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1

ON EXISTENCE OF SOLUTIONS FOR $\eta-$ GENERALIZED IMPLICIT VECTOR VARIATIONAL- LIKE INEQUALITIES

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ABSTRACT. In this work, we intend to introduce and study a class of η generalized implicit vector variational- like inequalities and a class of η generalized implicit strong vector variational -like inequalities in the setting of Hausdorff topological vector spaces.An equivalence result concerned with two classes of $\eta-$ generalized implicit vector variational-like inequalities is proved under the suitable conditions.By using $FKKM$ theorem, some new existence results of solutions for the η generalized implicit vector variational-like inequalities and $\eta-$ generalized strong implicit vector variational-like inequalities are obtained under some suitable conditions.

Key Words : Generalized parametric quasi-variational inclusions, sensitivity analysis, resolvent operator, Hausdorff metric.

1. INTRODUCTION

Vector variational inequality was first introduced and studied by Giannessi [1] in the setting of finite dimensional Eucledean spaces. since then,the theory with applications for vector variational like inequalities, vector problems, vector equilibrium problems, and vector optimization problems, have been studied and generalized by many authors(see, e.g.,[2-14] and refernces therein). Recently Yu et al [15] considered a more general form of weak vector variational inequalities and proved some new results on the existence of solutions of the new class of weak vector variational inequalities in the setting of Hausdorff topological vector spaces and Ahmed and Khan [16] introduced and considered weak vector variational like inequalities with $\eta-$ generally convex mapping and gave some existence results.

On the other hand, Fang and Huang [17] studied some existence results of solutions for a class of strong vector variational inequalities in Banach spaces which give a positive answer to an open problem proposed by Chen and Hou [18].

In 2008, Lee et al.[19] introduced a new class of strong vector variational type inequalities in Banach spaces. They obtained the existence theorems of solutions for the inequalities without monotonicity in Banach spaces by using Broweder Fixed point Theorem. Motivated and inspired by the work mentioned above , in this paper we introduce and study a class of $\eta-$ generalized implicit vector variational- like inequalities and a class of $\eta-$ generalized strong implicit vector variational -like inequalities in the setting of Haudorff toplogical vector spaces. We first show an equivalence theorem concerned with the two classes of $\eta-$ generalized implicit vector variational -like inequalities under suitable conditions. By ussing FKKM theorem, we prove some new existence results of solutions for

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the $\eta-$ generalized implicit vector variational like inequalities and $\eta-$ generalized strong implicit vector variational-like inequalities under some suitable conditions.The results presented in this paper improve and generalize some known results due to Ahmed and khan [16], Lee et al. [19], and Yu et al. [15].

2. Preliminaries

Let X and Y be two real Hausdorff topological vector spaces, $K \subset X$ a nonempty, closed, and convex subset, and $C \subset Y$ a closed, convex, and pointed cone with apex at the origin. Recall that the Hausdorff topological vector space Y is said to be an ordered Hausdorff toplogical vector space by (Y, C) if ordering relations are defined in Y as follows:

$$\forall x, y \in Y,\ x \leq y \Longleftrightarrow y - x \in C,$$

$$\forall x, y \in Y,\ x \nleq y \Longleftrightarrow y - x \notin C,$$

If the $intC \neq \phi$, then the weak ordering relations in Y is defined as follows:

$$\forall x, y \in Y,\ x < y \Longleftrightarrow y - x \in intC,$$

$$\forall x, y \in Y,\ x \nless y \Longleftrightarrow y - x \notin intC,$$

Let $L(X, Y)$ be the space of all continuous linear maps from X to$Y$ and'$T : X \to L(X, Y)$. We denote the value of $l \in L(X, Y)$ on $x \in X$by(l, x). throughout this paper, we assume that $C(x) : x \in K$ is a family of closed, convex, and pointed cones of Y such that $intC \neq \phi$ for all$x \in K, \eta$ is a mapping from $K \times K$ into Y.

In this paper, we consider the following two kinds of vector variational inequalities:- $\eta-$ *Generalized Implicit Vector Variational-Like Inequality* (in short,$\eta-$ GIVVLI): for each $z \in K$ and $\lambda \in (0, 1]$, find$x \in K$ such that

$$\langle T(\lambda x + (1 - \lambda)z), \eta(y, x)\rangle + f(y, g(x)) \notin -intC(x),\ \forall y \in K,$$

$\eta-$ *Generalized Strong Implicit Vector Variational-Like Inequality*(in short,$\eta-$ GSIVVLI): for each $z \in K$ and $\lambda \in (0, 1]$, find$x \in K$ such that

$$\langle x + (1 - \lambda)z), \eta(y, x)\rangle + f(y, g(x)) \notin -C(x) \setminus \{0\},\ \forall y \in K,$$

Definition 2.1 Let $T : K \to L(X, Y)$ and $\eta : K \times K \to K$ be two mappings and $C = \bigcap_{x \in K} C(x) \neq \phi$. T is said to be $\eta-$ monotone in C if and only if

$$\langle T(x) - T(y), \eta(x, y)\rangle \in C, \forall x, y \in K. \tag{2.1}$$

Definition 2.2 Let $T : K \to L(X, Y)$ and $\eta : K \times K \to K$ be two mappings. We say that T is $\eta-$ hemicontinuous if, for given any $x, y, z \in K$ and $\lambda \in (0, 1]$, the mapping $t \mapsto \langle T(\lambda(x + (1 - t)(y - x)) + (1 - \lambda)z), \eta(x, y)\rangle$ is continuous at 0^+.

Definition 2.3 A multivalued mapping Let $A : X \to X^Y$ is said to be upper semicontinuous on X if, for all $x \in X$ and for each open set G in Y with $A(x) \subset G$, there exist an open neighbourhood $O(x)$ of $x \in X$ such that $A(x^{'}) \subset G$ for all $x^{'} \in O(x)$.

Lemma 2.4([21]).Let (Y, C) be an ordered topological vector space with a closed, pointed convex cone C with $intC(x) \neq \Phi$. Then for any $y, z \in Y$, we have

(i) $y - z \in intC$ and $y \notin intC$ imply $z \notin intC$;
(ii) $y - z \in C$ and $y \notin intC$ imply $z \notin intC$;
(iii) $y - z \in -intC$ and $y \notin -intC$ imply $z \notin -intC$;
(iv) $y - z \in -C$ and $y \notin -intC$ imply $z \notin -intC$.

Lemma 2.5([22]).Let M be a nonempty closed , and convex subset of a Hausdorff topological space, and $G : M \to 2^M$ is a multivalued map.Suppose that for any finite $x_1, x_2, \cdots x_n \subset \bigcup_{i=1}^{n} G(x_i)$ (i.e F is a KKM mapping) $G(x)$ is closed for each $x \in M$ and compact for some $x \in M$, where we have conv deotes the convex hull operator.Then $\bigcap_{x \in M} G(x) \neq \Phi$.

Lemma 2.6([23]).Let X Hausdorff topological linear space, $A_1, A_2, \cdots A_n$ be nonempty, closed compact and convex subsets of X. Then $conv(\bigcup_{i=1}^{n} A_i)$ is compact.

Lemma 2.7([24]).Let X and Y be two topological spaces.If $A : X \to 2^Y$ is upper semicontinuous with closed values, then A is closed.

3. Main Results

Theorem 3.1. *Let K be nonempty set, closed and convex subset of Hausdorff topological vector space X , and $(Y, C(x))$ an ordered topological vector space with $intC(x) \neq \phi$ for all $x \in K$. Let $g : K \to K$,and let $\eta : K \times K \to X$ and $f : K \times K \to X$ be affine mappings such that $\eta(x, x) = f(x, g(x)) = 0$ for each $x \in K$. Let $T : K \to L(X, Y)$ be an $\eta-$ hemicontinuos mapping. If $C = \bigcap_{x \in K} C(x) \neq \phi$. and T is η monotone in C, then for each $z \in K, \lambda \in (0, 1]$, the following statements are equivalent*

(i) *find $x_0 \in K$ such that $\langle T_z(x_0), \eta(y, x_0)\rangle + f(y, g(x_0)) \notin -intC(x_0)$, for all $y \in K$;*
(ii) *find $x_0 \in K$ such that $\langle T_z(y), \eta(y, x_0)\rangle + f(y, g(x_0)) \notin -intC(x_0)$, for all $y \in K$;*

where $T_{(z)}$ is defined by $T_z(x) = T(\lambda x + (1 - \lambda)z$ for all $x \in K$.

Proof. Suppose that (i) holds. We can find $x_0 \in K$, such that

$$\langle T_z(x_0), \eta(y, x_0)\rangle + f(y, g(x_0)) \notin -intC(x_0), \ \forall\, y \in K \tag{3.1}$$

Since T is $\eta-$ monotone, for each $x, y \in K$, we have

$$\langle T(\lambda y + (1 - \lambda)z) - T(\lambda x + (1 - \lambda)z), \eta(\lambda y + (1 - \lambda)z, \lambda x + (1 - \lambda)z)\rangle \in C \tag{3.2}$$

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On the other hand, we know η is affine and $\eta(x,x)=0$. It follows that

$$\begin{aligned} &\langle T_z(y)-T_z(x),\eta(y,x)\rangle \\ &\qquad = \frac{1}{\lambda}\langle T(\lambda y+(1-\lambda)z)-T(\lambda x \\ &\qquad + (1-\lambda)z),\eta(\lambda y+(1-\lambda)z,\lambda x+(1-\lambda)z)\rangle\in C \end{aligned} \tag{3.3}$$

Hence T_z is also $\eta-$ monotone. That is

$$\langle T_z(x_0),\eta(y,x_0)\rangle-\langle T_z(y),\eta(y,x_0)\rangle\in -C\forall y\in K. \tag{3.4}$$

since $C=\bigcap_{x\in K}C(x)$, for all $y\in K$

$$\langle T_z(x_0),\eta(y,x_0)+f(y,g(x_0))\rangle-\langle T_z(y),\eta(y,x_0)\rangle-f(y,g(x_0))\in -C\subset -C(x_0). \tag{3.5}$$

By Lemma 2.4,

$$\langle T_z(x_0),\eta(y,x_0)\rangle+f(y,g(x_0))\notin -intC(x_0),\forall y\in K, \tag{3.6}$$

and so x_0 is a solution of (ii). Conversly suppose that (ii) holds. Then there exists $x_0\in K$ such that

$$\langle T_z(y),\eta(y,x_0)+f(y,g(x_0))\rangle\notin -intC(x_0),\forall y\in K. \tag{3.7}$$

For each $y\in K,t\in(0,1)$, we let $y_t=ty+(1-t)x_0$. Obviously, $y_t\in K$

$$\langle T_z(y_t),\eta(y_t,x_0)+f(y_t,g(x_0))\rangle\notin -intC(x_0), \tag{3.8}$$

Since f and η are affine and $\eta(x_0,x_0)=f(x_0,g(x_0))=0$, we have

$$\langle T(\lambda(ty+(1-t)x_0)+(1-\lambda)z),t\eta(y,x_0)\rangle+tf(y,g(x_0))\notin -intC(x_0), \tag{3.9}$$

That is

$$\langle T(\lambda(x_0+t(y-x_0))+(1-\lambda)z),\eta(y,x_0)\rangle+f(y,g(x_0))\notin -intC(x_0). \tag{3.10}$$

Considering the $\eta-$ hemicontinuity of T and letting $t\to 0^+$, we have

$$\langle T_z(x_0),\eta(y,x_0)+f(y,g(x_0))\rangle\notin -intC(x_0),\forall y\in K. \tag{3.11}$$

This completes the proof.

Remark If $C(x)=C$ and $f(y,g(x))=0$ for all $x,y\in K$, then Theorem 3.1 is reduced to Lemma 5 of [17].

Let K be a closed convex subset of a toppological linear space X, and $\{C(x):x\in K\}$ a family of closed, convex and a pointed cones of a topological space Y such that $intC(x)\phi$ for all $x\in K$. Throught this paper, we define set- valued mapping $\overline{C}:K\to L(X,Y)$ as follows:

$$\overline{C}=Y\setminus\{-intC(x)\},\forall x\in K. \tag{3.12}$$

Theorem 3.2. *Let K be nonempty, closed and convex subset of Hausdorff topological vector space X , and $(Y, C(x))$ an ordered topological vector space with $intC(x) \neq \phi$ for all $x \in K$. Let $g : K \to K$,and let $\eta : K \times K \to X$ and $f : K \times K \to X$ be affine mappings such that $\eta(x, x) = f(x, g(x)) = 0$ for each $x \in K$. Let $T : K \to L(X, Y)$ be an $\eta-$ hemicontinuos mapping. Assume the following conditions are satisfied*

(i) *If $C = \bigcap_{x \in K} C(x) \neq \phi$. and T is η monotone in C,*
(ii) *$\overline{C} : K \to 2^Y$ is upper semicontinuous set- valued mapping.*

Then for each $z \in K, \lambda \in (0, 1]$, there exists $x_0 \in K$ such that

$$\langle T(\lambda x_0 + (1 - \lambda)z), \eta(y, x_0)\rangle + f(y, g(x_0)) \notin -intC(x_0), \text{ for all } y \in K. \tag{3.13}$$

Proof. For each $y \in K$, we denote $T_z(x) = T(\lambda x + (1 - \lambda)z)$, and define

$$F_1(y) = \{x \in K : \langle T_z(x), \eta(y, x)\rangle + f(y, g(x))\},$$

$$F_2(y) = \{x \in K : \langle T_z(y), \eta(y, x)\rangle + f(y, g(x))\}$$

Then $F_1(y)$ and $F_2(y)$ are nonempty since $y \in F_1(y)$ and $y \in F_2(y)$. The proof is divided into the following three steps.
(I) First, we prove the following conclusion: F_1 is a KKM mapping. Indeed, assume that F_1 is not a KKM mapping; then there exist $u_1, u_2, \cdots u_m \in K, t_1 \geq 0, t_2 \geq 0, \cdots t_m \geq 0$ with $\Sigma_{i=1}^m t_i = 1$ and $w = \Sigma_{i=1}^m t_i u_i$ such that

$$w \notin \bigcup_{i=1}^{m} F_1(u_i), i = 1, 2, \cdots m \tag{3.14}$$

That is,

$$\forall i = 1, 2, \cdots m, \langle T_z(w), \eta(u_i, w)\rangle + f(u_i, g(w)) \in -intC(w). \tag{3.15}$$

since η and f are affine, we have

$$\begin{aligned} \langle T_z(w), \eta(u_i, w)\rangle + f(w, g(w)) &= \langle T_z(w), \eta(\Sigma_{i=1}^m t_i u_i, w)\rangle + f(\Sigma_{i=1}^m tiu_i, g(w)) \\ &= \Sigma_{i=1}^m t_i(\langle T_z(w), \eta(u_i, w)\rangle + f(u_i, g(w)) \in -intC(w). \end{aligned} \tag{3.16}$$

On the other hand,we know $\eta(w, w) = f(w, g(w)) = 0$ then we have

$$0 = \langle T_z(w), \eta(w, w)\rangle + f(w, g(w)) \in -intC(w).$$

It is impossible and so $F_1 = K \to 2^K$ is a KKM mapping.
(II) Further, we prove that

$$\bigcap_{y \in K} F_1(y) = \bigcap_{y \in K} F_2(y) \tag{3.17}$$

Infact, if $x \in F_1(y)$, then$\langle T_z(x), \eta(y, x)\rangle + f(y, g(x))) \notin -intC(x)$ from the proof of theorem 3.1, we know that T_z is $\eta-$ monotone in $C(z)$. it follows that

$$\langle T_z(y) - T_z(x), \eta(y, x)\rangle \in C, \tag{3.18}$$

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and so

$$(3.19)\langle T_z(x), \eta(y,x)\rangle + f(y,g(x)) - \langle T_z(y), \eta(y,x)\rangle - f(y,g(x)) \in -C \subset -C(x)$$

By Lemma 2.4, we have

$$\langle T_z(y), \eta(y,x)\rangle + f(y,g(x)) \notin -intC(x) \tag{3.20}$$

and so $x \in F_2(y)$ for each $y \in K$. That is, $F_1(y) \subset F_2(y)$ and so

$$\bigcap_{y\in K} F_1(y) \subset \bigcap_{y\in K} F_2(y) \tag{3.21}$$

Conversely suppose that $x \in \bigcap_{y\in K} F_2(y)$. Then

$$\langle T_z(y), \eta(y,x)\rangle + f(y,g(x)) \notin -intC(x) \forall y \in K. \tag{3.22}$$

it follows from Theorem 3.1 that

$$\langle T_z(x), \eta(y,x)\rangle + f(y,g(x)) \notin -intC(x) \forall y \in K. \tag{3.23}$$

That is, $x \in \bigcap_{y\in K} F_1(y)$. and so

$$\bigcap_{y\in K} F_2(y) \subset \bigcap_{y\in K} F_1(y), \tag{3.24}$$

which implies that

$$\bigcap_{y\in K} F_1(y) = \bigcap_{y\in K} F_2(y) \tag{3.25}$$

(III) We prove that $\bigcap_{y\in K} F_2(y) \neq \Phi$ Indeed, since F_1 is a KKM mapping, we know that, for any finite set $\{y_1, y_2, \cdots y_n\} \subset K$, one has

$$conv\{y_1, y_2, \cdots y_n\} \subset \bigcup_{i=1}^{n} F_1(y_i) \subset \bigcup_{i=1}^{n} F_2(y_{i)}) \tag{3.26}$$

This shows that F_2 is also a KKM mapping.
Now we prove that $F_2(y)$ is closed for all $y \in K$. Assume that there exists a net $\{x_\alpha\} \subset F_2(y)$ with $x_\alpha \to x \in K$. Then

$$\langle T_z(y), \eta(y,x_\alpha)\rangle + f(y,g(x_\alpha)) \notin -intC(x_\alpha) \tag{3.27}$$

Using the definition of $\overline{C}$, we have

$$\langle T_z(y), \eta(y,x_\alpha)\rangle + f(y,g(x_\alpha)) \notin -int\overline{C}(x_\alpha) \tag{3.28}$$

Since η and f are continuous, it follows that

$$\langle T_z(y), \eta(y,x_\alpha)\rangle + f(y,g(x_\alpha)) \to \langle T_z(y), \eta(y,x)\rangle + f(y,g(x)) \tag{3.29}$$

Since $\overline{C}$ is upper semicontinuous mapping with close values, by Lemma 2.7, we know that $\overline{C}$ is closed, and so

$$\langle T_z(y), \eta(y,x)\rangle + f(y,g(x)) \in \overline{C}(x). \tag{3.30}$$

This implies that

$$\langle T_z(y), \eta(y,x)\rangle + f(y,g(x)) \notin -intC(x) \tag{3.31}$$

, and so $F_2(y)$ is closed.Considering the compactness of K and closedness of $F_2(y) \subset K$ we know that $F_2(y)$ is compact. By Lemma 2.5, we have $\bigcap_{y \in K} F_2(y) \neq \Phi$, and it follows that $\bigcap_{y \in K} F_1(y) \neq \Phi$, that is, for each $z \in K$ and $\lambda \in (0,1]$,there exists $x_0 \in K$ such that

(3.32) $$\langle T(\alpha x_0 + (1-\alpha)z), \eta(y, x_0)\rangle + f(y, g(x_0)) \notin -intC(x_0) \forall y \in K.$$

, Thus, $\eta - GIVVLI$ is solvable. This complete the proof.
Remark If $C(x) = C$ and $f(y, g(x)) = 0$ for all $x, y \in K$ in theorem 3.3., the condition (ii) holds and condition (i)is equivalent to the $\eta-$ monotonicity of T. Thus, it is easy to see that Theorem 3.3 is a generalization of [17, Theorem 6].

In the above theorem, K is compact. In the following theorem, under some suitable conditions, we prove a new existence result of solutions for $\eta - GIVVLI$ without the conditions of compactness of K.

Theorem 3.3. *Let K be nonempty, closed and convex subset of Hausdorff topological vector space X , and $(Y, C(x))$ an ordered topological vector space with $intC(x) \neq \phi$ for all $x \in K$. Let $g : K \to K$,and let $\eta : K \times K \to X$ and $f : K \times K \to X$ be affine mappings such that $\eta(x, x) = f(x, g(x)) = 0$ for each $x \in K$. Let $T : K \to L(X, Y)$ be an $\eta-$ hemicontinuos mapping. Assume the following conditions are satisfied*

(i) *If $C = \bigcap_{x \in K} C(x) \neq \phi$. and T is η monotone in C,*
(ii) *$\overline{C} : K \to 2^Y$ is upper semicontinuous set- valued mapping.*
(iii) *there exists a nonempty compact and convex subset D of K and for each $z \in K, \lambda \in (0,1], x \in K \setminus D$, there exist $y_0 \in D$ such that*

(3.33) $$\langle T(\lambda y_0 + (1-\lambda)z), \eta(y_0, x)\rangle + f(y_0, g(x)) \in -intC(y_0).$$

Then for each $z \in K, \lambda \in (0,1]$, there exists $x_0 \in D$ such that

(3.34) $$\langle T(\lambda x_0 + (1-\lambda)z), \eta(y, x_0)\rangle + f(y, g(x_0)) \notin -intC(x_0), \text{ for all } y \in K.$$

Proof. By Theorem 3.1, we know that the solution set of the problem (ii) in Theorem 3.1 is equivalent to the solution set of the following variational inequality: find $x \in K$,such that

(3.35) $$\langle T(\lambda y + (1-\lambda)z), \eta(y, x)\rangle + f(y, g(x)) \notin -intC(x), \forall y \in K.$$

For each $z \in K$ and $\lambda \in (0,1]$, we denote $T_z(x) = T(\lambda x + (1-\lambda)z)$. Let $G : K \to 2^D$ be defined as follows:

(3.36) $$G(y) = \{x \in D : \langle T_z(y), \eta(y, x)\rangle + f(y, g(x)) \notin -intC(x), \forall y \in K.\}$$

Obviously , for each $y \in K$,

(3.37) $$G(y) = \{x \in D : \langle T_z(y), \eta(y, x)\rangle + f(y, g(x)) \notin -intC(x)\} \cap D.$$

Using the proof of Theorem 3.3, we obtain that $G(y)$ is a closed subset of D. Considering the compactness of D and closedness of $G(y)$, we know that $G(y)$ is compact.
Now we prove that for any finite set $\{y_1, y_2, \cdots y_n\} \subset K$, one has $\bigcap_{i=1}^{n} G(y_i) \neq \psi$ Let

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$Y_n = \bigcup_{i=1}^{n}\{y_i\}$. Since Y is a real Hausdorrf topological vector space, for each $y_i \in \{y_1, y_2, \cdots y_n\}$, $\{y_i\}$ is compact and convex. Let $N = conv(D \cup Y_n)$. By Lemma 2.6, we know that N is a compact and convex subset of K.
Let $F_1, F_2 : N \to 2^N$ be defined as follows:

$$F_1(y) = \{x \in N : \langle T_z(x), \eta(y,x)\rangle + f(y, g(x)) \notin -intC(x), \forall y \in N;\} \tag{3.38}$$

$$F_2(y) = \{x \in N : \langle T_z(y), \eta(y,x)\rangle + f(y, g(x)) \notin -intC(x), \forall y \in N.\} \tag{3.39}$$

Using the proof of Theorem 3.3, we obtain

$$\bigcap_{y\in N} F_1(y) = \bigcap_{y\in N} F_2(y) \neq \Phi, \tag{3.40}$$

and so there exists $y_0 \in \bigcap_{y\in N} F_2(y)$.
Next we prove that $y_0 \in D$. In fact, if $y \in K \setminus D$, then the assumption implies that there exists $u \in D$such that have

$$\langle T(\lambda u + (1-\lambda)z), \eta(u, y_0)\rangle + f(u, g(y_0)) \in -intC(u), \tag{3.41}$$

Which contradicts $y_0 \in F_2(u)$ and so $y_0 \in D$.
Since $\{y_1, y_2, \cdots y_n\} \subset N$ and $G(y_i) = F_2(y_i) \cap D$ for each $y_i \in \{y_1, y_2, \cdots y_n\}$ it follows that $y_0 \in \bigcap_{i=1}^{n} G(y_i)$. Thus for any finite set $\{y_1, y_2, \cdots y_n\} \subset K$, we have $\bigcap_{i=1}^{n} G(y_i) \neq \phi$. Considering the compactness of $G(y)$ for each $y \in K$, we know that there exists $x_0 \in D$ such that $x_0 \in \bigcap_{y\in K} G(y)\phi$. Therefore, the solution set of $\eta - GIVVLI$ is nonempty. This completes the proof.

In the following, we prove the solavability of $\eta - GSIVVLI$ under some suitable onditions by using $FKKM$ theorem.

Theorem 3.4. *Let Xbe a Hausdorff topological linear space, $K \subset X$ a nonempty, closed, and convex set, and $(Y, C(x))$ an ordered Hausdorff topological vector space with $intC(x) \neq \phi$ for all $x \in K$. Assume that for each $y \in K, x \to \eta(x,y)$ and $x \to f(g(x))$ are affine, $\eta(x,y) + \eta(y,x) = 0$, and $f(g(x), y) + f(y, g(x)) = 0$ for all $x \in K$, where $g : K \to K$. Let $T : K \to L(X,Y)$ be mapping such that*

(i) *for each $z, y \in K, \lambda \in (0,1]$, the set $\{x \in K : \langle T(\lambda x + (1-\lambda)z), \eta(y,x)\rangle + f(y, g(x)) \in -C(x) \setminus \{0\}\}$ is open in K;*
(ii) *there exists a nonempty compact and convex subset D of K and for each $z \in K, \lambda \in (0,1], x \in K \setminus D$, there exists $u \in D$ such that*

$$\langle T(\lambda x + (1-\lambda)z), \eta(y,x)\rangle + f(y, g(x)) \in -C(x) \setminus \{0\} \tag{3.42}$$

Then for each $z \in K, \lambda \in (0,1]$, there exists $x_0 \in K$ such that

$$\langle T(\lambda x_0 + (1-\lambda)z), \eta(y, x_0)\rangle + f(y, g(x)) \notin -C(x_0) \setminus \{0\}\forall y \in K. \tag{3.43}$$

Proof. For each $z \in K$ and $\lambda \in (0,1]$, we denote $T_z(x) = T(\lambda x + (1-\lambda)z)$. Let $G : K \to 2^D$ be defined as follows:

$$G(y) = \{x \in K : \langle T_z(x), \eta(y,x)\rangle + f(y, g(x)) \notin -C(x) \setminus \{0\}\}\forall y \in K. \tag{3.44}$$

Obviously, for each $y \in K$,

$$(3.45) \qquad G(y) = \{x \in D : \langle T_z(x), \eta(y,x)\rangle + f(y, g(x)) \notin -C(x) \setminus \{0\}\} \cap D.$$

Since $G(y)$ is closed subset of D, considering the compactness of D and closedness of $G(y)$ is compact.
Now we prove that for any finite set $\{y_1, y_2, \cdots y_n\} \subset K$, one has $\bigcap_{i=1}^n G(y_i) \neq \Phi$. Let $Y_n = \bigcup_{i=1}^n \{y_i\}$. Since Y is real Hausdorff topological vector space, for each $y_i \in \{y_1, y_2, \cdots y_n\}$, $\{y_i\}$ is compact and convex. Let $N = conv(D \cup Y_n)$. By Lemma2.6, we know that N is compact and convex subset of K. Let $F : N\ to 2^N$ be defined as folows:

$$(3.46) \quad F(y) = \{x \in N : \langle T_z(x), \eta(y,x)\rangle + f(y, g(x)) \notin -C(x) \setminus \{0\}\}, \forall y \in N.$$

We claim that F is KKM mapping.Indeed, assume that F is not a KKM mapping. Then there exist $u_1, u_2, \cdots u_m \in K, t_1 \geq 0, t_2 \geq 0, \cdots t_m \geq 0$ with $\sum_{i=1}^m t_i = 1$ and $w = \sum_{i=1}^m t_i u_i$ such that

$$(3.47) \qquad w \notin \bigcup_{i=1}^m F(u_i), i = 1, 2, \cdots m$$

That is,

$$(3.48) \qquad \forall i = 1, 2, \cdots m\ \langle T_z(w), \eta(u_i, w)\rangle + f(u_i, g(w)) \in -C(w) \setminus \{0\}.$$

Since η and f are affine, we have

$$\begin{aligned} \langle T_z(w), \eta(w,w)\rangle + f(w, g(w)) &= \langle T_z(w), \eta(\sum_{i=1}^m t_i u_i, w)\rangle + f(\sum_{i=1}^m t_i u_i, g(w)) \\ (3.49) \qquad &= \sum_{i=1}^m t_i \langle T_z(w), \eta u_i, w)\rangle + f(u_i, g(w)) \in -C(w) \setminus \{0\}. \end{aligned}$$

On the other hand, we know that $\eta(w,w) = f(w, g(w)) = 0$, and so

$$(3.50) \qquad 0 = \langle T_z(w), \eta(w,w)\rangle + f(w, g(w)) \in -C(w) \setminus \{0\}.$$

which is impossible. Therefore, $F : N \to 2^N$ is a KKM mapping. Since $F(y)$ is a closed subset of N, it follows that $F(y)$ is compact. By Lemma 2.5, we have

$$(3.51) \qquad \bigcap_{y \in N} F(y) \neq \Phi.$$

Thus, there exists $y_0 \in \bigcap_{y \in N} F(y)$. Next we prove that $y_0 \in D$. In fact, if $y \in N \setminus D$, then the condition (ii) implies that there exists $u \in D$ such that

$$(3.52) \qquad \langle T(\lambda y_0 + (1-\lambda)z), \eta(u, y_0)\rangle + f(u, g(y_0)) \in -C(y_0) \setminus \{0\},$$

which contradicts $y_0 \in F(u)$ and so $y_0 \in D$. Since $\{y_1, y_2, \cdots y_n\} \subset N$ and $G(y_i) = F(y_i) \cap D$ for each $y_i \in \{y_1, y_2, \cdots y_n\}$. Thus, for any finite set $\{y_1, y_2, \cdots y_n\} \subset K$, we have $\cap_{i=1}^n G(y_i) \neq \Phi$. Considering the compactness of $G(y)$ for each $y \in K$, it is easy

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to know that there exists $x_0 \in D$ such that $x_0 \in \cap_{y \in K} G(y) \neq \Phi$. Therefore, for each $z \in K, \lambda \in (0,1]$, there exists $x_0 \in K$ such that

$$\langle T(\lambda x_0 + (1-\lambda)z), \eta(y, x_0)\rangle + f(y, g(x_0)) \in -C(x_0) \setminus \{0\}, \forall y \in K. \tag{3.53}$$

Thus, $\eta-$ GSIVVI is solvable. This completes the proof. Remark 3.8. If K is compact, $C(x) = C, g = I$ and $\lambda = 1$, then Theorem 3.7 is reduced to Theorem 2.1 in [20].

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