International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 3 Issue 7, July - 2014

Numerical Solution to the Crack Detection in Cracked Beam using Fuzzy Logic

[1.] Rane Harshal S., [2.] Barjibhe R. B., [3.] Patil A. V.
[1.] Research scholar,
[2.] PG-coordinator,
[3.] Head of Mechanical Engg. Dept.,
S. S. G. B. College of Engineering and Technology, Bhusawal, Maharashtra, India

***Abstract*-Most of the structures and the mechanical systems are failed due to excessive vibrations in their working life. Since most human activities involve vibration in one form or other. The vibrations having certain amplitude and frequency may be reliable for human being. But excessive vibrations may lead to structural or mechanical failure of the system. Thus, study of vibration has more importance in order to avoid failures. The study of vibrations is necessary to understand the vibration behaviour of the system.**

The phenomenon of vibration can be applied to identify the crack size and location. Modern NDT techniques like dye penetrant testing, ultrasonic testing, acoustic emission techniques are hold the position of crack identification but such techniques require more time interval for processing. Thus it is necessary to develop new techniques for crack identification.

The improved Fuzzy logic techniques and curve fitting in Matlab are the alternatives for NDT techniques. These techniques give approximately solution for the problems of cracks. This paper contains certain knowledge about fuzzy logic techniques applied for different beam models.

Keywords-* *vibrations, NDT techniques, crack identification, Fuzzy logic techniques

I. INTRODUCTION

Any motion which repeats itself after a certain interval of time is called vibration. The swing of pendulum is a typical example of vibration. The theory of vibration deals with study of oscillatory motions of bodies and the forces associated withthem. A vibration can caused due to external unbalanced forcealso. A vibratory system, in general, includes elastic memberfor storing potential energy, a mass or inertia member forstoring kinetic energy and damper by which gradual loss ofenergy takes place. A simple pendulum as shown in figure 1 isan example of vibration system. Pendulum has a string forelastic nature, mass of bob acts as a means for kinetic energy.Like pendulum, spring-masssystem, vehicle suspensionsystem, simply supported and cantilever beam, lateralvibrating string, vibration due to unbalance reciprocating orrotating force, etc. are the examples of vibrating system.

A. Lateral vibrations of a cantilever beam

The vibrations in which particles of the systemvibrates in the direction perpendicular to the axis of system isknown as lateral vibration. A large number of practicalsystems can be described using finite number of degree offreedom; such a system is shown in figure 1but some systems,especially those involving continuous elastic members, have

an infinite number of degree of freedom. As an example,consider a cantilever beam shown in figure .2. Since the beamhas infinite number of masses, we need infinite number ofcoordinates to specify the deflected configuration. Thus,cantilever beam is infinite degree of freedom system and it isnecessary to study lateral vibrations of cantilever beam.

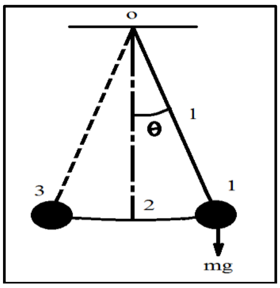

Figure 1 Vibrations of Simple Pendulum

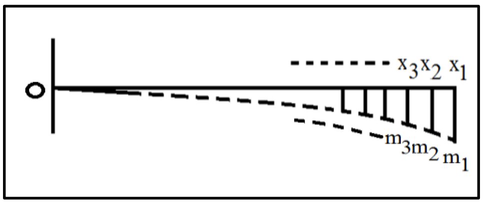

Figure 2 Cantilever Beam as Multi Degree of Freedom System

B. Crack identification in cantilever beam structure

Vibration-based methods have been proved as a fastand inexpensive means for crack identification. A crack in astructure induces a local flexibility which affects the dynamicbehaviour of the whole structure to a considerable degree. Itresults in reduction of natural frequencies and changes inmode shapes. An analysis of these changes makes it possibleto determine the position and depth of cracks. Most of theresearches used in their studies are open crack models, that is,they assume that a crack remains always open duringvibration. The assumption of an open crack leads to a constantshift of natural frequencies of vibration.

The decrease in experimental natural frequencies willlead to an underestimation of the crack depth. Harish andParhi, 2009 have performed analytical studies on freevibrations of cracked beam models and obtains the appropriateresults.

II. LITERATURE REVIEWS

Different researchers have discussed damage detection of vibrating structures in various ways. They are summarized below.

Free and forced vibration analyses of a cracked beam were performed by S Orhan et al. in order to identify the crack in a cantilever beam. Single- and two-edge cracks were evaluated. The study results suggest that free vibration analysis provides suitable information for the detection of single and two cracks, whereas forced vibration can detect only the single crack condition.

F Leonard, J Lanteigne, S Lalonde and Y Turcotte et al. proposed a study based on cracks that occurred in metal beams obtained under controlled fatigue-crack propagation. Spectrograms of the free-decay responses showed a time drift of the frequency and damping: the usual hypothesis of constant modal parameters is no longer appropriate, since the latter are revealed to be a function of the amplitude.

An experimental investigation has been carried out byM. Karthikey and R. Tiwari et al. to establish an identification procedure for the detection, localization, and sizing of a flaw in a beam based on forced response measurements. The experimental setup consisted of a circular beam, which was supported by rolling bearings at both ends.

Sensibility analysis of the inverse problem of the crack parameters (location and depth) determined by M. B. Rosales, C P Filipich and F S Buezas et al. An efficient numerical technique is necessary to obtain significant results. Two approaches are herein presented: The solution of the inverse problem with a power series technique (PST) and the use of artificial neural networks (ANNs).

An analytical, as well as experimental approach by H. Nahvi and M. Jabbari et al. to the crack detection in cantilever beams by vibration analysis is established.

A model-based approach is developed by Zhigang Yu and Fulei Chu et al. to determine the location and size of an open edge crack in an FGM beam. The *p*-version of finite element method is employed to estimate the transverse vibration characteristics of a cracked FGM beam. A rational approximation function of the stress intensity factor (SIF) with crack depth and material gradient as independent variables is presented in order to overcome the cumbersomeness and inaccurateness caused by the complicated expression of the analytical SIF solution in crack modelling.

An analytical approach forcrack identification procedure in uniform beams with an open edge crack, based on bending vibration measurements, is developed by N. Khaji, M. Shafiei and M. Jabalpur et al. The method is based on the assumption that the equivalent spring stiffness does not depend on the frequency of vibration, and may be obtained from fracture mechanics. The results provide simple expressions for the characteristic equations, which are functions of circular natural frequencies, crack location, and crack depth.

III. THEORETICAL FORMULATIONS

A Governing equation for free vibration of beam

The cantilever beam with a transverse edge crack is clamped at left end, free at right end and has same cross section and same length like model in Figure 3 and 4. The Euler- Bernoulli beam model is assumed for the theoretical formulation. The crack in this particular case is assumed to be an open surface crack and the damping is not being considered in this theory. Both single and double edged crack are considered for the formulation.The free bending vibration of a beam of a constant rectangular cross section having length l, width b, and depth w is given by the Euler's beam theory as follows:

If the cross sectional dimensions of beam are small compared to its length, the system is known as Euler-Bernoulli beam. Only thin beams are treated in it. The differential equation for transverse vibration of thin uniform beam is obtained with the help of strength of materials. The beam has cross section area A, flexural rigidity *EI* and density of material ρ. Consider the small element dx of beam is subjected to shear force Q and bending moment M, as shown in figure 5.

While deriving mathematical expression for transverse vibration, it is assumed that there are no axial forces acting on the beam and effect of shear deflection is neglected. The deformation of beam is assumed due to moment and shear force.

The net force acting on the element,

$$Q - \left(Q + \frac{\partial Q}{\partial x} dx\right) = dm * acceleration$$

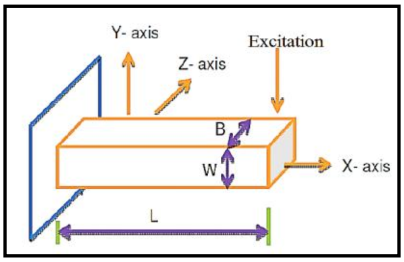

Figure 3 Uncracked Cantilever Beam Model

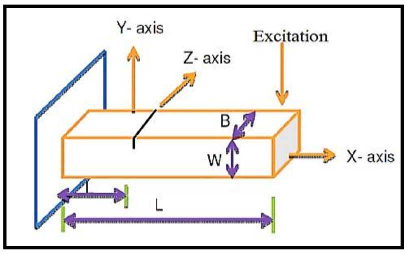

Figure 4 Cracked Cantilever Beam Model

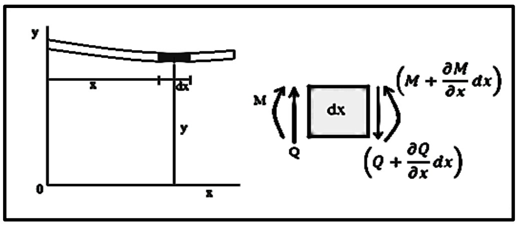

Figure 5 Shear Force and Bending Moment acting on Beam Element

$$-\frac{\partial Q}{\partial x}dx = (\rho A dx)\frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial Q}{\partial x} + \rho A\frac{\partial^2 y}{\partial t^2} = 0 \quad \textbf{(\textit{Equation 1})}$$

Considering the moments about A, we get

$$M - \left(M + \frac{\partial M}{\partial x}dx\right) + \left(Q + \frac{\partial Q}{\partial x}dx\right)dx = 0$$

$$-\frac{\partial M}{\partial x} + Q + \frac{\partial Q}{\partial x}dx = 0$$

So $Q = \frac{\partial M}{\partial x}$ higher order derivatives are neglected here $\left(\frac{\partial Q}{\partial x}dx = 0\right)$

or $\frac{\partial Q}{\partial x} = \frac{\partial^2 M}{\partial x^2}$ (***Equation 2***)

From the above two equations 1 and 2, we get

$$\frac{\partial^2 M}{\partial x^2} = \rho A\frac{\partial^2 y}{\partial t^2} \quad \textbf{(\textit{Equation 3})}$$

We know from strength of materials that

$$M = -EI\frac{\partial^2 y}{\partial x^2}$$

$$\frac{\partial^2 M}{\partial x^2} = -EI\frac{\partial^4 y}{\partial x^4} \quad \textbf{(\textit{Equation 4})}$$

Comparing equation 3and 4 we get,

$$\frac{\partial^4 y}{\partial x^4} + \left(\frac{\rho A}{EI}\right)\frac{\partial^2 y}{\partial t^2} = 0 \quad \textbf{(\textit{Equation 5})}$$

This is the general equation for transverse vibration. Thus the natural frequency can be found out by this theory (Appendix A) as,

$$\omega_n = C * \sqrt{\frac{EI}{\rho A l^4}} \quad \textbf{(\textit{Equation 6})}$$

Where,
E= Young's modulus of the material,
I= Moment of inertia,
A= Area of cross section,
l=length of the beam,
C= Constant depending mode of vibration,
C_1=0.56 for first mode,
C_2= 3.51 for second mode,
C_3= 9.82 for third mode.

The moment of inertia can be found out by relation,

$$I = \frac{bd^3}{12} \textbf{\textit{Equation 7}})$$

Where,
b= width of the cantilever beam section,
d= depth of the cantilever beam section.

Due to presence of crack, moment of inertia of the beam changes and correspondingly the natural frequency also changes. For a constant beam material and cross section the reduced moment of inertia will be found by relation below.

$$I_1 = I - I_c \quad \textbf{(\textit{Equation 8})}$$

Where,
I_1=Moment of inertia of a cracked beam,
I= Moment of inertia of Uncracked beam,
I_c= Moment of inertia of cracked beam element.

Thus by the use of equation 6, 7 and 8 we can find out the different modes of natural frequencies for the cantilever beam. Instead of using crack depth and location, the terms RCD and RCL are considered and calculated by the formula,

$$\text{RCD} = \frac{depth\ of\ the\ crack}{depth\ of\ the\ beam} \quad \textbf{(\textit{Equation 9})}$$

$$\text{RCL} = \frac{distance\ \ of\ crack\ grom\ fixed\ end}{length\ of\ the\ beam} \quad \textbf{(\textit{Equation 10})}$$

The theoretical calculations are carried out by using above theory. Here total 10 models have been studied for natural frequency analysis having different crack depth and location. The beam models no. 1 to 5 is of rectangular cross section while beam model from 6 to 10 are of square cross section. It is assumed that the natural frequency changes due to the change in moment of inertia only. The beam made up of structural steel material by extrusion process. The Young's modulus for the beam material of length 700 mm is 210×10^9 N/m^2.

From the theory it is predicted that presence of crack in structure reduces the natural frequency. The natural frequency is the function of flexural rigidity and inversely proportional to the density of material and length of the beam.

The calculations show the natural frequency and mode shapes for first three beam models. Similarly, the natural frequency and different mode shapes for remaining beam models are calculated and tabulated in Table 1.

Table 1 THEORETICAL NATURAL FREQUENCY FOR THE BEAM MODELS

Beam model no.	RCD	RCL	First Natural Frequency	Second Natural Frequency	Third Natural Frequency
1	0	0	8.53	53.63	149.61
2	0.2	0.25	8.49	53.40	148.98
3	0.4	0.25	8.25	51.88	144.24
4	0.2	0.5	8.49	53.40	148.98
5	0.4	0.5	8.25	51.88	144.24
6	0	0	17.06	107.22	299.11
7	0.2	0.25	17.04	107.14	298.82
8	0.4	0.25	16.98	106.76	297.84
9	0.2	0.5	17.04	107.14	298.82
10	0.4	0.5	16.98	106.76	297.84

IV. FINITE ELEMENT FORMULATIONS

Finite element analysis has been carried out byANSYS12 software. ANSYS is a general-purpose finiteelementmodelling package for numericallysolving a widevariety of mechanical problems. These problems includestatic/dynamic, structural analysis (both linear and nonlinear),heat transfer, and fluidproblems, as well as acoustic and
electromagnetic problems.

In general, a finite-element solution may be brokeninto the following three stages.

(1) **Pre-processing**: defining the problem

The major steps in pre-processing are

(i) Define keypoints/lines/areas/volumes,

(ii) Define element type and material/geometric properties,and (iii) mesh lines/areas/ volumes as required.

The amount of detail required will depend on thedimensionality of the analysis,i.e., 1D, 2D, axisymmetric, and3D.

(2) **Solution**: assigning loads, constraints, and solving. Here, itis necessary to specify the loads (point or pressure),constraints (translational and rotational), and finally solve theresulting set of equations.

(3) **Post processing**: Further processing and viewing of theresultsIn this stage one may wish to see (i) lists of nodaldisplacements, (ii) elementforces and moments, (iii) deflection plots, and (iv) frequencies and temperature maps.

Following steps show the guidelines for carrying outModal analysis.

Define Materials

1. Set preferences. (Structural)
2. Define constant material properties.

Model the Geometry

3. Follow bottom up modelling and create/import thegeometry

Generate Mesh

4. Define element type.
5. Mesh the area.

Apply Boundary Conditions

6. Apply constraints to the model.

Obtain Solution

7. Specify analysis types and options.
8. Solve.

The ANSYS 12 finite element program was used forfree vibration of thecracked beams. For this purpose, the total10 models are created at various crack positions in CADsoftware (CATIA) and imported in ANSYS (.stp file). Thebeammodel wasdiscretised into no. of elements with N nodes.Cantilever boundary conditions can also be modelled byconstraining all degrees offreedoms of the nodes located onthe left end of the beam. The subspace mode extraction method was used to calculate the natural frequencies of the beam.

The results of finite element analysis for the beams have first natural frequencies are tabulated in Table 2.

Table 4.1 Finite Element Analysis Result of Beam Models

V. FUZZY LOGIC TECHNIQUE

Beam Model No.	RCD	RCL	First Natural Frequency	Second Natural Frequency	Third Natural Frequency
1	0	0	8.34	53.20	146.32
2	0.2	0.25	8.31	53.11	145.90
3	0.4	0.25	8.23	52.88	144.97
4	0.2	0.5	8.33	52.09	146.34
5	0.4	0.5	8.31	51.43	146.30
6	0	0	16.65	104.25	291.46
7	0.2	0.25	16.62	104.25	291.38
8	0.4	0.25	16.55	104.25	291.13
9	0.2	0.5	16.64	104.20	291.40
10	0.4	0.5	16.62	104.06	291.46

A fuzzy inference system (FIS) essentially defines a nonlinear mapping of the input data vector into a scalar output, using fuzzy rules. The mapping process involves input/output membership functions, FL operators, fuzzy if–then rules, aggregation of output sets, and defuzzification. An FIS with multiple outputs can be considered as a collection of independent multiinput, single-output systems. A general model of a fuzzy inference system (FIS) is shown in Figure 6. The FLS maps crisp inputs into crisp outputs. It can be seen from the figure that the FIS contains four components: the fuzzifier, inference engine, rule base, and defuzzifier. The rule base contains linguistic rules that are provided by experts. It is also possible to extract rules from numeric data. Once the rules have been established, the FIS can be viewed as a system that maps an input vector to an output vector. The fuzzifier maps input numbers into corresponding fuzzy memberships. This is required in order to activate rules that are in terms of linguistic variables. The fuzzifier takes input values and determines the degree to which they belong to each of the fuzzy sets via membership functions. The inference engine defines mapping from input fuzzy sets into output fuzzy sets. It determines the degree to which the antecedent is satisfied for each rule. If the antecedent of a given rule has more than one clause, fuzzy operators are applied to obtain one number that represents the result of the antecedent for that rule. It is possible that one or more rules may fire at the same time. Outputs for all rules are then aggregated. During aggregation, fuzzy sets that represent the output of each rule are combined into a single fuzzy set.

Fuzzy rules are fired in parallel, which is one of the important aspects of an FIS. In an FIS, the order in which rules are fired does not affect the output. The defuzzifier maps output fuzzy sets into a crisp number. Given a fuzzy set that encompasses a range of output values, the defuzzifier returns one number, thereby moving from a fuzzy set to a crisp number. Several methods for defuzzification are used in practice, including the centroid, maximum, mean of maxima, height, and modified height defuzzifier. The most popular defuzzification method is the centroid, which calculates and returns the centre of gravity of the aggregated fuzzy set. FISs employ rules. However, unlike rules in conventional expert systems, a fuzzy rule localizes a region of space along the function surface instead of isolating a point on the surface. For a given input, more than one rule may fire. Also, in an FIS, multiple regions are combined in the output space to produce a composite region. A general schematic of an FIS is shown in Figure 7.

The result of fuzzy logic or outputs from fuzzy logic has been tabulated in Table 2. Here input for fuzzy logic is different modes of natural frequencies and output will be relative crack depth and relative crack location. The natural frequency are obtained from finite element formulation are taken for developing fuzzy model.

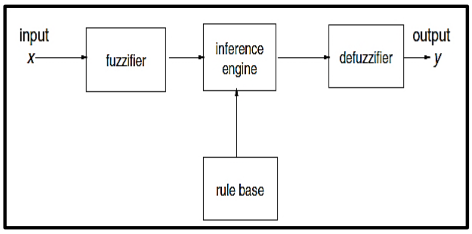

Figure 6 Block Diagram of Fuzzy Interface System

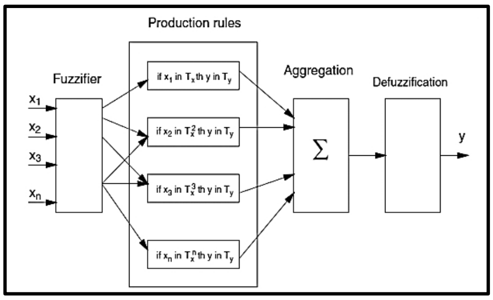

Figure 7 Schematic Diagram of a Fuzzy Interface System

TABLE 2 COMPARISON OF FUZZY LOGIC PARAMETERS WITH THEORETICAL PARAMETERS

Beam model no.	**Relative Crack Depth**		**Relative Crack Location**	
	Fuzzy Logic	**Theoretical**	**Fuzzy Logic**	**Theoretical**
1	0.052	0	0.065	0
2	0.191	0.2	0.239	0.25
3	0.34	0.4	0.25	0.25
4	0.2	0.2	0.433	0.5
5	0.212	0.4	0.424	0.5
6	0.16	0	0.25	0
7	0.167	0.2	0.25	0.25
8	0.3388	0.4	0.25	0.25
9	0.1608	0.2	0.348	0.5
10	0.3416	0.4	0.427	0.5

VI. CONCLUSIONS

The present investigation based on the theoretical Analysis, FEA Analysis and experimental analysis draws the following conclusions.

Inputs for FEA and FFT are crack location and crack depth and outputs are natural frequency for different modes of vibration. Significant changes in natural frequency observed at the vicinity of crack location. When the crack location is constant but the crack depth increases, the natural frequency of the beam decreases. When the crack depth is constant and crack location from the cantilever end varied, natural frequencies of first, second and third modes are also decreased. The results show that the values of natural frequencies by theory, ANSYS and FFT are close to the agreement.

The presence of crack in beam models reduces the natural frequency of all the beam models, but the beams with square cross section area have approximately same longitudinal as well as transverse frequencies so it become hard to select beams with square cross section. Thus it is better to use rectangular cross section beams for appliances, since they have larger transverse frequency than longitudinal.

The fuzzy controller is developed with triangular membership function for inputs and output and results shows that the triangular MF predicts the output parameters of crack. Crack depth and crack location of a beam can be predicted by fuzzy controller is within nanoseconds. Hence it saves considerable amount of computation time. By Comparing the Fuzzy results with the theoretical results it is observed that the developed Fuzzy Controller can predict the relative crack depth and relative crack location in a very accurate manner. Certain precision and skilled operating is required to develop Fuzzy controller. Results based on fuzzy techniques are not much accurate as it depends on some training pattern of fuzzy controller, whereas in ANSYS, it is much accurate as it is based on finite elements. But it is practically suitable that natural frequency can be obtained but crack location and crack depth are not possible as they are very small values.

Here an approach can be suggested which is based on the combination of both ANSYS and Fuzzy, in which natural frequency obtained in ANSYS can be used as input for fuzzy controller for determination of accurate value of crack depth and crack location.

ACKNOWLEDGEMENT

The author would like to thank Prof. R. B. Barjibhe (Guide, Associate Professor) ShriSantGadage Baba College of Engineering and Technology, Bhusawal, Maharashtra, India. I also thankful to Prof. A. V. Patil (HOD, Associate Professor) for valuable guidance.

The blessing of Family, Teacher‟s and my friends is the main cause behind the successful completion of this paper. I wish to acknowledge great moral support given by management of ShriSantGadage Baba College of Engineering and Technology, Bhusawal, Maharashtra, India.

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