W\int_{-\infty}^{\infty}F_2\left[\frac{h}{\sigma}\right]dx$$

10. NUMERICAL SIMULATION RESULTS:

Table-1

Rotation of cam shaft	Min film thickness	
Frequency	Nose	Flank
1200rpm (20Hz)	0.095 μm	2.16 μm
3000rpm(50Hz)	0.20 μm	3.46 μm

i. variation of film thickness with base circle radius

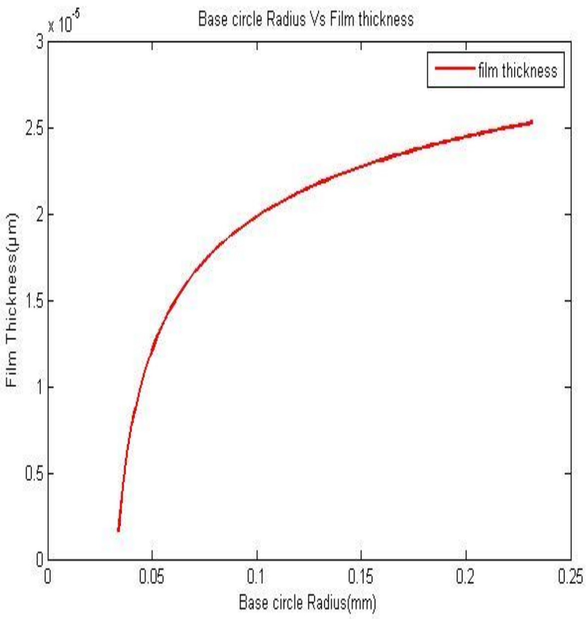

Fig.4 base circle vs film thickness.

ii. Variation of cam radius with cam angle

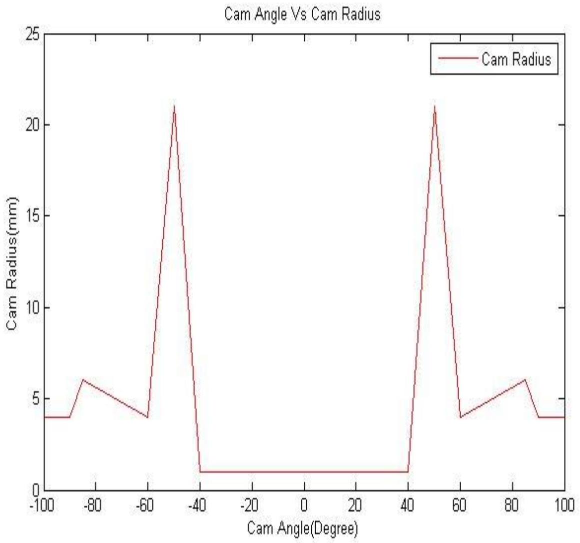

Fig.5. cam angle vs cam radius

iii. Variation of velocity with cam angle:

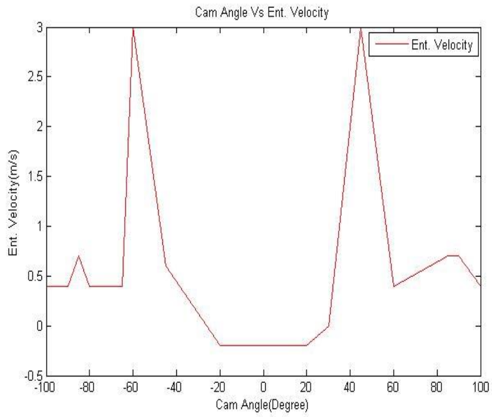

Fig.6. cam angle vs velocity

iv. Variation of cam angle with thickness:

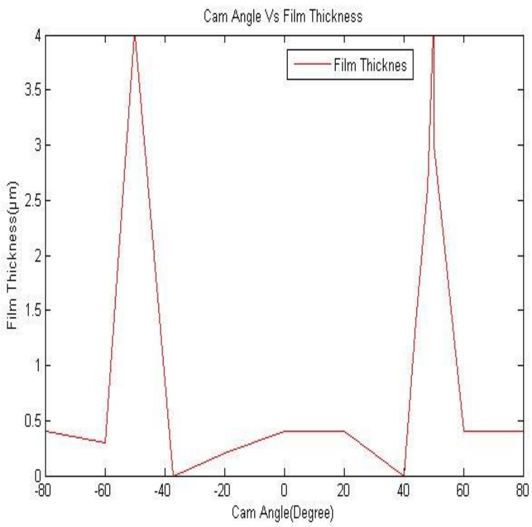

Fig.7. cam angle vs film thickness

11. CONCLUSION:

The cam and follower is solved numerically and the data is plotted between various physical parameter and it is found that the initially the film thickness is lower and gradually it goes on increasing and after some time it became stable with change in base circle radius, means lubricant film thickness is not only depend upon the base circle radius, nose radius and frequency which is clearly shown in table (1) There will be good lubrication in the cam follower mechanism when base circle increase. it gives more efficiency compare to lower base circle radius cam.cam angle vs. cam radius graph is plotted between -100 degree to 100 degree, cam angle and found to be maximum at -40 degree and 40 degree. It shows there should be a lower cam angle compare to the cam radius for easy working of cam. The velocity is also important parameter of the cam follower mechanism, when the cam angle increases there will be sudden increase in the velocity at certain point and it gives the jerk to the system. Film thickness also get maximum at certain interval when cam angle attain maximum. So effect of physical parameter on cam and follower is analyzed. All the above validated the classical result of design of cam and follower motion. So design consideration is the main parameter for cam and follower mechanism, mainly its efficiency depend upon the cam radius and the lubricant film thickness.

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