Numerical Simulation of Supersonic Expansion in Conical and Contour Nozzle

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Abstract: **Supersonic flow through the rocket nozzle has been simulated using numerical method. The parameters like Mach number, static pressure and shocks are observed for conical and contour nozzles using axi-symmetric model in ANSYS FLUENT 14® software. The occurrences of shocks for the conical nozzles were observed along with the other parameters for various divergent angles. The parameters under observation are compared with that of contour nozzle for respective divergent angles by maintaining the inlet, outlet and throat diameter and lengths of convergent and divergent portions as same. The convergent portion and throat diameter are kept constant across the cases. The phenomenon of shock was visualized and the results showed close resemblance in formation of Mach disk and its reflection patterns as reported in various experimental studies on expansion in conical CD nozzles with lower divergent angles. No occurrence of shocks is observed with higher divergent angles. Results depicted higher exit velocity and higher degree of flow separation with contour nozzles compared to that of with corresponding conical nozzles.**

Keyword: Rocket nozzle, FVM, shock, Mach Disk, Mach number, sonic, sub-sonic, super-sonic, conical nozzle, contour nozzle.

Nomenclature: Ae=Nozzle exit area in m2; At=Nozzle throat area in m2; a= Ambient conditions; m=Mass flow rate Kg/s; M=Mach number; α= Semi divergence angle; T= Total temperature in K; r=Local Radius in m; V= Actual velocity in m/s; C= Sound Velocity in m/s.

1. INTRODUCTION

A rocket engine is a jet engine that uses specific propellant mass for forming high speed propulsive exhaust jet. Rocket engines are reaction engines and obtain thrust in accordance with Newton's third law. Rocket engines produce thrust by creating a high-speed fluid exhaust. This fluid is generally always a gas which is created by high pressure (10 - 200bar) combustion of solid or liquid propellants, consisting of fuel and oxidizer components, within a combustion chamber. The fluid exhaust is then passed through a supersonic propelling nozzle which uses heat energy of the gas to accelerate the exhaust gases to a very high speed, and the reaction to this pushes the engine in the opposite direction. Hence the major function of rocket nozzle is to give the maximum outlet velocity and meet the thrust requirements, for this to be accomplished the divergent angle must be optimally set considering occurrence of shocks and phenomenon of flow separation at the walls.

To overcome the difficulty in experimental methods, numerical methods are adopted and the transport equations are solved mathematically using software assistance. The problems are simulated using CFD techniques in ANSYS FLUENT 14 software and the transport equations are mathematically solved. Flow instabilities if created due to formation of shocks would reduce the exit Mach number as well as the thrust of the engine. The performance thus depends on the divergent angle and hence the conical nozzle has been thus far tested at 7˚, 13˚and 15˚ divergent angles keeping the inlet and outlet diameters same and dependent parameters such as the Mach number, static pressure and shocks are observed. The geometry of the conical nozzle have adopted in similarity with the reference [2]. The sectional profile is also changed to observe the performance by keeping the inlet, outlet and throat diameters and lengths of convergent and divergent portions same, the parameters under observation are captured for contour nozzles. Creation of contours in converging and diverging portions of the nozzle are done using third order polynomials and the contour profile of the convergent portion is retained the same across the cases. Cases were solved by varying the contour profile of the diverging portion with an extent to 5% on either side of the 2nd and 3rd order coefficients of the polynomial and the case resulting in producing best Mach number is presented in each of the configurations. The simulated results obtained on conical nozzle are in acceptable proximity with the results presented in few of the CFD works [2]. Results visualized on conical nozzles with lesser divergent angles resembled with experimental Schlieren photographs and averaged shadowgraph images of supersonic expansion through CD nozzles [3][4].

2. MATHEMATICAL MODEL

The Mathematical model used for present work is one of Reynolds Average Navier- Stoke (RANS) model called the K-ε model a widely used transport model and the equation for turbulent kinetic energy K is:

$$\frac{DK}{Dt} = \frac{\partial K}{\partial t} + u_j \frac{\partial K}{\partial x_j}\left[\frac{V_t}{\sigma_k}\frac{\partial K}{\partial x_j}\right] + P - \varepsilon$$

= Rate of increase of K+ Convective transport = diffusive transport + Rate of production-Rate of destruction

The model equation for the turbulent dissipation ε is:

$$\frac{D\varepsilon}{Dt} = \frac{\partial \varepsilon}{\partial t} + u_j \frac{\partial \varepsilon}{\partial x_j}\left[\frac{V_t}{\sigma_k}\frac{\partial \varepsilon}{\partial x_j}\right] + C_{\varepsilon 1}\frac{P\varepsilon}{K} - C_{\varepsilon 2}\frac{\varepsilon^2}{K}$$

= Rate of increase of ε+ Convective transport = diffusive transport + Rate of production-Rate of destruction

The standard values of all the model constants are $C\mu = 0.09$; $\sigma k = 1.00$; $\sigma\varepsilon = 1.30$; $C\varepsilon 1 = 1.44$; $C\varepsilon 2 = 1.92$ [1]

Now the Reynolds stresses are found out using: $-\rho u'_i u'_j = \frac{2\rho}{3}K\delta_{ij} + \mu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$

And the eddy-viscosity is evaluated as: $\mu_t = \rho C_\mu \frac{K^2}{\varepsilon}$

3. COMPUTATIONAL PROCEDURE

A 2-D Geometry nozzle was created using ANSYS WORKBENCH ®14 and Analysis was carried out on ANSYS FLUENT ® 14. The dimensions and the boundary condition at inlet as shown in below table 1 were adopted from experimental data used as per the work presented in reference [2].

Inlet width (in m)	1.000
Throat width (m)	0.304
Exit width (m)	0.861
Throat radius of curvature (m)	0.228
Convergent Length (m)	0.640
Convergent angle(°)	30˚
Divergent angle(°)	15˚
Mass flow rate (Kg/s)	826
Inlet Temperature (K)	3400

TABLE 1: DIMENSIONS AND THE BOUNDARY CONDITIONS

The created geometry has been imported to the meshing workbench, meshed using Quad mesh with Map face fine mesh and refined to the third degree using refinement option. Grid independence study has been conducted for two dimensional numerical simulations. Results presented are captured with the grid size with which they were found to be insensitive with further refinements of mesh. The set up of the cases was done as per table 2.

General Setup	Solver type: Density Based Velocity formulation: Absolute Time: Steady 2D Space- Axi-Symmetric
Models	Energy Equation: ON Viscous Models: Standard K-ε model.
Material	Air- Ideal gas
Cell zone boundary condition	Fluid Domain
Boundary condition	Inlet- Mass flow rate-826 Kg/sec Inlet –Temperature-3400 K Axis-Axis boundary condition Outlet- Pressure outlet

TABLE 2: SETUP IN FLUENT

The change in angle gives rise to change in lengths of the diverging portion. The polynomial profile for convergent portion is retained as common for all the divergent angles. The coefficients of polynomials of the respective angles for contour profiles are listed below.

	1st	2nd	3rd	4th
Convergent	1.804	-3.045	0.113	1.001
7˚	-0.040	0.152	-0.015	0.278
13˚	-0.009	-0.029	0.414	0.071
15˚	-0.035	0.074	0.458	0.015

TABLE 3: POLYNOMIAL COEFFICIENTS FOR DIVERGENT CONTOUR PROFILE

4. RESULTS AND DISCUSSIONS

Case1.1: Conical Nozzle - Divergent angle =7°

1.1.1 Mach Number v/s Position

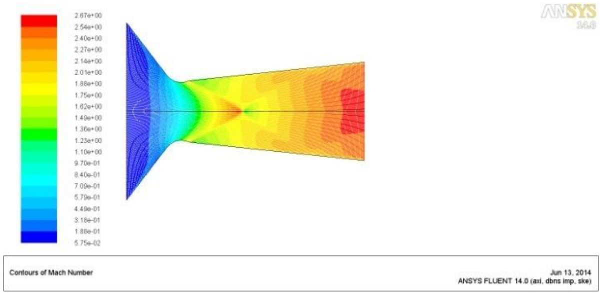
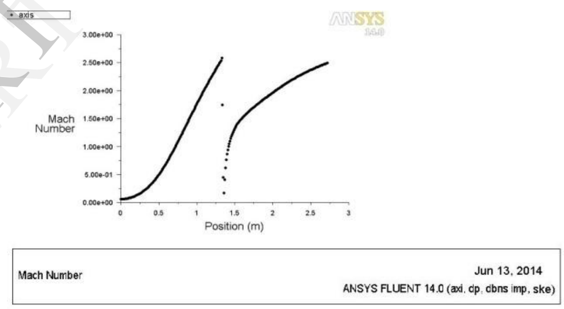


Figure 4.1: Mach number for conical nozzle with 7˚ divergent angle

In this case, from the plot it is evident that a single shock has occurred in contrast with the two shocks of lower magnitudes observed with lower divergent angles. At the throat velocity varies from 9.70e-01 and 1.10Mach. Across the shock the velocity drops from 2.5 Mach to 1.5 Mach, resulting in a marginal increase (2.55Mach) in the velocity at the exit when compared to the velocity before shock. The position of shock can be found the mach plot as in figure 4.1 and Graph 4.1 and it is observed that the shock occurs at 1.25m from the inlet.



Graph 4.1: Plot of Mach number v/s Position

It can also be observed that the position of occurrence of shock has shifted towards the exit when compared to the same with lower divergent angles.

Case1.2: Contour Nozzle – length of the divergent portion same as that of conical nozzle with Divergent angle of 7°.

1.2.1 Mach Number v/s Position

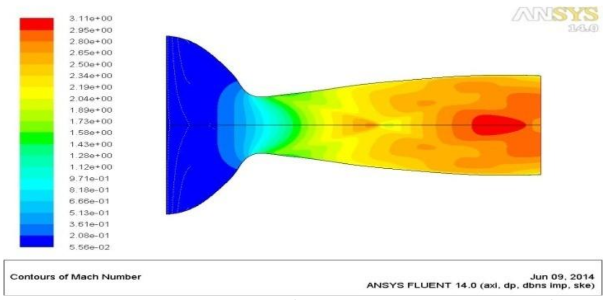
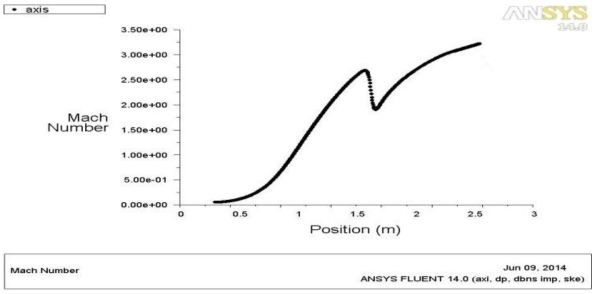


Figure 4.2: Mach number for Contour Nozzle – length of the divergent portion same as that of conical nozzle with Divergent angle of 7°.



Graph 4.2: Plot of Mach number v/s Position

In this case, across the shock the velocity drops from 3Mach to 1.7Mach. It can be observed the there is substantial increase in velocity at the exit when compared to the conical nozzle with same length of divergence portion. The position of shock can be found from the mach plot as in figure 4.2 and graph 4.2 and it is observed that the shock occurs at 1.5m from the inlet. The exit Mach number is found to be 2.95.

1.1.2 Static pressure: Conical nozzle with 7˚ divergent angle

The static pressure is found to be 3.46e+06Pa at the inlet section as observed in figure 4.3. The pressure dropped to about 3.11e+06Pa at the throat section and continues to decrease to a value of 1.85e+06Pa. At the position of shock it has increased to 1.31e+06Pa. Then the static pressure again drops and it reaches a very low value of 4.85e+04Pa at the exit section.

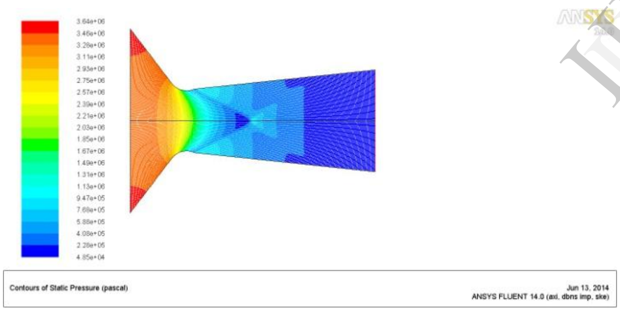


Figure 4.3: Static Pressure for conical nozzle with 7˚ divergent angle

1.2.2 Static Pressure: Contour Nozzle – length of the divergent portion same as that of conical nozzle with Divergent angle of 7°.

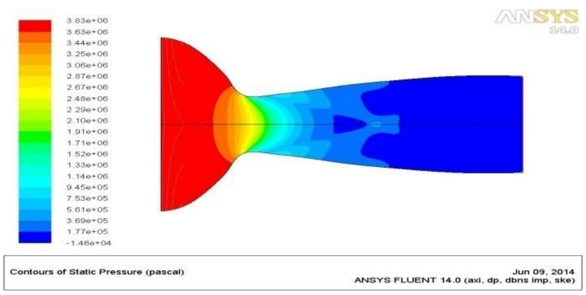


Figure 4.4: Static Pressure for contour nozzle with 7˚ divergent angle

It is observed from figure 4.4 that the pressure dropped from 3.44e+06 to 1.91e+06Pa at the throat section. Up to the position of shock pressure decreases to 1.77e+05 and suddenly increases to 5.61e+05pa. Further it reduces to -1.46e+04 at the exit.

Case2.1: Conical Nozzle - Divergent angle = 13°

2.1.1 Mach Number v/s Position

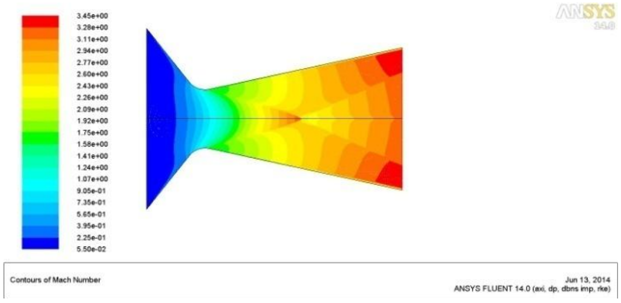
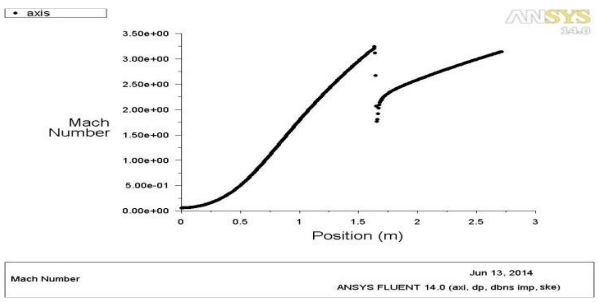


Figure 4.5: Mach number for conical nozzle with 13˚ divergent angle

As seen in figure 4.5, at the throat velocity varies from 2.25e-01 to 9.05e-01 Mach which is lesser in range when compared to 7˚ conical nozzle. Across the shock the velocity drops from 3.28 to 2.43Mach, resulting in a marginal increase (3.32Mach) in the velocity at the exit when compared to the velocity before shock. The position of shock can be found from the mach plot as in fig 4.3 and it is observed that the shock occurs at 1.65m from the inlet. It can also be observed that the position of occurrence of shock has shifted towards the exit when compared to the same with lower divergent angles.



Graph 4.3: Plot of Mach number v/s Position

Case2.2: Contour Nozzle – length of the divergent portion same as that of conical nozzle with Divergent angle of 13°.

2.2.1 Mach Number v/s Position

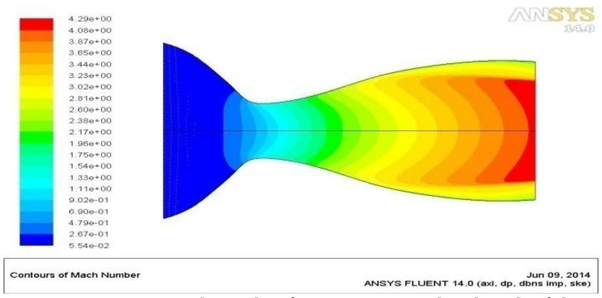
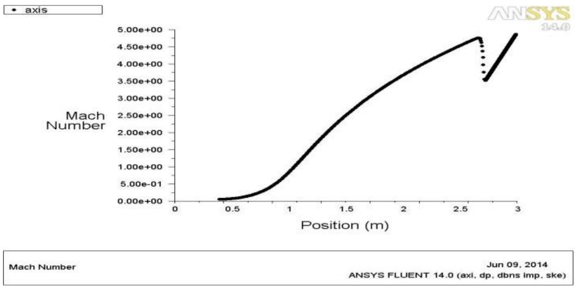


Figure 4.6: Mach number for Contour Nozzle – length of the divergent portion same as that of conical nozzle with Divergent angle of 13°.



Graph 4.4: Plot of Mach number v/s Position

In this case across the shock the velocity drops from 4.6Mach to 3.5Mach. It can be observed from figure 4.6 and graph 4.4 that there is substantial increase in velocity at the exit when compared to the conical nozzle with same length of divergence portion. The position of shock can be found from the mach plot as in fig 4.3 and it is observed that the shock occurs at 2.3m from the inlet. The exit Mach number is found to be 4.29.

2.1.2 Static pressure: Conical nozzle with 13˚ divergent angle

The static pressure is found to be 3.65e+06Pa at the inlet section as seen in Figure 4.7. The pressure dropped from 3.45e+06 to 1.90e+06Pa at the throat section. Further the pressure drops to 1.45e+05 till the occurrence of the shock where it suddenly increases to 9.23e+05. Then the static pressure again drops and it reaches a very low value of -4.93e+04Pa at the exit section.

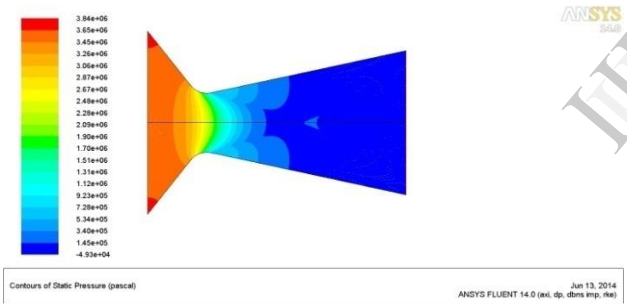


Figure 4.7: Static Pressure for conical nozzle with 13˚ divergent angle

2.2.2 Static Pressure: Contour Nozzle – length of the divergent portion same as that of conical nozzle with Divergent angle of 13°.

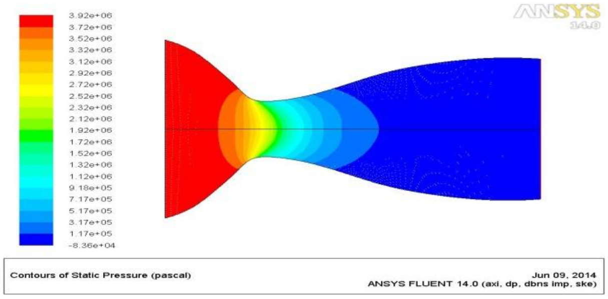


Figure 4.8: Static Pressure for contour nozzle with 13˚ divergent angle

It can be observed from figure 4.8 that the pressure dropped from 3.72e+06 to 1.92e+06Pa at the throat section. With no occurrence of shock, the pressure drops further to -8.36e+05 till the exit.

Case3.1: Conical Nozzle - Divergent angle = 15°

3.1.1 Mach Number v/s Position

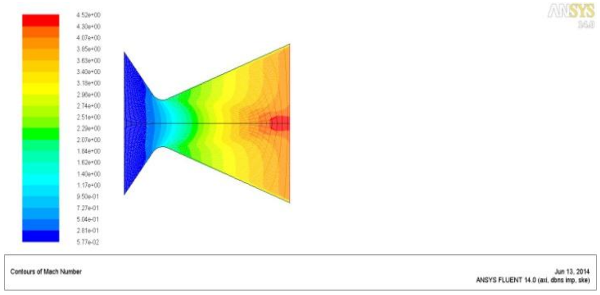
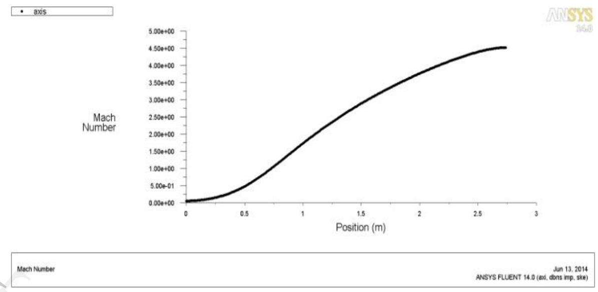


Figure 4.9: Mach number for conical nozzle with 15˚ divergent angle



Graph 4.5: Plot of Mach number v/s Position

At the throat velocity varies from 2.81e-01 to 1.4Mach which is higher in range when compared to 13˚ conical nozzle. In this case, it can be observed from figure 4.9 and graph 4.5 that there is no formation of Mach Disk and also absence of shock as observed in lower divergent angles. The Mach number, after continuous increase reaches a maximum of 4.32Mach. Higher velocities are observed around the axis throughout the divergent portion up to the exit.

Case3.2: Contour Nozzle – length of the divergent portion same as that of conical nozzle with Divergent angle of 15°.

3.2.1 Mach Number v/s Position

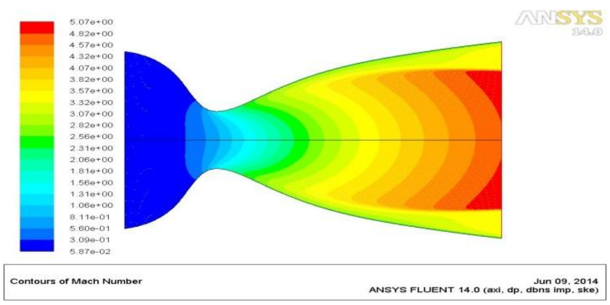
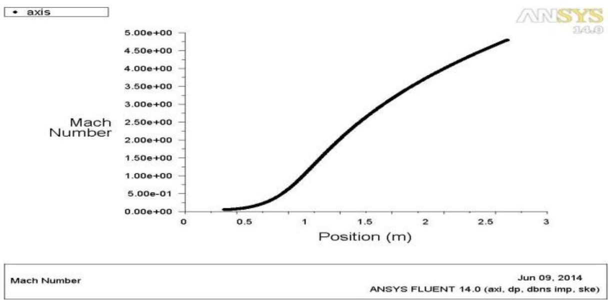


Figure 4.10: Mach number for Contour Nozzle – length of the divergent portion same as that of conical nozzle with Divergent angle of 15°.



Graph 4.6: Plot of Mach number v/s Position

In this case, as in conical nozzle with same end geometry no shock is observed, resulting in higher exit velocity of 4.82Mach around the axis. It can also be observed from figure 4.10 and graph 4.6 that although exit Mach number is fairly high, a higher degree of flow separation occurs from almost 1.8m from the inlet, which is far sooner than when compared to previous case of contour nozzle.

3.1.2 Static pressure: Conical nozzle with 15˚ divergent angle

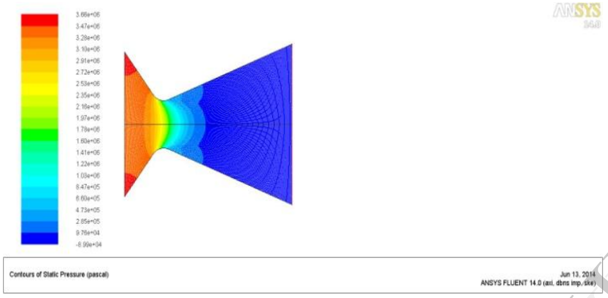


Figure 4.11: Static Pressure for conical nozzle with 15˚ divergent angle

The static pressure is found to be 3.47e+06Pa at the inlet section. The pressure dropped from 3.28e+06 to 1.78e+06Pa at the throat section, which is slightly lower in range compared to conical nozzle with 13˚ divergent angle. In this case, it can be observed from figure 4.11 that no occurrence of shock. The expansion almost completely happens to a value of -8.99e+04Pa near to the throat and there is no much further expansion towards the exit.

3.2.2 Static Pressure: Contour Nozzle – length of the divergent portion same as that of conical nozzle with Divergent angle of 15°.

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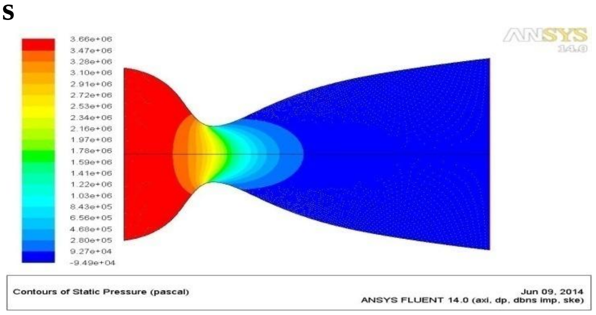

Figure 4.12: Static Pressure for contour nozzle with 15˚ divergent angle

It can be observed from figure 4.12 that the pressure dropped from 3.47e+06 to 1.59e+06Pa at the throat section, which is larger in magnitude when compared with 15˚ conical. The expansion almost completely happens to a value of -9.49e+04Pa near to the throat and there is no much further expansion towards the exit as in the case of 15˚ conical, hence having higher degree of expansion.

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