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Odd-graceful labeling of corona graph C_{2n}*K_1.

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Abstract

The research in Graph theory had lead to one of the important area which involves labeling of graphs. There are different types of labelings such as graceful labeling, magic labeling, prime labeling etc applied to various classes of graphs. In this paper, odd-gracefulness of the corona graph C_{2n}*K_1 for $n \geq 2$ is obtained.

Keywords:

Graph theory, Graceful graph, labeling of graphs, corona graph, odd-graceful labeling.

Introduction

Definition 1:Graph: A graph G is a pair (V(G) , E (G)) where V (G) is a nonempty finite set of elements known as vertices and E (G) is family of unordered pairs of elements of V (G) known as edges.

Figure 1: Different graphs.

Definition 2: Corona graph: The corona

G_1* G_2 of two graphs G_1 and G_2 is a graph G obtained by taking one copy of G_1 which has p_1-vertices and p_1-copies of G_2 and then joining i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

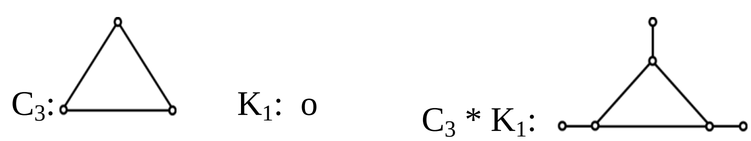

Figure 2: Corona C_3* K_1

Definition 3: Difference vertex labeling: A difference vertex labeling of graph G is an assignment f of labels to the vertices of G that induces for each edge uv the weight ∣ f(u) - f(v) ∣ .

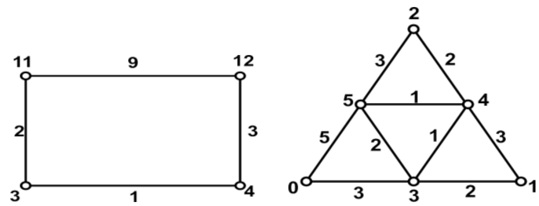

Figure 3: Difference vertex labeling.

Definition 4: Labeled graph: When the dots or lines in a graph are labeled with numbers we call it a '**labeled graph**'. Labeling of the vertices of a graph G is assignment of distinct natural numbers to vertices of G. This labeling induces a natural labeling of the edges called edge labels or edge weights.

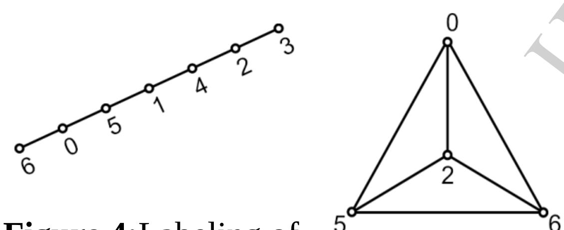

Figure 4: Labeling of graphs.

Definition 5: Graceful graph: Let G be a graph with q edges. Let f be labeling of G such that the set of labels of vertices is a subset of { 0,1,2,3,......,q} and the set of the edge labels is from set { 1,2,3,......,q} Then the labeling f is said to be graceful and graph G is called graceful graph .

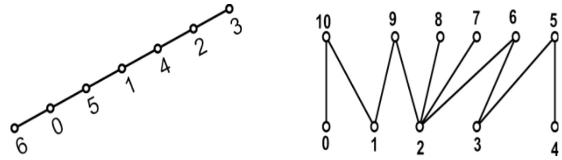

Figure 5: Graceful graph.

Definition 6: Odd-graceful graph: A difference vertex labeling of graph G of size n is odd-graceful if f is an injection from V(G) to {0,1,……..,2n-1} such that the induced weights are {1,3,…..,2n-1}. The graph with odd-graceful labeling is called odd-graceful graph.

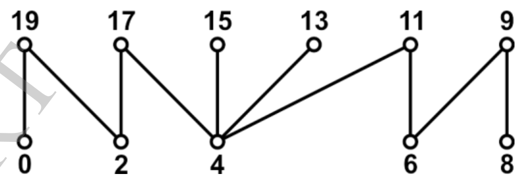

Figure 6: Odd-graceful graph.

Odd-gracefulness of C_{2n}*K_1.

Theorem : The graphs C_{2n}*K_1 are odd graceful, for $n \geq 2$.

Proof :- Number of vertices of C_{2n}*K_1 = $p(C_{2n}$*$K_1) = 4n$

Number of edges of C_{2n}*K_1 = $q\ (C_{2n}$*$K_1)$ = 4n

Let vertex set of C_{2n}*K_1 be $V(C_{2n}$*$k_1)$ = $\{u_1,u_2,\ldots\ldots\ldots,u_{2n}\ ;\ v_1,v_2,\ldots\ldots\ldots,v_{2n}\}$ where vertices $u_1,u_2,\ldots\ldots..,u_{2n}$ are vertices of cycle C_{2n}. 'v_i' is the pendant vertex adjacent to u_i ; $1 \leq i \leq 2n$.

There are two cases : (i) $n \equiv 0 \pmod 2$ (ii) $n \equiv 1 \pmod 2$

In both the cases we show that edge weight set is W = {1,3,5,……,8n-1} with vertex set labeling{0,1,2,3,4,5,… ,8n-1}.

In both cases, the labeling function f is given in two parts viz Part -I and Part -II.

Part- I describes the labeling function for the vertices (u_i,v_i) where $1 \le i \le n$ and Part -II describes labeling function for the vertices (u_i, v_i) where $n + 1 \le i \le 2n$. Further in each case, Part- II is divided into three subparts namely S-1, S-2 and S-3.

Part -I : The labeling function f for vertices u_i and v_i where $1 \le i \le n$ is given as follows:

$f(v_{2i-1}) = 4i - 4$ and $f(u_{2i-1}) = 8n - (4i-3)$

for $1 \le i \le \frac{n}{2}$ when $n \equiv 0 \pmod 2$

and

for $1 \le i \le \left[\frac{n}{2}\right] + 1$ when $n \equiv 1 \pmod 2$

$f(v_{2i}) = 8n - (4i-1)$ and $f(u_{2i}) = 4i - 2$

for $1 \le i \le \frac{n}{2}$ when $n \equiv 0 \pmod 2$

and

for $1 \le i \le \left[\frac{n}{2}\right]$ when $n \equiv 1 \pmod 2$

The edge weights covered in Part- I are all odd numbers in descending order from 8n – 1 to 4n + 3 i.e. from 8n -1 to 8n – (4n - 3).

Part -II :- As mentioned earlier, Part -II is divided into three subparts S-1, S-2 and S-3.

Subpart S-1 : In this subpart we have to consider only two vertices u_{n+1} and v_{n+1}. The labeling function for this subpart is as follows:

$f(u_{n+1}) = 2n - 1$ and $f(v_{n+1}) = 6n - 2$ when $n \equiv 0 \pmod 2$

$f(u_{n+1}) = 6n - 2$ and $f(v_{n+1}) = 2n - 1$ when $n \equiv 1 \pmod 2$

Subpart S-2 : The labeling function f in this subpart is defined for vertices u_{n+2}, u_{n+3}, ……, u_{2n-1} and for vertices v_{n+2}, v_{n+3},……..,v_{2n-1}.

$f(u_{2i}) = 4i - 2$ and $f(v_{2i}) = 8n - (4i - 1)$

for $\frac{n}{2} + 1 \le i \le n - 1$

when $n \equiv 0 \pmod 2$

and

for $\left[\frac{n}{2}\right] + 2 \le i \le n-1$

when $n \equiv 1 \pmod 2$

$f(u_{2i+1}) = 8n - (4i + 1)$ and $f(v_{2i+1}) = 4i$

for $\frac{n}{2} + 1 \le i \le n - 1$

when $n \equiv 0 \pmod 2$

and

for $\left[\frac{n}{2}\right] + 1 \le i \le n - 1$

when $n \equiv 1 \pmod 2$

Remark :i) When $n \equiv 0 \pmod 2$, in subpart S-2, for the validity of the range of the parameter i, we need $n \ge 4$. For $2 \le n < 4$, we have only one such value of n which is 2. When n =2, the set of vertices belonging to S-2 is an empty set.

ii) When $n \equiv 1 \pmod 2$, in subpart S-2, for the validity of the range of the parameter i , we need $n \ge 3$. For $3 \le n < 5$ we have only one value of n which is 3. When n = 3 the set of vertices belonging to S-2 are u_5 and v_5.

Subpart S-3 : In this subpart there are only two vertices viz ; u_{2n} and v_{2n} with labeling function as follows:

$f(u_{2n}) = 4n - 2$ and $f(v_{2n}) = 1$
if $n \equiv 0 \pmod 2$ and $n \equiv 1 \pmod 2$

All odd edge weights in descending order from 8n-1 to 4n + 3 are covered in Part -I. Remaining odd edge weights from 4n +1 to 1 are covered in Part- II as follows :-

1. $| f(u_1) - f(u_{2n}) | = 4n + 1$
2. $| f(v_{n+1}) - f(u_{n+1}) | = 4n - 1$
3. $| f(u_{2n}) - f(v_{2n}) | = 4n - 3$
4. For $\left[\frac{n}{2}\right]+1 \leq i \leq n-1$ edge weights covered are from 4n – 5 to 7 in descending order.
5. $| f(u_{2n-1}) - f(u_{2n}) | = 5$
6. $| f(u_{n+2}) - f(u_{n+1}) | = 3.$
7. $| f(u_{n+1}) - f(u_n) | = 1$

Remark : i)In case of $n \equiv 0 \pmod 2$, when n = 2, edge weights covered in Part-I are from 8n-1 to 4n +3 .i.e. 15, 13 and 11, then in subpart S-1 edge weights covered are 1 and 7. Since subpart S-2 is empty for n = 2, we get
$| f(u_{2n}) - f(u_{n+1}) | = | 4n - 2 - (2n-1) | = | 2n-1 | = 3.$
Thus, edge weights covered in subpart S-3 are 5 & 9.

ii) In case of $n \equiv 1 \pmod 2$, when n =3, the edge weights covered in Part -I are from 8n-1 to 4n +3 .i.e. 23,21,19,17,15. In subpart S-1 edge weights covered are 3 & 11. In subpart S-2, edge weights covered are 1,7 and 5. In subpart S-3, edge weights covered are 9 and 13.

Thus, the labeling function f is injective and that each odd edge weight from 1 to 8n-1 is covered exactly once. Hence, the graph $C_{2n}*K_1$ is odd-graceful for $n \geq 2$.

Illustration1:

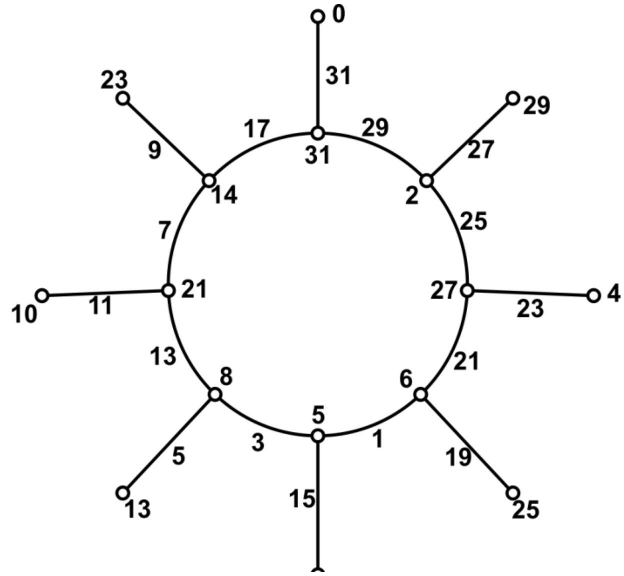

Figure 7: Odd-gracefulness of C_8*K_1

Illustration 2:

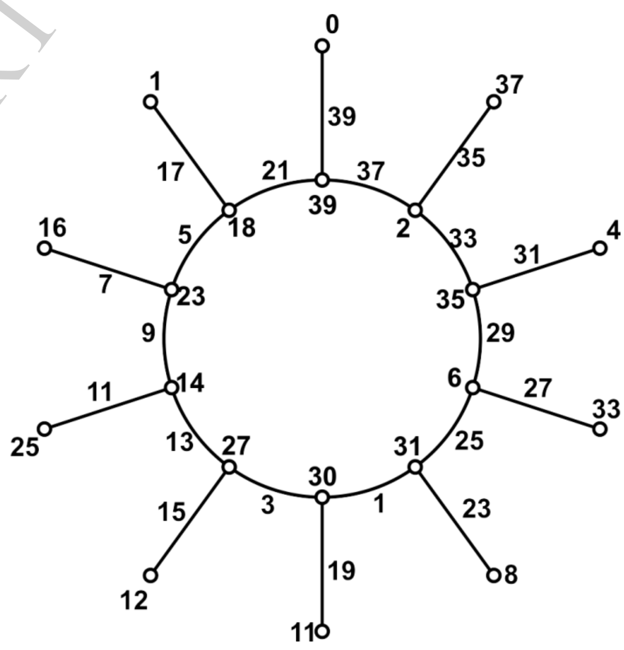

Figure 8: Odd-gracefulness of $C_{10}*K_1$

Conclusion

The odd graceful labeling is presented to the corona graph $C_{2n}*K_1$.The corona graph $C_{2n}*K_1$ is thus a edge-odd graceful graph. It is interesting to apply this

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labeling to certain classes of graph. It is also area of interest to make computer programmes for the given labeling.

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